PRACTICAL

ELECTRICAL TESTING

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IN

PHYSICS AND ELECTRICAL ENGINEERING

BEING

A COURSE SUITABLE FOR FIRST AND SECOND YEAR STUDENTS AND OTHERS

BY

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WITH 231 ILLUSTRATIONS AND 20 TABLES OF CONSTANTS, USEFUL FIGURES, ETC.

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PREFACE

THIS work is intended to form a systematic course of instruction in "Electrical Testing" connected with Physics and Electrical Engineering. It is difficult, if not well-nigh impossible, to draw any distinctive line between a vast number of experimental investigations in that branch of Physics termed "Electricity and Magnetism" and the more elementary portions of Electrical Engineering. In fact, the latter may be regarded as the development of the former and almost entirely dependent on it. Thus the author has little hesitation in recommending the following course of experimental work as eminently suitable for constituting the electrical laboratory practice in the first and second years of a complete course in Electrical Engineering as well as in Physics.

The author includes in a separate work, "Electrical Engineering Testing," a large variety of tests in advanced electrical engineering work, such as might well constitute the latter part of a complete course of instruction in this branch of industry. As far as has been practicable, the experimental investigations have been arranged in the order in which they may be worked, but exceptions have arisen owing to the advisability of keeping certain tests of a similar nature together. The arrangement adopted is in a measure similar to that which has been in use for some years past at the Central Technical College of the City and Guilds of London, but is a considerable extension of that arrangement. The author has endeavoured to make the several tests as complete and descriptive as possible, and in addition to the "Introduction" of each, which contains the principal theoretical considerations pertaining to the test, condensed as much as possible, a complete digest of the method of performing the experiment, with a diagram of apparatus and connections, represented symbolically, and the most convenient and suitable

form in which the results should be tabulated, are given, together with "*Inferences*" to be deduced. These latter, if conscientiously worked out, are calculated to cause the experimenter to think and reason for himself. Following the series of tests is an Appendix containing the algebraical solutions of the various formulæ used in the tests, and these the student is strongly recommended not to refer to until he has tried by all the means in his power to solve the inference for himself. The Appendix also contains complete descriptions and sketches of almost all the apparatus which may be employed in carrying out the tests, and it is such as will be found, to a large extent, in almost every college and testing-room. Useful tables and data, which are constantly needed in physical and electrical engineering work, are added at the end of the book.

It is sincerely hoped that the general arrangement of the present work will be found both helpful and conducive to systematic and valuable results.

A considerable amount of the apparatus illustrated has been constructed by the mechanical assistants, Messrs. John Watkinson and Herbert Addy, of the Physical and Electrical Engineering Departments respectively of the University.

In conclusion, I wish to express my sincere thanks to my valued friend Mr. Charles Mercer, M.A., for the very considerable amount of trouble he has taken in producing the greater proportion of the photographs from which the illustrations are obtained, to Dr. John Henderson for permission to use the tables of squares and reciprocals of numbers, to Mr. S. Joyce for allowing me to use his tables of sines and tangents, to Messrs. Kelvin & James White for permission to use the table of doubled square roots, to His Majesty's Stationery Office for allowing me to use the tables of logarithms and antilogarithms, and also to Messrs. Nalder Bros. & Co., R. W. Paul, Kelvin & James White, and W. & J. George for their kindness in lending me the blocks of some of the illustrations of the very excellent apparatus made by them.

G. D. A. P.

THE UNIVERSITY, LEEDS,
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PRACTICAL ELECTRICAL TESTING

1. Curve Plotting.

Introduction.—It is of paramount importance in a great many kinds of work, but perhaps more especially in physics and electrical engineering, to thoroughly grasp, firstly, the great convenience, advantage, and practical utility of representing the results of any experiment, when possible, graphically (as well as in tabular form), by means of a curve, which will show the variation of one quantity with another; secondly, the method of accurately and rapidly plotting such curves to the most convenient scales for future reference.

By means of a curve, the way in which two quantities vary together can not only be seen at a glance far more easily than from the table of results, but there is the enormous advantage that any intermediate values between those actually obtained from experiment can be easily, quickly, and accurately deduced, providing the curve is drawn as it should be. In order to more fully illustrate the method and principles involved in plotting curves, tables of results obtained from two totally different experiments, together with their corresponding curves drawn on squared curve paper, are shown in Figs. 1 and 2.

Instructions.—(1) After having neatly entered up the results of the experiment in a "finished" table, *note carefully* the magnitudes of the two sets of quantities to be plotted, and on which axes OA and OB (Figs. 1 and 2) each is to be plotted. Distances measured from OB along lines parallel to OA are called "*ordinates*," those parallel to OB being termed "*abscissæ*."

(2) Choose the numerical value of the lengths of the axes so

as to obtain as large a curve as possible, for this will enable it to be drawn more accurately, and will magnify experimental errors and the desired result.

(3) **Number the axes at well-defined and equal intervals, and at no other points,** choosing as convenient a scale as possible for readily reckoning intermediate values.

(4) **Write along each axis the denomination of the quantity plotted thereon**, as shown in Figs. 1 and 2.

(5) Plot the points by mentally following out the right axes to their point of intersection, using a convenient notation for the points. Thus, if several curves are to be plotted on the same sheet of curve paper, they may or may not be very close together, and even cross one another. Hence, to avoid confusion, the following notation for points might be used :—

× × × × × ×
⊙ ⊙ ⊙ ⊙ ⊙ ⊙
⊗ ⊗ ⊗ ⊗ ⊗ ⊗
⊡ ⊡ ⊡ ⊡ ⊡ ⊡
◬ ◬ ◬ ◬ ◬ ◬

(6) Draw a *probable* or *mean curve* through as many of the points as possible in the way shown (Figs. 1 and 2), endeavouring to get as many of the erroneous points on one side of the curve as on the other, but, of course, as many on the curve itself as possible.

Note.—The curve line should be thin and clear, and, unless otherwise authorized, the curve sheet must bear *no other numbers* than those mentioned in (3) above.

On no account should any other curve than the *mean* one be drawn. Some of the points, all of which, whether erroneous or not, must be plotted, are sure to be experimental errors, and would give an irregular and erroneous curve if the latter was drawn through all of them. Thus we see that a curve corrects experimental observations, and in addition enables the nature of the law of the instrument to be observed.

Fig. 1 shows the relation between the deflection and current producing it in a certain galvanometer.

Fig. 2 shows that between the magnetic induction produced in a sample of iron and the magnetizing force causing it.

The first-named is what is called the "*relative*" calibration curve of the galvanometer. If, however, the values of current, c,

were in amperes, it would be the "*absolute*" calibration curve of the instrument.

TABLE I.

Current, C.	Deflection, D.
0·00025	1·25
0·00050	2·50
0·00100	4·25
0·00143	7·00
0·00200	10·50
0·00250	12·50
0·00333	17·50
0·00400	19·15
0·00450	22·50
0·00500	25·50

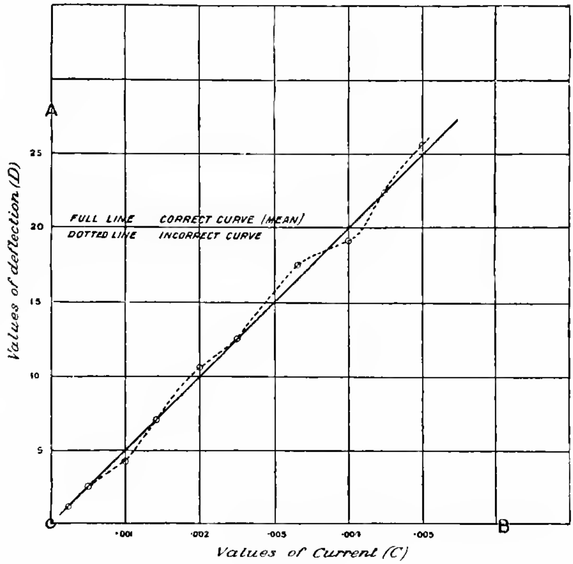


FIG. 1.

TABLE II.

Induction, B.	Magnetizing force, H.
5,000	3·2
6,000	3·9
7,000	6·0
8,000	5·4
9,000	6·3
10,000	6·1
11,000	8·5
12,000	10·6
12,500	10·0
13,000	14·4
13,500	20·0
14,000	22·5
14,500	32·0
15,000	37·5
16,000	58·0
16,500	88·0

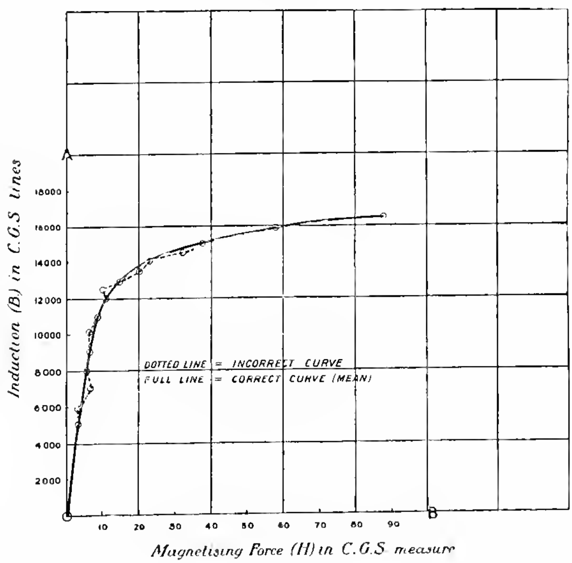


FIG. 2.

2. Delineation of Lines of Magnetic Force (Compass-Needle Method).

Introduction.—In the medium surrounding any magnetic body whatsoever a "magnetic field" of force always exists, and the direction in which a free north pole would move when acted on by such a field is termed a "*line of magnetic force.*" Now, the magnetic field in the vicinity of, for instance, a bar magnet consists of a large number of "lines of force," each forming a closed curve or loop, and the intensity of such a field will be represented by the number of lines emerging from one-half of the magnet which pass through the surrounding air and enter the other half of the magnet, thus completing their paths through the magnet. Thus, a line of force denotes the direction of the force acting on a north pole placed in the field, and to find this direction, *i.e.* that in which a north pole would move if free to do so, we may proceed as follows, assuming that the "law of inverse squares" holds good, namely, that the force exerted between two magnetic poles, whether of attraction or repulsion, is inversely proportional to the square of the distance between them.

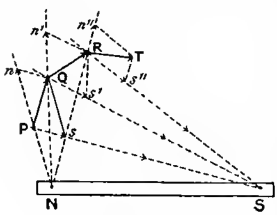


FIG. 3.

Then, referring to Fig. 3, the force acting on the free north pole at P due to N is $\propto \frac{1}{(NP)^2}$, and may be taken as Pn, direction of which will be along NP produced. Now, S exerts on the north pole at P an attractive force $\propto \frac{1}{(SP)^2}$, which must be denoted by a line $= Pn \cdot \frac{(NP)^2}{(SP)^2} = Ps$. Hence, the free pole at P being acted on by two forces, Pn and Ps, will have a resultant force represented, in magnitude and direction, by the diagonal PQ of the parallelogram, of which Pn and Ps are adjacent sides, and so on for the rest. Thus PQ, QR, RT . . . form elements of the line of force, which will eventually reach the south pole of the magnet. We cannot, however, separate a north pole from its conjugate south pole in practice; but, notwithstanding

this, a short compass needle will take up positions represented by those elements of the line of force with which its magnetic axis will coincide, and hence enable the whole line to be mapped out. It must be clearly understood that the earth's field will modify the distribution of the field due to the magnet, as also would any other field, and the object of the present experiment is to determine the distribution of the lines of force when so acted upon.

Apparatus.—Short bar magnet, small compass needle, large sheet of drawing-paper, and drawing-pins.

Observations.—(1) Pin the drawing-paper to the table or a drawing-board, with one of its sides parallel to the magnetic meridian of the earth.

(2) Place the bar magnet near one side of the paper with its magnetic axis parallel to the meridian, and its N. pole pointing towards the N. geographical pole of the earth, and make a pencil line round it at a distance of say about 1 cm.

N.B.—The magnetic N. pole of the earth, though not actually coincident, is practically the same as the S. geographical pole.

(3) Divide the long pencil lines into about six equal parts, and the end or short lines into about four equal parts, and place the compass so that one end (call it A) is just opposite or over one of the marks, 0. Then make a pencil point or mark, 1, just opposite the other end (call it B) of the needle when floating freely.

(4) Next slide the compass along in the direction it points until end A is over mark 1; then make another fresh mark, 2, just opposite end B, and so on until the whole curve 0 1 2 3 4 . . . is mapped out. Then draw a neat curve through all these points.

(5) Repeat (2)–(4), starting from each of the other marks on the rectangular line.

(6) Repeat (2)–(5) with the magnet placed near the other side of the paper, but reversed so that its S. pole now points toward the N. geographical pole of the earth.

Inferences.—What can you infer from the respective distributions of the lines? Point out their bearing on fundamental principles involved with magnetic poles.

3. Distribution of Magnetism in Bar Magnets (Method of Oscillations).

Introduction.—From the remarks and results of observations in the last experiment, we may gather that every uniformly magnetized magnet possesses a region, approximately midway between its ends, from which no lines of force emerge into the air, *i.e.* a region at which there is no free magnetism, which is called the *equator* of the magnet. The intensity of magnetization or the number of lines of force inside the magnet is greatest here at this point, and diminishes towards the ends, where the amount of free magnetism is a maximum. Hence any external magnetic effect of the magnet on neighbouring bodies will be a minimum at the equator and a maximum at the poles. Now, suppose a short magnetic needle, *ns*, is suspended in a frame of brass, E, which can slide along a bar magnet, NS, and be clamped in any part by a screw, *c*. Then from p. 221, we see that if the needle oscillates in front of NS, which is placed vertically so that the magnetic axis of *ns* cuts it, and is also parallel to the magnetic meridian, then—

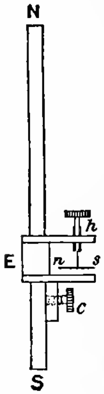


FIG. 4.

$$T_1 = 2\pi\sqrt{\frac{K}{M(H_m \pm H_E)}}, \text{ or } M(H_m \pm H_E) = \frac{4\pi^2 K}{T_1^2}$$

where T_1 = time of one complete oscillation in seconds of the needle having a moment of inertia K and magnetic moment M, H_E being the horizontal intensity of the earth's field, and H_m being the horizontal component of the magnet's total field; for manifestly the earth's field is superimposed on that of NS, resulting in a total field $(H_m \pm H_E)$ acting on *ns*. The + and − sign denotes whether the two fields are acting *summationally* or *differentially* respectively on the needle, which will depend on what half of NS the needle is opposite. If the north magnetic pole of the earth is on the left, and the south magnetic pole of the earth on the right of Fig. 4, then, when the needle is below the equator of NS, the north pole of it will point as shown, and we evidently have a differential effect of the fields $(H_m - H_E)$, and the south magnetism

of NS will preponderate over the north magnetism of the earth. If the needle is *above the equator*, we have a summational effect of the fields $(H_m + H_E)$. If n_1 = number of "transits" of the needle per second, *i.e.* the number of times one end swings across the magnetic meridian per second, $T_1 = \frac{2}{n_1}$, and the above formula becomes—

$$(H_m \pm H_E) = \frac{4\pi^2 K n_1^2}{4M} = \frac{\pi^2 K}{M} . n_1^2, \text{ or } (H_m \pm H_E) \propto n_1^2$$

If ns is very close to the magnet, H_m will be proportional to the amount of *free magnetism* at any point, and H_E will be equal to $\frac{\pi^2 K}{M} n^2$, where n = number of oscillations per second in the earth's field alone. Then, for two different positions of ns on the magnet on the same side of its equator, we have for summational effect of magnet and earth on the needle, $H_{m1} = \frac{\pi^2 K}{M}(n_1^2 - n^2)$, and for summational effect of magnet and earth on the needle, $H_{m2} = \frac{\pi^2 K}{M}(n_2^2 - n^2)$, where n_1 and n_2 = number of transits per second in the two positions. Hence—

$$\frac{H_{m1}}{H_{m2}} = \frac{n_1^2 - n_2}{n_2^2 - n_2}$$

For the differential effect—

$$\frac{H_{m1}}{H_{m2}} = \frac{n_1^2 + n^2}{n_2^2 + n^2}$$

In other words, the strengths of two single fields are proportional to the squares of the number of oscillations performed in each by the same suspended needle.

Thus, by placing the needle at different parts of the magnet, and obtaining the number of vibrations in the same time at each, the squares of these will be proportional to the free magnetism at each.

Observations.—(1) Remove the magnet from its supporting clamp (not shown), and take off the clamp E carefully. Then, bringing the magnet up to a compass card or other galvanometer needle, note the effect, and thus determine which is the north pole.

(2) Replace the magnet in its clamp with the N. pole uppermost (say). Place E at one end, and reduce the needle to complete rest.

(3) Give the needle a *motion of rotation* of about 20° by bringing an outside magnet or piece of iron near one end. Now remove this and all other magnets to a distance, and count the number of transits, N of one end in say two minutes, t, and repeat this two or three times, and take the mean as being more accurate.

(4) Repeat (2) and (3) for about sixteen or twenty positions of the clamp on the magnet right up to the other end, and differing by about equal amounts, carefully noting *where* the *needle turns round.*

(5) Remove the clamp E and needle to a position quite free from the influence of everything except the earth's field, and note the number of transits in the same time, and tabulate your results as follows:—

Magnet: Length = cms.; section = ; transits in earth's field, n, = per sec.

Distance along magnet.	Total transits, N.	Time, t secs.	$n_1, n_2, n_3, \ldots = \frac{N_1}{t}, \frac{N_2}{t}, \frac{N_3}{t} \ldots$	$Hm \propto (n_1{}^2 \mp n^2)$.

(6) Plot a curve having values of $(n_1^2 \pm n^2)$ as ordinates (with due regard to their sign, which must be reckoned $+^{ve}$, say when E is above the equator of the magnet, and $-^{ve}$ when below), and distances along the magnet as abscissæ.

Inferences.—What sources of error is the method liable to, and from your curve state where the poles of the magnet are?

4. Distribution of Magnetism in Bar Magnets (Induction Ballistic Method).

Introduction.—Coulomb's vibration method of finding the distribution of magnetism in a bar magnet by the rapidity of oscillation of a small magnetic needle placed close to and at different parts of the magnet, is not an accurate one, owing (a) to

the close proximity of needle to magnet altering the distribution; (*b*) to inductive action of latter on the needle temporarily altering its strength, thereby making strong and weak fields relatively stronger and weaker respectively than they really are. The following method, depending on Faraday's principle of induction, was originally used by *Rees*, but is commonly called Rowland's method.

Apparatus.—Low-resistance ballistic mirror galvanometer, G; resistance, R; earth inductor, E (p. 359); magneto-inductor, I (p. 334), consisting of a bobbin wound with a large number of turns of insulated wire of low resistance, and capable of being moved rapidly through a fixed distance along the bar magnet, M, in a frame which may be clamped in different positions on M. If G, E, R, and I are in simple series, then on suddenly turning E through 180° so as to cut either the vertical or horizontal component of the earth's field, F_1, the whole quantity of electricity in the transient current set up $Q_1 = \frac{2N_1A_1F_1}{R_1} = K \sin \frac{1}{2}\theta_1{}^\circ$, causing a first throw d_1 scale-divisions, where N_1 = number of turns on E. A_1 = their mean area in square cms.; R_1 = total circuit resistance; K = "ballistic constant;" and $\theta_1{}^\circ$ = angular throw in degrees. If I is now suddenly slipped along, it will cut a field, F_2, consisting of the lines of force which emerge from the surface of the magnet at the part over which I passes. Hence $Q_2 = \frac{N_2F_2}{R_2} = K \sin \frac{1}{2}\theta_2{}^\circ$, where $N_2 R_2 \theta_2{}^\circ$ have the same meaning as before. $\therefore F_2 = F_1 \frac{2N_1A_1}{N_2} \times \frac{d_2}{d_1}$ (in absolute measure), since $R_1 = R_2$.

Observations.—(1) Connect up as mentioned, using a long flexible twin lead to connect G, R, and E to I, which must be placed vertically some distance off. Adjust the spot of light to zero.

(2) Make sure, by trial, at the start that the first throw (d_2) on G, produced by letting I slip in its frame by its own weight at the strongest part of the magnet, is just on the scale; alter the resistance R to get this if necessary.

(3) Clamp the frame of I close up to one end of M, and observe twice over for this position the first throw on G when I falls. Note the mean (d_2) and the position (L) of I from one end.

In each case the spot of light must be absolutely at rest prior to slipping the magneto inductor coil.

(4) Repeat (3) about every 4 or 5 cms. along M, and tabulate as follows:—

$N_1 =$ turns; $N_2 =$ turns; $F_1 =$ C.G.S. units; $A_1 =$ sq. cms.; $d_1 =$ divs.; $K_1 = \dfrac{2N_1A_1F_1}{N_2d_1}$

Position on magnet, L.	First throws.			Stray field, $F_2 = K_1d_2$.	Sign of deflection, + or −.
	First.	Second.	Mean, d_2.		

(5) Plot a curve with values of L as abscissæ and F_2 as ordinates.

(6) Find the position of the poles and distance between them by projecting the centre of gravity of each area on to the abscissæ.

Inferences.—What can be inferred from your results?

5. Measurement of Magnetic Dip (Dip Circle Method).

Introduction.—The magnetic dip at any place is the angle which the magnetic axis of a magnetized needle makes with a horizontal plane when it is free to turn about a horizontal axis perpendicular to the magnetic meridian.

A magnet freed from all forces except magnetic ones would in the earth's magnetic field tend to point in the direction of a line of force at the place. This line of force, and consequently the magnet, will only be horizontal at or near the earth's equator. Hence, at any place—for instance, in the British Isles—the magnet will "dip" considerably.

Errors to be avoided.— (*a*) Error from eccentricity, *i.e.* axis of rotation of needle may not coincide with centre of vertical circle—eliminated by reading both ends of the needle; (*b*) magnetic axis may not coincide with axis of figure—eliminated by reversing the position of the axis of rotation of the needle. This is accomplished by rotating the box of the dip circle through 180°; (*c*) C.G. may not coincide with axis of rotation needle—eliminated by reversing the magnetic polarity of the needle;

(*d*) rolling friction—counteracted or minimized by *gently* tapping the base of the instrument. It is usually very small.

We will now consider these errors more in detail as follows:—

Error (*a*).—In Fig. 5, I., the true dip is AOn or BON, and if there is eccentricity, what we read off at one end is AOn', and at the other BON′; but, since NN′ = nn', hence the mean of the readings at both ends gives the dip corrected for this error. The correction is more important in the "dip circle" than in most other instruments, because, owing to the needle *rolling* on its pivots, the axis shifts its position along a horizontal diameter of the graduated circle.

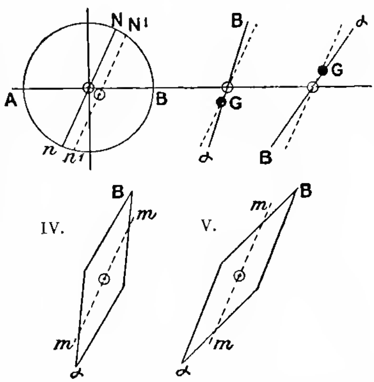


Fig. 5.

Error (*b*).—In Fig. 5, IV. and V., let αB represent the axis of figure, and *mm* the magnetic axis. Now, it is *mm* which sets itself in the "line of dip," whereas it is αB which is observed. Therefore, in IV. we get too large a value for the dip. If, however, we rotate the instrument in azimuth through 180° about a vertical axis, we shall be looking at the other face, and the dip will now be too small, as seen by V. Hence, the mean of the two gives the position of the magnetic axis *mm*, which is what we require.

Error (*c*).—In Fig. 5, II. and III., αB is the needle, G its centre of gravity, and O the axis of rotation. Hence, in II. it will be obvious we shall get too large an angle of dip. If, however, the polarity of the needle is reversed so that the end B now dips, as shown in III., instead of α as before, the centre of gravity will now be above O, and hence the dip will be too small. The *mean* consequently gives the dip corrected for error (*c*).

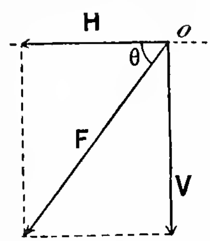


Fig. 6.

If a magnetic needle is pivoted on a horizontal axis passing through its centre O, and is free to rotate in a

vertical plane parallel to the magnetic meridian of the earth, then, if F = the total force, and H and V the horizontal and vertical components of this force due to the earth—

$$T = \frac{H}{\cos \theta^\circ}, \text{ and } V = H \tan \theta^\circ$$

where θ° = angle of dip.[1]

Observations.—(1) Carefully level the instrument, if necessary, by means of the levelling screws and spirit level. Lower the needle on to the knife edges if it is resting on the forked clamps.

(2) Turn the instrument in horizontal azimuth until, on *gently* tapping it, the needle lies vertically, pointing to 90° on the scale. Note the reading on the horizontal scale.

(3) Now turn it in horizontal azimuth through 90° from this position in (2), when the needle will then be swinging in the magnetic meridian.

(4) Read both ends of the needle to 0·8° to avoid error (*a*).

(5) Turn the instrument in azimuth through 180° and repeat (4) to avoid error (*b*).

(6) Reverse the polarity of the needle to correct for error (*c*), by means of a solenoid and current, and repeat (1)–(5).

N.B.—Before commencing the observations it is as well to first strongly re-magnetize the needle, in order to ensure maximum sensibility, and in all cases great care is required in removing it from the instrument for this purpose, in order to *avoid bending* or otherwise *damaging* the axis or *pivots*.

(7) Tabulate all your results as follows:—

Face of instrument pointing	Reading of		Magnetism reversed.		Means.	Mean of all the observations.
	Upper end.	Lower end.	Reading of upper end.	Reading of lower end.		
East West						

Therefore angle of "inclination" or "dip" at = .

[1] For accurate work most elaborate forms of "dip circles" can be used, for a description of which the reader may refer to Gordon's "Treatise on Electricity and Magnetism," vol. i. The one used in this experiment is that shown on p. 335 of this book.

6. Magnetic Inclination or Dip by the Induction Magnetometer.

Introduction.—The magnetic dip or inclination at any place is the angle which the magnetic axis of a magnetized needle makes with a horizontal plane, when it is free to turn about a horizontal axis perpendicular to the magnetic meridian. It may be found by means of a "dip circle," or by the following method, which possesses several advantages over the dip circle, and depends on measuring the relative values of the horizontal and vertical components of the earth's magnetic force. This is done by comparing the throws produced on a ballistic galvanometer, G, connected up to an earth inductor, E, arranged to cut the above components when suitably rotated.

To obtain an accurate result, the following precautions must be used :—

(*a*) The speed of turning should in all cases be about the same.

(*b*) The coil should be turned through exactly 180°, though an error of 1° produces no sensible error in the results, as cos 1° = 1·000 very approximately.

(*c*) *In cutting the horizontal component* the initial and final positions of the plane of the coil should be perpendicular to the magnetic meridian, an error of 5° making 0·8 per cent. error in the result.

(*d*) Axis of rotation of the coil should be in a vertical plane perpendicular to the meridian. This might cause the most serious error of any, as an error of 1° produces nearly 5 per cent. error in the result, since cos 69° = 0·358 and cos 70° = 0·342.

(*e*) *In cutting the vertical components* the axis should be horizontal and in the magnetic meridian, 1° error in this producing 0·3 per cent. resultant error.

Observations.—(1) Connect up and adjust the galvanometer needle to zero.

(2) *Horizontal components.*—Correct for error (*d*) by making axis of E vertical by means of a plumb line, and placing packing under the base if necessary.

(3) With the spot absolutely at rest and the plane of E

perpendicular to the magnetic meridian, quickly rotate it through 180°, and note the first throw d_H on G.

(4) Repeat (3) several times at the same speed for slightly different positions of E, turned in azimuth, and note its position giving maximum throw. This will show that the plane is truly perpendicular to the meridian.

(5) With the base of E fixed in this position take about twelve or fifteen throws, and note the mean d_H.

(6) *Vertical components.*—Place E with both its plane and axis horizontal and in the meridian. Rapidly rotate it through 180°, noting the throw. Repeat this about twelve or fifteen times, and note the mean d_V.

(7) Calculate the angle of dip $\delta°$ from the relation $V = H \tan \delta$, and tabulate all your readings in a convenient form.

Inferences.—Show by a sketch the meaning of the angle of dip, and state any advantages this method may possess over other methods. Prove the relation given in (7) above, using the fundamental principles of the earth indicator.

7. Measurement of the Magnetic Moment of a Bar Magnet (Deflection Method).

Introduction.—This method can be performed in one of two ways, according to the relative positions of the magnet tested, and the magnetic meridian of the earth.

(A) The magnetic axis of the bar magnet pointing east and west, and passing through the centre of the suspended needle. This is called the "A. position of Gauss."

(B) The magnetic axis of the bar magnet pointing east and west, but the line joining its centre with that of the needle lies in the magnetic meridian. This is called the "B. position of Gauss."

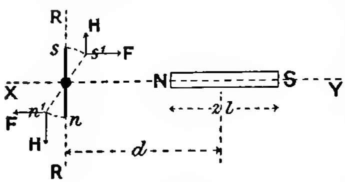


Fig. 7.

We will now consider the first-named position, which is symbolically represented in Fig. 7.

The magnetic moment M is defined as the product of the

strength m of either pole, multiplied by the length $2l$ of the magnet in centimetres, or $M = 2ml$.

This quantity is of considerable importance, for several reasons as follows: The strength of the magnetic field in the vicinity of a magnet is directly proportional to the magnetic moment of the latter. Again, from the above relation $m = \frac{M}{2l}$; hence, since 4π lines of force emerge from unit magnetic pole, the number N emanating from a magnet of strength m is $N = 4\pi m = \frac{4\pi M}{2l}$. And, further, if the cross-sectional area of the magnet is A sq. cms., the "magnetic induction" in it $B = \frac{4\pi M}{2lA}$ lines per square centimetre. Referring to Fig. 7, it will be observed that whereas N is attracting s and repelling n, S is doing just the opposite. Thus the actual deflection of ns is due to the differential effect of N and S on it. It may further be remarked that a magnet whose length is great compared with its breadth can be supposed to have two poles equal in strength, and situated at its ends.

Apparatus.—Magnet to be tested; magnetometer (p. 336), consisting of a suspended "short" magnetic needle, the deflections of which can be read on a circular scale. The metre scale is so placed that when the magnet slides to and from the needle, its axis passes through the centre of the needle.

A. Position: Observations.—(1) Level the baseboard of the magnetometer by means of the levelling screws, so that the centre of the needle is just over that of the circular scale, and at the same time adjust the pointer to zero by slightly turning the instrument.

N.B.—All magnets must be removed to a distance while doing this.

(2) Bring the magnet to be tested, and place it at the side of the scale with its north end eastwards and centre at, say, 100 on the scale. Note the deflection θ° (or $\tan\theta$) and distance d cms. of its "centre" from that of the needle.

(3) Repeat (2) with the south end of the magnet pointing eastwards, and with the value of d the same. Take the mean of the values of $\tan\theta$ obtained in (2) and (3) for this value of d.

(4) Repeat (2) and (3) for about nine values of d, decreasing

10 cms. at a time, and calculate the magnetic moment M of the magnet from the formula—

$$M = \frac{(d^2 - l^2)^2}{2d} H \tan \theta$$

where l = half the length of magnet in centimetres, and H = horizontal intensity of the earth's magnetism in C G.S. units.

(5) From the mean value of M calculate m, N, and B, and tabulate your results as follows :—

d cms.	Values of				Mean, tan θ.	M.
	North pole pointing east.		South pole pointing east.			
	θ.	tan θ.	θ.	tan θ.		

Inferences.—Prove the formula given in (4), and state what assumptions are made in obtaining it.

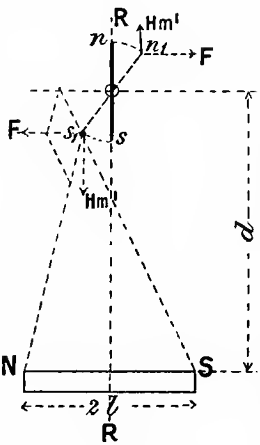


FIG. 8.

B. Position: Observations.—(1) Arrange the magnetometer as in Fig. 8, and repeat A. (1) above.

(2) Place the magnet NS across the scale with its centre at some convenient distance d, say, 100 cms. from ns, if not too great to obtain a suitable steady deflection $\theta°$ (or tan θ).

(3) Repeat (2) with the magnet reversed in position, *i.e.* position of its poles interchanged, and with the value of d the same. Take the mean of the values of tan θ obtained in (2) and (3) for this value of d.

(4) Repeat (2) and (3) for about nine values of d, decreasing by equal amounts at a time, and calculate the magnetic moment M of the magnet from the relation—

$$M = (d^2 + l^2)^{\frac{3}{2}} H \tan \theta$$

where l = half the length of the magnet in centimetres and H as in A. (4) above.

(5) From the mean value of M calculate m, N, and B, and

tabulate in an exactly similar manner to that shown in A. (5) above.

Inferences.—Prove the relation given in (4), and state any assumption made in obtaining it.

8. Comparison of the Magnetic Moments of Two Magnets (Deflection Method).

Introduction.—From reference to the preceding tests, it will be apparent that the magnetic moments of two magnets can be readily compared by observing the deflections they produce on a magnetometer needle at the *same* distance off, d, in each case. We then have—

$$\frac{M_1}{M_2} = \frac{\tan \theta_1}{\tan \theta_2} \text{ for each position of Gauss}$$

N.B.—The student should read through the preceding tests carefully.

Apparatus.—Same as for the last experiments.

Observations.—(1) Apply those of the preceding experiment with each of the magnets to be compared, using both A. and B. positions in each case.

(2) Calculate the ratio of the magnetic moments from the above relation, and tabulate as follows for each position:—

Distance, d.	Deflections.				$\frac{M_1}{M_2} = \frac{\tan \theta_1}{\tan \theta_2}$
	θ_1.	θ_2.	$\tan \theta_1$.	$\tan \theta_2$.	

Inferences.—What can you infer from your results?

9. Determination of the Magnetic Moment of a Bar Magnet (Vibration Method).

Introduction.—The method depends on a well-known principle in mechanics, namely, that when a body oscillates through a *small* angle θ, from its position of rest, about a given axis O, under the influence of a force $F\theta$, which tends to reduce it to

rest, the time of one complete oscillation T, in seconds, or, as it is more frequently termed, its periodic time of oscillation $T, = 2\pi\sqrt{\frac{K}{F}}$, where K = moment of inertia of the body about the given axis O.

To apply this to the present instance. Let NS be a magnet suspended by a *torsionless* fibre attached to its centre O, such that its magnetic axis lies perpendicular to the magnetic meridian AB. Then, if m is the strength of each of its poles, and H the horizontal intensity of the earth's magnetic field, there are two forces each = Hm, but acting in opposite directions perpendicular to and at the ends of NS. If $2l$ = the length of magnet, then the moment of Hm at N about O = Hml, and the moment of Hm at S about O = Hml, and hence the total moment about O = $2Hml$ = the moment of the couple about O. But $2ml$ = M, the magnetic moment of the magnet, therefore the force acting on the magnet tending to reduce it to rest = Hml = HM. Hence, from the above reasoning, it follows that the time of one complete vibration of this magnet in the earth's field only is $T = 2\pi\sqrt{\frac{K}{MH}}$, when the oscillations are very small, and where K = moment of inertia of the magnet, depending on its shape, mass, and dimensions.

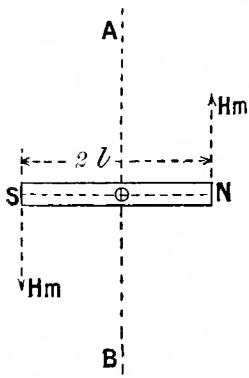


Fig. 9.

If W = weight of the magnet in grammes, $2l$ its length in centimetres, r = its radius (if it is of circular cross-section), and b = its breadth, measured horizontally, if it is of rectangular cross-section, then we have—

For a cylindrical bar suspended from the mid-point of its horizontal axis—

$$K = W\left(\frac{(2l)^2}{12} + \frac{r^2}{4}\right)$$

For a rectangular or square bar suspended in the same way—

$$K = W\left(\frac{(2l)^2 + b^2}{12}\right)$$

We finally have the magnetic moment $M = \frac{4\pi^2 K}{T^2 H}$, from which relation we see that the horizontal component H must be known to be able to apply the method.

Apparatus.—Small magnetic rod about 3 inches long (and cylindrical, say); stop-watch beating quarter seconds; instrument for suspending the rod (p. 335).

Observations.—(1) Place the magnet in the stirrup of the suspension, and make it hang motionless. Replace the glass shade very carefully so as not to jar the instrument.

(2) Since the motion of the magnet must be one of pure rotation, without any motion of translation, bring an auxiliary magnet up to such a distance from the suspended one as to cause it to rotate through about 8° to 10° of arc, and remove all outside magnets or iron to a distance.

(3) Count the number of *transits*, N, *i.e.* the number of times one end of the needle crosses the magnetic meridian or its position of rest in, say, two minutes.

(4) Repeat (2) and (3) about six times; then, if the time is two minutes, the number of transits per second, $n = \frac{N}{120}$, and since one complete period of oscillation T = two transits, therefore $\frac{1}{2}n$ = number of periods per second—

$$\therefore M = \frac{4\pi^2 K}{T^2 H} = \frac{4\pi^2 K n^2}{4H} = \frac{\pi^2 n^2 K}{H}$$

where $T = \frac{2}{n}$.

(5) Calculate the magnetic moment from this relation, and from the mean, the values of m, N, and B, where m = strength of magnet's pole, N = number of lines emerging from them, and B = magnetic induction in the magnet, and tabulate as follows:—

H = C.G.S. units. ; wt. of magnet, W, = grms. ; length, l, = cms. ; breadth, b, = cms. K, = or radius, r, = ,,

No. of magnet.	Time, t secs.	Transits, N.	$\frac{N}{t} = n$	M.	Mean value of M.

N.B.—The way in which m, N, and B can be found will be seen by reference to the "Deflection Method," p. 15.

Inferences.—What can you infer from your results?

10. Comparison of the Magnetic Moments of Two Magnets (Vibration Method).

Introduction.—It will be obvious that without knowing H, the horizontal component of the earth's field, the magnetic moments of any two magnets can be readily compared with one another by subjecting each to the preceding test. In other words, the magnetic moments of two magnets can be compared by observing their respective periods of oscillation in the *same* magnetic field H, for we have—

$$M_1 = \frac{4\pi^2 K_1}{T_1^2 H} = \frac{\pi^2 n_1^2 K_1}{H}$$

$$\text{and } M_2 = \frac{4\pi^2 K_2}{T_2^2 H} = \frac{\pi^2 n_2^2 K_2}{H}$$

$$\text{hence } \frac{M_1}{M_2} = \frac{n_1^2 K_1}{n_2^2 K_2} = \frac{T_2^2 K_1}{T_1^2 K_2}$$

Apparatus.—Same as for the last experiment.

Observations.—(1) Apply those of the last experiment with each of the magnets to be compared.

(2) Calculate the ratio of the magnetic moments from the preceding relation, and tabulate as follows:—

Magnet No. ; weight, W_1, = grms. ; length, $2l_1$, = cms. ; r_1 = cms. ; K_1 =
Magnet No. ,, W_2, = ,, ,, $2l_2$, = ,, r_2 = ,, K_2 =

Times in seconds.		No. of transits.		Transits per second.		$\frac{M_1}{M_2}$	Mean, $\frac{M_1}{M_2}$
t_1.	t_2.	N_1.	N_2.	$n_1 = \frac{N_1}{t_1}$	$n_2 = \frac{N_2}{t_2}$		

Inferences.—What can you infer from your results?

11. Distribution of the Lines of Force of the Magnetic Field in a Dynamo (Iron Filings Method).

Introduction.—The present experiment is arranged with a view to determining, as far as possible, the general configuration of

the magnetic field in and around a dynamo, and to see approximately how this is affected by the magnitude of the magnetizing force and magnetic induction in the iron parts. It is, of course, well known that the particular form given to the field magnets and armature of a dynamo, together with the sectional areas of the various parts, very materially affect the relative distributions of the lines of force in the various parts of the machine and the space surrounding it. The principle upon which this method of investigation depends is based on the fact that whenever "free magnetism" is existent, *i.e.* whenever lines of force emerge from a magnetic body and traverse the surrounding non-magnetic space, their presence, in a greater or less extent, can be detected by iron filings placed in this "stray field," and setting themselves along the lines of force. In this connection, it should be remembered that an absence of filings in any part indicates either a complete absence of lines of force, or that these latter are contained solely in the magnetic material, and therefore are unable to produce external influence on the filings.

Apparatus.—A sheet of stiff paper, or large, slightly milky glass plate; box of iron filings, with perforated lid; ammeter, A; carbon rheostat, R (p. 307); secondary battery, B (p. 337); model dynamo, D, to be experimented upon (p. 360); switch, S.

Observations.—(1) Connect up as in Fig. 10, using the two outside terminals of D, and adjust the pointer of A to zero if necessary.

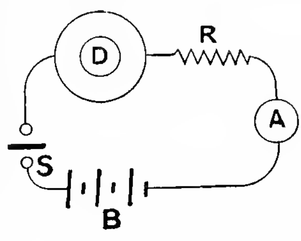


Fig. 10.

(2) With S *open*, and *no iron rings* between the poles, place the stiff sheet of paper or glass provided over D, and carefully sift iron filings uniformly all over it, very gently tapping the paper so as to assist the filings into position, too many of which should not be used. Copy the diagram so formed into your note-book, with their relative denseness well defined. Lastly, return the filings to the sifter.

(3) Still with no iron rings between the poles, and with R quite loose, close S, and adjust the current A to, say, $\frac{1}{10}$ maximum value, the sheet of stiff paper being in position, begin to sift, etc., repeating the latter part of (2) above.

(4) Repeat (3) with maximum current allowable.

(5) Repeat (3) and (4) with each of the iron rings in position between the pole faces of the field magnets.

Inferences.—State very carefully and concisely all the inferences which can be drawn from your experimental results, and point out their bearing on the construction of magnetic circuits.

12. Measurement of the "Horizontal Intensity" of the Earth's Magnetism (Method of Oscillations and Deflections).

Introduction.—The following method of determining the horizontal component "H" of the earth's magnetic force is a combination of the vibration and deflection methods for finding the magnetic moment of a magnet (*vide* pp. 14 and 18). Some refinements and precautions are here necessary which were not of paramount importance in the above-named experiments. The magnetometer should be a more delicate one, and preferable of the form shown on p. 337. In the present case, its deflections will be observed by means of a small telescope and illuminated scale divided in millimetres. In this case, if D = scale-deflection in millimetres and $\theta°$ = angular deflection of the magnetometer needle, then, since the angular motion of the reflected ray is double that of the mirror with its attached magnet, we have—

$$\tan 2\theta = \frac{D}{L}, \text{ or } \tan \theta = \frac{D}{2L}$$

where L = distance from mirror to scale in scale divisions, and θ is a very small angle, which generally will not exceed $5°$ or $6°$ of arc.

It is extremely important to have no moving pieces of magnetic material about, and to avoid all draughts, which might affect the magnetometer needle. For very accurate work other corrections are needed, namely, for the magnitude of the arc of oscillation in diminishing the time T of vibration, for the torsion of the suspensions, air churning and damping, and variations in temperature altering the magnetic moment during the test. These are, however, all very small quantities, and no account need be

taken of them, with fairly delicate means at hand in testing, in order to obtain an approximate value for H. The value of this so obtained will be the value which holds in the room the test is made in, and may be somewhat different from the values obtained for the year from tables. Using the same magnet in the two cases, we have by *oscillations*—

$$MH = \frac{4\pi^2 K}{T^2}$$

and by *deflections*—

$$\frac{M}{H} = \frac{(d^2 - l^2)^2}{2d} \tan \theta$$

using the "*A. position.*"

Hence, dividing, we get—

$$H^2 = \frac{4\pi^2 K 2d}{T^2(d^2 - l^2)^2 \tan \theta}$$

and if D = the scale-deflection corresponding to an angular motion $\theta°$ of the needle, then—

$$H = \sqrt{\frac{4\pi^2 K 2d \cdot 2L}{T^2(d^2 - l^2)^2 D}} = \frac{4\pi}{T(d^2 - l^2)}\sqrt{\frac{KdL}{D}}$$

NOTE.—$T = \frac{2}{n}$, where n = the number of transits per second.

Apparatus.—Delicate magnetometer with accessories (p. 337); instrument for taking vibrations (p. 335); micrometer gauge sensitive chemical weighing balance; stop-watch; three or four similar permanently magnetized steel needles about 10 cms. long and $\frac{1}{4}$ cm. diameter.

Observations.—(1) Place the vibration instrument on the magnetometer table and a needle in the stirrup. Bring the needle perfectly to rest in the magnetic meridian, then replace the glass shade *carefully* so as not to jar the needle, and make some convenient mark on the shade opposite one end.

(2) With all iron and magnets removed to a distance, give to the needle a motion of pure rotation of an arc, not exceeding about 7° or 8° by means of an outside magnet, and note the time in seconds of 100 transits (N) of the end of the needle past the mark.

(3) Repeat (2) about three or four times, and take the mean, t seconds. Then $n = \frac{N}{t} = \frac{100}{t}$ transits per second.

(4) Place the magnetometer in position on the table, level it, seeing that the needle is quite free to deflect, and adjust the spot of light to zero by slightly moving the scale if necessary.

(5) Place the same magnetic needle, which in the meanwhile has been carefully guarded against vibration, on the sliding carriage (capable of moving along the magnetometer bench due east and west), with its magnetic axis due east and west. Then set the table so that the centre of the magnet and magnetometer needle are d cms. apart, causing a convenient deflection D scale-divisions of the latter.

(6) Reverse the needle so that the other end now points towards the magnetometer, and at the same distance d note the deflection D again.

(7) Repeat (5) and (6) with the magnet at the same distance d on the other side of the magnetometer, and take the *mean* of the four deflections D.

(8) Repeat (5)–(7) with the same magnet at about five or six different distances, d.

(9) Repeat (1)–(8) for each of the other magnets, and tabulate all your results as follows:—

Distance between scale and mirror, L, = scale-divisions.

No. of magnet.	Weight, W grms.	Length, $2l$ cms.	Radius, r cms.	K.	Transits.		Time in secs.		Scale deflection, D.	Distance, d cms.	H.	Mean, H.
					N.	Per sec. $n = \frac{N}{t}$	t.	$T = \frac{2}{n}$				

(10) Carefully weigh and measure the magnets.

13. Proof of Ohm's Law (Galvanometer Method).

Introduction.—The above-named is one of the most important fundamental laws of electricity. It states that the difference of potential (P.D.), which we will call V, at the ends of any conductor *at constant temperature* and carrying a current C, always

bears a constant ratio to that current. This constant ratio is called the *resistance* R of that conductor. In symbols, therefore, we have $R = \frac{V}{C}$ = constant (at constant temperature), or, as it is common to write it, $C = \frac{V}{R}$. Either an electrometer (which passes no current) or a high-resistance galvanometer (which will pass only a very small current), may be used to measure the P.D., and in this method the latter will be used. If such an instrument has a very high resistance compared with that between the points to which it is applied, its indications will be a correct measure of the P.D. between those two points. It is important to carefully distinguish between the E.M.F. and P.D. respectively at the terminals of any source of electricity. The E.M.F. is the *total force* tending to send a current round the whole circuit, whereas the P.D. denotes the *available force* for the external circuit alone, after a certain deduction, which depends on the current, is made for the potential lost in the generator itself of internal resistance b ohms, or we may put $E = V + Cb$. If now a high-resistance galvanometer is used, it will pass only a very small current C, making the term Cb negligible, whence its deflections will be proportional to E or V.

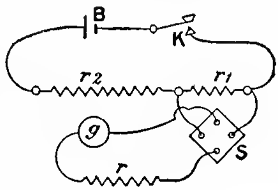


FIG. 11.

Apparatus.—High resistance mirror galvanometer, g (p. 281); two variable known resistance boxes, r_1 and r_2; reversing switch, S (p. 329); spring tapping-key, K; and three Daniell's cells, B.

Observations.—(1) Connect up as shown in Fig. 11, and adjust the galvanometer needle to zero.

(2) With one Daniell's cell in circuit make $r_1 = 1$ ohm (say) and $r_2 = 4999$ ohms. Press K, and note the deflections θ_1 and θ_2 either side of zero by turning S.

(3) Repeat (2) for about six different values of r_1, altering r_2 each time so as to keep $r_1 + r_2$ constant and equal to 5000 ohms.

(4) Repeat (2) and (3) with two and three cells respectively in circuit, and tabulate as follows:—

No. of cells used.	r_1.	r_2.	$r_1 + r_2$.	Deflection, θ_1.	Deflection, θ_2.	Mean, θ.	Value of $\frac{\theta}{r_1}$

(5) Plot a curve having values of θ as ordinates and r_1 as abscissæ.

Inferences.—State clearly all the inferences which you can draw from the results of your experiment. On what does the constancy of the figures in the last column depend?

14. Proof of Ohm's Law (Electrometer Method).

Introduction.—The above-named, as already stated in the preceding test, is one of the most important fundamental laws of electricity, and it may be enunciated thus—

The difference of potential (P.D.) measured electrostatically, which we will call V, at the ends of any conductor *at constant temperature*, and carrying a current C, always bears a constant ratio to that current. This constant ratio is called the *resistance* R of that conductor. In symbols, therefore, we have $R = \frac{V}{C}$ = constant (at *constant* temperature), or, as it is more commonly written, $C = \frac{V}{R}$. In the present method an electrometer (which passes no current at all) will be used in place of the galvanometer of the preceding one, and it may be used in one or other of two ways: (*a*) ideostatically, *i.e.* with the needle maintained at an initial high potential by means of a "dry pile" or other independent suitable source of E.M.F., in which case the deflection caused by placing an E.M.F. across the quadrants $\propto$ P.D. between them; (*b*) heterostatically, *i.e.* with the needle merely connected to one pair of quadrants, in which case the deflection $\propto$ (P.D.)^2. These two relations follow at once from the following formula for the quadrant electrometer:—

If N = P.D. between the needle and earth or framework of the instrument,

Q_1 = P.D. between one pair of quadrants and earth or framework of the instrument,

and Q_2 = P.D. between the other pair of quadrants and earth or framework of the instrument,

then the deflection of the spot of light on the scale is—

$$d \propto (Q_1 - Q_2)\{N - \tfrac{1}{2}(Q_1 + Q_2)\}$$

from which we see that d is more nearly $\propto Q_1 - Q_2$, and that the sensibility of the instrument becomes greater, as N increases. A reversing key should always be used with an electrometer, and should be so arranged that when the instrument is not in use the quadrants are short circuited.

Apparatus.—Reflecting electrometer, V (p. 289), and its reversing key, K_1 (p. 298); source of potential, P, for charging the needle; galvanometer, G (p. 271), merely for indicating the current; battery, B, of about 12 cells of fairly constant E.M.F.; key, K; platinoid resistance, R.

Observations.—(1) Connect up as in Fig. 12, where N is the terminal of the needle of V and Q_1, Q_2 those of the two pairs of quadrants. If P is a battery, the negative pole should be either earthed, *i.e.* connected to the nearest gas or water pipe, or connected to the case of V. Adjust both V and G to zero.

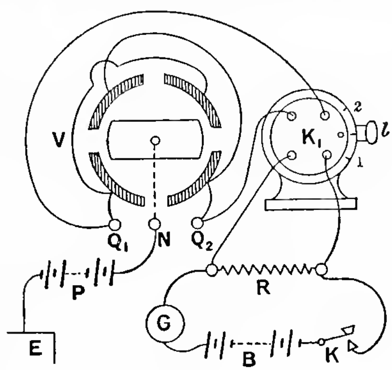


FIG. 12.

(2) Using only two cells for B, put l of K_1 to 1 and close K by continued tapping, so as to "nurse" the deflections up to their steady full values. In this way large flings and waste of time in waiting for the deflection to become steady will be avoided. Note the steady deflection D on V and of d on G. Repeat this with l to 2, K still being closed, and note the deflections again, using the mean.

(3) Repeat (2), using 4, 6, 8, 10, and 12, or more cells successively, and calculate the ratio $\frac{D}{d}$ for each. Tabulate as follows:—

Number of cells used.	Approximate potential of needle.	Current through R, d.	P.D. across R $\propto$ to deflections to			Constant $\frac{D}{d}$
			Right D.	Left D.	Mean D.	

(5) Plot a curve having values of D as ordinates and d as abscissæ.

Inferences.—What can you infer from your results?

15. Sensitiveness of Galvanometers (Figure of Merit).

Introduction.—The sensitiveness, or "*figure of merit*," of a galvanometer, as it is variously termed, is reckoned in one or other of two ways: (a) as the *current* in amperes required to produce unit deflection; (b) as the *resistance* of the circuit containing the instrument through which 1 volt P.D. will cause unit deflection. The former is manifestly the most natural way of expressing it, and is the one which the author prefers to adopt, though the latter way is the one most frequently adopted commercially.

It should be carefully noted in all cases that for the figure of merit to be of future use, the position of the scale and needle of the galvanometer must be fixed relatively to one another, and the position of the controlling magnet (if there is one) carefully noted, as both will affect the figure of merit.

Apparatus.—Galvanometer, G, to be tested; high resistance, r; key, K; Daniell's cell, B, preferably newly made up. Its E.M.F. may be taken to vary between 1·04 and 1·12 volts, the mean 1·08 being assumed for the experiment.

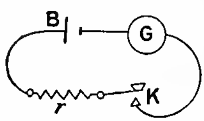


FIG. 13.

Observations.—(1) Connect up as in Fig. 13, and adjust the galvanometer needle to zero with the controlling magnet in one extreme position.

(2) Adjust r so that on pressing K a steady deflection of about one-tenth of the available part of the scale is obtained. Note this deflection d and the value of r.

(3) Repeat (2) for about ten different deflections, rising by about equal increments to the maximum scale reading.

(4) Repeat (2) and (3) for the other extreme position of the controlling magnet, and tabulate as follows :—

Resistance	; galvanometer, G, =	ohms ; battery, B, =		ohms.
Position of controlling magnet.	r.	Total circuit resistance, $R = (r + B + G)$.	Deflection, d.	Figure of merit, $\frac{1 \cdot 08}{dR}$

Inferences.—What can be inferred from your results?

16. Relation between Deflection and Current in Mirror Galvanometers.

Introduction.—In experimental work it is of great importance to know the "relative" (and in some cases the "absolute") calibration curve of some form of galvanometer, *i.e.* the way in which the deflections vary with the strengths of the various currents producing them. In many types, particularly in non-reflecting instruments, the deflections are only proportional to the currents for a very small angular motion of the needle, so that though the current producing a deflection of eight might be just double that giving four, one producing sixty might be much greater than fifteen times the current giving four. The following experiment will enable the relation, in the case of mirror galvanometers, whether of the suspended needle or coil type (for a detailed description on the construction of which see p. 265, *et seq.*), to be determined, and also show the way in which a galvanometer can be calibrated (relatively) by the "resistance method."

Apparatus.—Mirror galvanometer, G, to be tested; resistance box, R ; spring tapping-key, K_2 ; Leclanché cell, B ; reversing key, K_1.

Observations.—(1) Connect up as indicated in Fig. 14, and adjust the spot of light to zero.

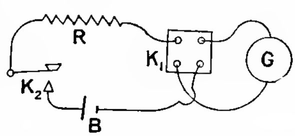


FIG. 14.

(2) Close K_1 and K_2, and adjust R so as to get a deflection of about one-twelfth of the maximum scale-reading. Note this and the value of R, and by means of K_1 note the deflection on the other side of zero.

(3) Repeat (2) for about twelve different deflections, rising by

about equal increments to the maximum obtainable, and tabulate as follows :—

Form of galvanometer used ; resistance, g, = ohms ; battery resistance, b, = ohms.

Box resistance, R.	Total resistance, $(R+g+b)=R_T$	Current, $C=\frac{E}{R_T} \propto \frac{1}{R_T}$	Deflections.		
			Right.	Left.	Mean, d.

(4) Plot a "calibration" curve having values of C as abscissæ and d as ordinates.

Inferences.—What inferences can you draw from the resnlts of your experiment, and how could the "absolute calibration curve" of the galvanometer be obtained?

17. Calibration of a "Sine" Galvanometer (Constant Current Method).

Introduction.—The following method, due to Mr. T. Mather, is a very simple and convenient one for calibrating a galvanometer throughout the whole range of its scale on the "sine" principle. Any galvanometer can be used as a sine galvanometer providing (a) the controlling field or force acting on the needle is constant in magnitude and direction; (b) the deflecting force, although variable in its direction in space, must act in a fixed direction relatively to the needle; (c) it can turn in a horizontal plane about a point which is coincident with the centre of the needle.

If now a certain current, which we will call C, deflects the needle through $\theta_1°$, and with it still flowing and constant in strength, the galvanometer with its scale is turned round so that the pointer again points to its original position on the scale, then, on breaking the circuit, the needle will swing back into the magnetic meridian, describing an arc of $\theta_2°$ to its position of rest, and we shall now have—

$$C \propto \sin \theta_2°$$

Apparatus.—The galvanometer, G, capable of rotating in a

horizontal azimuth about its centre; source of constant E.M.F., E; key, K.

Observations.—(1) Connect up the above apparatus in series, and remove any neighbouring magnets to a distance.

(2) Close K and turn G round so that the pointer is at 10° on the scale, which we will call $\theta_1°$. Open K, and note the position on the scale $\theta_2°$, at which the pointer comes to rest.

(3) Repeat (2) every 10° up to 70° or 80°, and calculate the values of $\dfrac{\sin\theta_1°}{\sin(\theta_1 - \theta_2)}$. Tabulate your results in a convenient manner.

(4) Plot the calibration curve of the galvanometer, having values of deflection θ_1 as abscissæ, and $\dfrac{\sin\theta_1}{\sin(\theta_1 - \theta_2)}$, which is proportional to current strength, as ordinates.

Inferences.—If the galvanometer is fixed with its pointer at O, when no current passes, show that $\dfrac{\sin\theta_1}{\sin(\theta_1 - \theta_2)} \propto$ current strength causing a deflection $\theta_1°$ from O.

18. Calibration of a Galvanometer by Comparison with a Standard Galvanometer.

Introduction.—It may sometimes be necessary to know, when using a particular galvanometer, what relation exists between various deflections and the currents producing them. This can be at once found in a very simple manner by comparing the galvanometer deflections with those produced by the same currents on either a tangent or mirror or other galvanometer, the relation between the deflections and currents in which is accurately known. The present experiment is arranged for the purpose of so calibrating the unknown galvanometer.

Apparatus.—Standard galvanometer, G; galvanometer to be calibrated, g; continuously variable resistance, Rh (p. 309); key, K_1; battery, B; and reversing key, K_2 (p. 329).

Observations.—(1) Connect up as in Fig. 15, and adjust the needles of both G and g to zero.

(2) Close K_1 and K_2 and alter Rh so that a deflection d_1 of about one-tenth of the full scale is obtained on g. Note this and the reading of the standard galvanometer D.

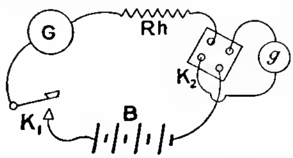


FIG. 15.

(3) Reverse the current through g by turning K_2 round, with K_1 still closed, and G reading the same as in (2). Now note the deflection d_2 on g on the opposite side of zero.

(4) Repeat (2) and (3) for about nine or ten different deflections on g, rising by about equal increments to the maximum scale reading, and tabulate your results as follows :—

Standard galvanometer used ; type				
Deflection, D.	Current, D, or tan D.	d_1.	d_2.	Mean or true deflection on $g = \frac{1}{2}(d_1 + d_2)$.

(5) Plot a calibration curve having values of current strengths as obtained from the standard as ordinates and values of $\frac{1}{2}(d_1 + d_2)$, or deflections on galvanometer to be tested as abscissæ.

Inferences.—How could G be adapted to the test if its sensibility was much greater than that of g? What would be the result if it was so much more sensitive?

19. Measurement of the Strength of the Magnetic Field due to a Constant Current flowing round a Circular Coil.

Introduction.—It is a matter of considerable importance to be able to predict the way in which the strength of the magnetic field, or, in other words, the *sensitiveness* of a galvanometer, will vary with the number of turns in the coils, its diameter, and its position with respect to the needle. The following experiment will enable this to be determined experimentally, and is divided into three distinct parts, A, B, and C.

A is to determine the effect of altering the number and radius of the convolutions.

B is to compare the strength of field at different points *in the plane* of the coils.

C is to compare the strength of field at different points *along the axis* of the coils.

Apparatus.—An auxiliary galvanometer, g, merely for the purpose of showing that the current flowing in the circuit is constant in strength; spring tapping-key, K; an adjustable resistance, R; battery, B; arrangement of coils and magnetic needle, G, to experiment upon, consisting of two bobbins and needle, as follows (*vide* p. 270): On the large bobbin there are two distinct circuits of 10 and 30 turns of wire respectively, having a mean radius r of 20 cms. On the small bobbin there is one circuit of 10 turns, having a mean radius r of 10 cms. By means of pins, grooves, and scales the bobbins can be placed in any desired position relatively to the needle.

Observations.—**A.** (1) Adjust the pointer to zero by slightly turning the baseboard, and be careful not to move it during the experiments.

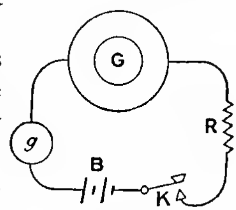


FIG. 16.

(2) Place the large and small bobbins so that their centres coincide with the centre of the magnetic needle, and their planes contain its axis.

(3) Send a convenient current round the 10 turns on the small bobbin, and note the deflection θ° produced.

(4) Send an equal current round 10, 20, 30, and 40 turns successively of the large bobbin, and note the corresponding deflections.

Tabulate your results as follows :—

Coil or bobbin.	Current in circuit.	Deflection, θ°.	$\tan\theta$.	No. of turns, n.	Values of $\frac{n}{r}$	$\frac{\tan\theta}{\frac{n}{r}}$

(5) Plot a curve having for abscissæ the values of $\frac{n}{r}$ and for ordinates $\tan\theta$.

(6) Observe how nearly a deflection 0° is produced by passing the same current round 20 turns of the large coil in one direction, and 10 turns of the small coil in the opposite direction.

B. (1) Take away the small coil, and arrange the large one so as to be capable of moving *in* its own plane.

(2) Keep the current constant round 20 turns on the large coil, and note the deflections θ produced when the centres of the coil and magnet are at different distances d cms. apart. Six readings should be taken between the extreme positions possible with (*a*) the magnet inside the coil, (*b*) with it outside the coil.

Tabulate as follows :—

Magnetic needle inside the coil.				Magnetic needle outside the coil.			
Distance, d cms.	Current.	Deflection, θ.	tan θ.	Distance, d cms.	Current.	Deflection, θ.	tan θ.

(3) Plot curves having values of d as abscissæ and tan θ as ordinates.

C. (1) Place the large coil in its initial position, and arrange it so as to be capable of moving *perpendicular* to its own plane.

(2) Keep the current constant round 40 turns of the large coil, and note the deflections θ° produced when the centres of the coil and magnet are at about eight different distances d cms. apart, increasing by equal increments from 0 to the maximum possible.

Tabulate as follows :—

Distance, d cms.	Current.	Deflection, θ.	tan θ.	Values of $\frac{r^2}{(\sqrt{r^2+d^2})^3}$

(3) Plot a curve having values of $\frac{r^2}{(\sqrt{r^2+d^2})^3}$ as abscissæ and tan θ as ordinates.

D. Repeat experiments B and C with the small coil, and tabulate your results in like manner.

Inferences.—Write out clearly all the inferences which can be drawn from the above experiments, and point out their bearing on the construction of galvanometers. Prove theoretically that the strength of the magnetic field produced is proportional to

$$\frac{r^2}{(\sqrt{r^2+d^2})^3}$$

What precautions ought to be taken in the above experiments?

20. Resistance of a Uniform Wire at Constant Temperature proportional to its Length (Direct Experimental Proof).

Introduction.—The following is a very simple and convenient means of proving the important relation stated above by means of Ohm's law. Any uniform wire may be used, but the most convenient arrangement will be to use the stretched wire on a metre bridge, as different lengths of it can be at once measured by the scale over which it is stretched.

Apparatus.—The metre bridge, PQ; high-resistance galvanometer, G; rheostat, R (p. 309); key, K; battery of constant E.M.F., such as a secondary cell, B.

Observations.—(1) Connect up as in Fig. 17, and adjust the galvanometer needle to zero on the scale.

(2) Set the sliding key K_1 of the bridge at or near the end Q of the stretched wire, and adjust R so as to obtain nearly a full-scale deflection *d* on the galvanometer; when K_1 is closed after K is pressed, note the position D of K_1 on the scale and the deflection *d* of the galvanometer.

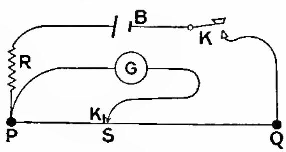


Fig. 17.

(3) Repeat (2) every 10 cms. down the scale to the other end.

Note.—If a tangent or sine galvanometer is used, then A will be $\propto$ to tan *d* or sin *d* as the case may be, where *d* is in degrees. If it is a mirror galvanometer, then A $\propto$ *d* simply.

Tabulate your observations as follows :—

Galvanometer resistance, G, = ohms ; type used

Distance, D.	Deflection, *d*.	Current A through galvanometer.	$\frac{D}{A}$

(4) Plot a curve having values of D as abscissæ and A as ordinates.

Inferences.—Why should the galvanometer have a high resistance? and how would the test be affected if it had a low one? Is the method based on any assumptions? If so, state them.

21. Measurement of Resistance by the Substitution Method.

Introduction.—This is the simplest of all methods of measuring electrical resistance, and is an application of Ohm's law. The equality of two currents of electricity, as indicated on a suitable galvanometer, is the only thing observed, the actual value of a certain standard known resistance producing this being read off, and giving, without any calculation, the value of the unknown resistance.

Apparatus.—Leclanché cell, B; sensitive galvanometer, G (p. 287); two-way key, K; resistance, r, to be measured; standard box of known resistances, R (p. 315); resistance box, S, for use as a shunt to G in such a way that if G is too sensitive, and gives too large a deflection with r in circuit, the resistance S can be reduced so as to shunt more current past G, and so diminish its deflection.

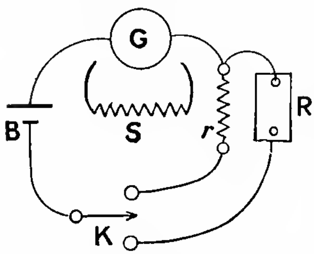


FIG. 18.

Observations.—(1) Connect up as indicated in Fig. 18, and adjust the needle of G to zero.

(2) Turn K so as to place r in circuit, and adjust S (if necessary at all) to get about a quarter-scale deflection d.

(3) Leaving S as in (2), turn K so as to place R in circuit; now adjust R so as to exactly reproduce the deflection d again. Then r = R so found.

(4) Repeat (2) and (3) for each of the unknown resistances provided.

(5) Repeat (2)–(4) for a half and three-quarter-scale deflection, and tabulate your results as follows:—

Resistance tested.	For reference only.		Unknown resistance, R = r.	Mean, r.
	Shunt, S.	Deflection, d.		

Inferences.—State clearly any assumptions made in the

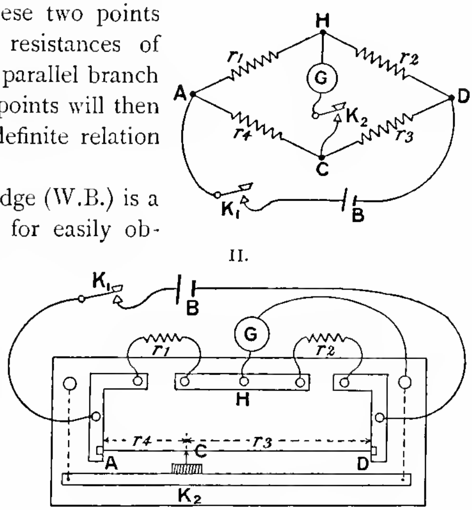
experiment, and show clearly how the above relation between r and R is arrived at.

22. Measurement of Resistance by the Metre Form of Wheatstone Bridge.

Introduction.—This is by far the most accurate and expeditious method of measuring resistance, and it depends on the fact that if any point, H, be taken in one of two conductors, AHD and ACD, in parallel, and carrying a current, then some point, C, can always be found in the other having exactly the same potential as H. Hence no current will flow in a conductor joining these two points H and C; but the resistances of the two parts of each parallel branch either side of these points will then bear a simple and definite relation to one another.

A Wheatstone bridge (W.B.) is a special arrangement for easily obtaining this relation, and consists of six conductors joined three and three together at four points. The four conductors joining the points consecutively are called the "arms" of the W.B., and the two remaining conductors contain a galvanometer, G, and battery, B, respectively (Fig. 19, I.). Two arms, r_3 and r_4, can be formed by a box of proportional coils, or, as in the present case, by a platinum-iridium wire, AD (Fig. 19, II.), 1 metre long, and of *uniform* cross-section, stretched over a metre scale. The other two arms consist respectively of a resistance box, r_2, and unknown resistance, r_1, to be measured. If the resistances, r_1, r_2, r_3, r_4, of

I.



II.

FIG. 19.

the arms (taken consecutively) have the relation $r_1 r_3 = r_2 r_4$, called the "Law of the W.B.," then no current will pass through G, and therefore there will be no deflection when the currents in the arms have reached their steady values. This "balance" will still hold good, even if the positions of G and B be interchanged, and they are therefore called conjugate positions. The *battery key* K_1 must always be *pressed before the galvanometer key* K_2 to get over any inductive action in r_1 (if present). In other words, the battery key must be pressed *first* for just long enough to allow the currents to arrive at their steady values in all parts of the bridge before the galvanometer key is closed. If this is not done when measuring resistances of the nature of an electromagnet, balancing to obtain no deflection on the galvanometer will be impossible, owing to the extra kick due to the phenomena known as self-induction. K_2 is a sliding key capable of making contact with any point of AD, and it must on *no account be moved when pressed.* The sensibility of the metre B is increased by lengthening AD, and any error made in the position of K_2 will have least effect on the calculated resistance when K_2 is in the middle portions of AD. The method has the great advantage of being a null one, *i.e.* one in which no deflection has to be obtained. If any thermo-current effects are observed, they can be eliminated by inserting a reversing key in the battery circuit.

Apparatus.—Metre bridge complete with its sliding key, K_2; sensitive reflecting galvanometer, G; spring tapping-key, K_1; battery, B, of one or more Leclanché cells; standard known adjustable resistance, r_2; resistance, r_1, to be tested.

Observations.—(1) Connect up as indicated (Fig. 19, II.), and adjust the galvanometer to about zero.

(2) Insert a suitable resistance, as r_2, and balance the bridge by pressing K_1 first and then moving K_2 until no deflection occurs on pressing it. Note the value of r_2 and the lengths of AD each side of K_2, which are directly proportional to r_3 and r_4.

N.B.—At all times, except when balance is almost obtained, the keys should be closed for the shortest possible time, so as not to allow the current time to warm up the resistances, and so alter their value. To see if a balance is possible, tap K_2 for an instant *gently* near each end of the scale, and if the galvanometer deflects to opposite sides of zero, there will be a point of balance somewhere between them.

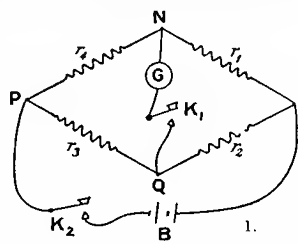
(3) Repeat (2) for about six different values of r_2, and do the same for each of the other resistances to be tested, tabulating your results as follows:—

Form of resistance tested.	Standard, r_2.	r_3.	r_4.	Unknown, $r_1 = \frac{r_4}{r_3} \times r_2$	Mean value, r_1 ohms.

Inferences.—Prove from Ohm's law that $r_1 r_3 = r_2 r_4$ if no current flows through G. On what does the accuracy of this method depend?

23. Measurement of Resistance by the "Post Office" Pattern of the Wheatstone Bridge.

Introduction.—It is assumed that the metre form of Wheatstone bridge (W.B.) has already been used. The Post Office (P.O.) pattern (Fig. 20, II.) is merely a specially arranged and compact form of W.B. placed in a suitable box for portable purposes. If the principle and action of a W.B. is understood at all, and the stamping opposite the various terminals in the P.O. form observed, it ought to be impossible to couple up incorrectly. Each of the "proportional arms" r_3 and r_4 consists of three resistance coils of 10, 100, and 1000 ohms respectively, hence the ratio $\frac{r_3}{r_4}$ or $\frac{r_4}{r_3}$ can be made a very simple number. QRST (Fig. 20, II.) is the "adjustable arm" r_2, and it consists of sixteen different coils and one infinity plug either at Q, R, or S. The value of r_2 can be made anything from 1 to 11,110 ohms. Opposite two of the terminals, N and T, is marked "Galvanometer, Line" and "Line,



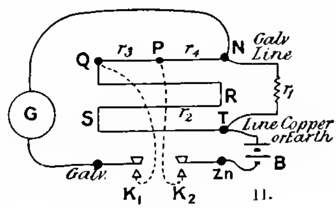


FIG 20.

Copper, or Earth" respectively. This is because the P.O. form is primarily intended for measuring the metallic and insulation resistance of telegraph lines, and hence in the first case that line would be joined to N and T, and in the second case only one end to N, the other being free and insulated, T then being put to earth. As therefore we are measuring metallic resistance r_1, it is put between N and T. The terminals to which the battery B must be connected are equally obvious. The white dotted lines on the top show where the under contacts of the keys K_1 and K_2 are joined to inside the box. In any form of W.B. variation of the battery E.M.F., or its resistance, or that of the galvanometer G, has no effect on the accuracy of the measurement. The sensitiveness of the test, though principally depending on that of G, can be increased within limits by using a larger E.M.F. and making r_1, r_2, r_3, and r_4 as nearly equal as possible. *The battery key* K_2 *must always be pressed before the galvanometer key* K_1, to allow the currents in the arms to become steady before pressing K_2. The battery key should be pressed for as short a time as possible, so as to prevent the coils being heated by the current and their resistance thereby altered. In inserting plugs *press in lightly and give about one-eighth of a turn* to insure good electrical contact. Reverse this operation when removing them. The ends of all connecting wires should be scraped clean. For a description of the P.O. form of Wheatstone bridge, *vide* p. 319.

Observations.—(1) Connect up as indicated in Fig. 20, II., and adjust the galvanometer needle to zero.

(2) Note once for all the direction in which G deflects when r_2 (Fig. 20, II.) is too large to give balance (done by taking out "Inf" in r_2 with, say, $r_3 = r_4 = 10$).

NOTE.—Until balance is nearly obtained, only tap K_1 for a fraction of a second.

(3) Make $r_3 = r_4 = 10$, and balance the bridge by altering r_2 so as to get no deflection on pressing K_2 and then K_1. If it is impossible to get exact balance, note the steady deflection when r_2 is just too large and too small, and calculate the correct intermediate resistance to give balance, by proportion.

(4) Repeat (3) for $r_3 = r_4 = 100$ and 1000 respectively, and then for all possible permutations of the coils in r_3 and r_4 taken two together, one out of each proportional arm.

NOTE.—If the unknown resistance r_1 is greater than 11,110 ohms, then r_4 will be greater than r_3, but if r_1 is less than 11,110 ohms, then r_4 may be either equal or less than r_3.

Tabulate as follows :—

Resistance tested.	Proportional arms.		Adjustable arm, r_2.	Unknown resistance, $r_1 = \frac{r_4}{r_3} \times r_2$	Mean, r_1.
	r_3.	r_4.			

NOTE.—The limits of the P.O. bridge are $(\frac{10}{1000} \times 1 =)$ 0·01 ohm and $(\frac{1000}{10} \times 11{,}110 =)$ 1,111,000 ohms; but measurements become less accurate as they approach these limits.

24. Measurement of Resistance by the Differential Galvanometer.

Introduction.—The differential galvanometer method of measuring resistance is simple, accurate, and in many cases a convenient one. It has, in common with the Wheatstone bridge (W.B.) method, the advantages of being a zero method, and also of being independent of the constancy of the working battery, though for some reasons the W.B. method is to be preferred. The galvanometer must accurately fulfil two conditions: (*a*) its two coils must be of exactly *equal* resistance; (*b*) they must have exactly *equal magnetic effects* on the needle. To obtain the maximum magnetic effect or sensibility the resistance of each of the two coils should be about one-third of that to be tested. The range of utility of the galvanometer can be considerably extended, and, in fact, made equivalent to that of a W.B. by the use of shunts. Thus, if the two galvanometer coils are shunted by resistances S_1 and S_2 respectively, then $\left(1 + \frac{g}{S_2}\right)x = \left(1 + \frac{g}{S_1}\right)R$, from which we see that if x was known to be large compared with R, we should only use one shunt $S_1 = \frac{g}{9}, \frac{g}{99}$, or $\frac{g}{999}$ (say), whereas if x was small compared with R, we should only use the one shunt S_2 of similar value as above.

Apparatus.—Sensitive reflecting differential galvanometer, *g* (p. 281), with its shunts, S_1, S_2; battery, *b*; key, K; adjustable known resistance box, R (p. 315); and the resistance, *x*, to be measured.

Observations.—(1) Adjust the galvanometer as follows: (*a*) needle to zero by means of the controlling magnet; (*b*) by turning the levelling screws so that on sending a current through the two coils connected up in *opposing series, no deflection* is produced.

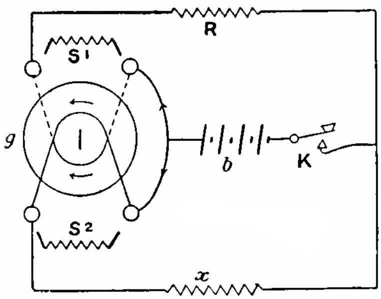


FIG. 21.

(2) Connect up as shown in Fig. 21, and adjust R so that on pressing K there is no deflection on *g*. Note the value of R, which therefore = *x*.

N.B.—If it is impossible to obtain no deflection with the available values of resistances in R, adjust R by the smallest possible amount, so as to get deflections on each side of zero, then calculate by proportion the true resistance which it would be necessary to insert at R so as to obtain no deflection.

(3) Repeat (2) for different values of shunts applied separately to each galvanometer coil, and also to both together, and tabulate as follows:—

Resistance of each galvanometer coil, *g*, = ohms.

Shunt, S_1.	Shunt, S_2.	Right.		Left.		True res. R ohms.	$x = \frac{1+g/S_1}{1+g/S_2} R$
		R.	Deflection.	R.	Deflection.		

Inferences.—Prove the formula given in the last column of the table, and state any assumptions made in deducing it.

25. Measurement of High Resistance by the Substitution Method.

Introduction.—The Wheatstone bridge is quite unsuitable for measuring very high resistances amounting to a megohm and

upwards. This is due to the fact that the bridge is not being used under the most sensitive conditions, and that even a delicate high-resistance Thomson galvanometer would not be sensitive enough to indicate an accurate balance. The following is a simple and accurate method of measuring very high resistances. It will almost always be impossible to reproduce the deflection which was obtained with the unknown resistance alone in circuit, by altering the known, as very high adjustable known resistances are seldom available. In such cases the respective deflections are used in calculating the unknown, and these should be as nearly equal as possible to give maximum accuracy.

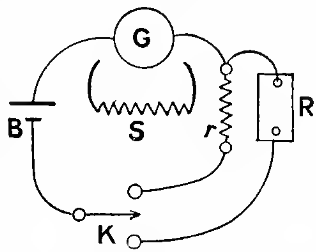


FIG. 22.

NOTE.—To avoid damaging the galvanometer, which is a very delicate one, the shunt box provided with it must always be used in the way indicated below.

Apparatus.—Delicate high-resistance Thomson galvanometer, G (p. 284); shunt box, S (p. 323); known high resistance, R (p. 314); unknown resistance, r; battery, B; two-way, high-insulation key, K (p. 327).

Observations.—(1) With the short-circuit key of S down, connect S up to G first, and then the rest of the circuit as indicated.

(2) With it still down, gently tap K for a fraction of a second so as to complete circuit through r; then, if the deflection is inappreciable, release K, plug up the $\frac{1}{999}$ shunt, and remove the short circuit. Again tap as before, and if still too small, release K, plug up $\frac{1}{99}$, and so on until a convenient steady deflection d_r is obtained. Note this and the shunt S_r in use at the time (if any).

N.B.—The key K must always be released before altering the shunt S.

(3) Repeat (2) with R, noting the deflection d_R and shunt S_R (if any).

(4) Repeat (2) and (3) for about three or four pairs of deflections in different parts of the scale, with R and r, by altering S,

and calculate the unknown resistance r from the formula (*see below*). Tabulate as follows :—

Galvanometer resistance, G, = ohms at ° C. ; form of unknown resistance

No. of cells used.	Standard known resistance.			Unknown resistance.		
	In ohms, R.	Deflection, d_R.	Shunt, S_R.	Deflection, d_r.	Shunt, S_r.	r ohms.

Note.—If $S = \frac{1}{9}$ (say), then—

$$\frac{S}{S + G} = \frac{1}{10}, \text{ or } \left(1 + \frac{G}{S}\right) = 10$$

Inferences.—Prove the formula mentioned in (4), and state what assumptions are made in deducing it.

$$d_R\left\{R\left(1 + \frac{G}{S_R}\right) + G\right\} = d_r\left\{r\left(1 + \frac{G}{S_r}\right) + G\right\}$$

26. Measurement of Very High or Insulation Resistance (Loss of Charge Method).

Introduction.—The following is a simple and convenient method of measuring very high resistances, such as that of the insulating covering of joints and samples of electric light cable, providing the condenser used has a sufficient capacity and the resistance to be measured is not too high. In this particular case a galvanometer is used to measure the condenser discharge.

Apparatus.—Delicate high-resistance reflecting ballistic galvanometer, G (p. 285) ; battery, B, capable of giving a sufficient E.M.F. ; condenser, C (p. 354) ; high insulation charge and discharge key, K (p. 328) ; *well-insulated* trough, T, of water containing the cable to be tested as the high resistance.

Note.—All connections and apparatus must be insulated with great care, all insulating parts being quite clean and free from dust or grease. Avoid the use of keys when possible unless they are very highly insulated ones. If the cable is an electric light one, B should give at least the *working pressure* for which the cable is intended to be used.

Observations.—(1) Carefully prepare the *free end* F of the cable as described on p. 46, and train it well out of the water as shown. Adjust the galvanometer to zero, and connect up as in Fig. 23.

(2) With a disconnection somewhere, indicated at s and t, press K to b and charge C fully for a few seconds, then immediately raise to a so as to discharge the condenser through G. Note the first throw d_1 corresponding to a quantity K_1 coulombs.

(3) Recharge C *fully* after connecting at s and t, and note the exact time t_1 on gently releasing K so as to *rest between* blocks a and b, the upper contact a having been previously raised to allow of this.

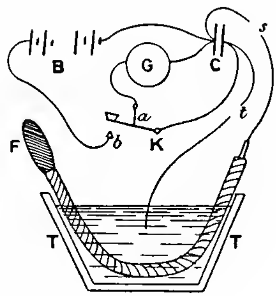


Fig. 23.

(4) Allow the condenser to thus slowly discharge through the high resistance R to be tested in the trough T; then, after a few minutes, close K to a, noting the throw d_2 corresponding to K_2 coulombs discharge from the condenser.

Note.—It is best to arrange the time so that $d_2 = \frac{1}{3}d_1$ (about).

(5) Repeat (2)–(4) for about six values of capacity C, and calculate the insulation or high resistance from the formula—

$$R = \frac{0{\cdot}4343 \cdot t}{C \log \frac{K_1}{K_2}} = \frac{0{\cdot}4343 \cdot t}{C \log \frac{d_1}{d_2}}$$

where $t = t_2 - t_1$ in seconds.

Tabulate your results as follows:—

Test made after minutes' electrification of cable; cable soaked for hours before test.

Deflections.		Time.		Time $(t_2 - t_1)$ in seconds, t.	Capacity used, C microfarads.	Insulation resistance, R megohms.
d_1.	d_2.	t_1.	t_2.			

If R is very large indeed, d_1 and d_2 will be nearly equal, unless t is made inconveniently long. This may be obviated by using a large E.M.F. and discharging through G (shunted) to get

d_1, and again discharging after t secs. through G (unshunted) to get d_2. In the former case, its sensibility can be reduced to any extent, whereas in the latter it is used to the best advantage. Since, however, the effect of shunting a ballistic galvanometer is somewhat indefinite, this method will not be accurate unless the correction mentioned on p. 113 is used, or—

$$\text{True ratio} = \frac{d_1}{d_2} \times \text{true multiplying power of the shunt used.}$$

27. Measurement of Very High or Insulation Resistance (Loss of Charge Method).

Introduction. — The following "loss of potential" method is suitable for measuring very high resistances, such as is frequently met with in either long or short lengths of well-insulated cables, and consists in joining the two terminals of a charged condenser by the very high resistance, and measuring by means of a sensitive electrometer the "time rate of fall" of potential, which may be more or less rapid, depending on the magnitude of this resistance and the capacity of the condenser.

Apparatus.—Sensitive reflecting quadrant electrometer, E, having well-insulated quarter cylinders and other portions; well-insulated trough, T, containing water and the highly insulated cable to be tested; battery, B, with two flexible wires, which will just reach to the well-insulated condenser, C (p. 354).

N.B. — Great care must be taken that all connections and apparatus are highly insulated and free from dust, etc., and when possible keys should be avoided for this reason, unless they are very good. The free end F of the cable should be carefully prepared by paring off about 1 inch of the outer covering or layer of insulating material, then drying this part carefully over a spirit or naphtha lamp, and when quite dry, painting it over repeatedly with melted paraffin wax, until the end and about $1\frac{1}{2}$ inch of the cable is coated with about $\frac{3}{8}$ inch of wax.

Observations.—(1) First connect up C (only) to E, and adjust the spot of light to zero. Now touch the condenser terminals with the battery wires a and b for a few seconds only, and when the deflected spot of light is stationary on the scale,

note whether the deflection falls appreciably on removing a and b. If it does not, the quadrants of E are well insulated enough for the test.

(2) Connect up as in Fig. 24 now, and again touch C with a and b for a few seconds, and immediately the spot of light is stationary remove a and b, noting simultaneously the deflection d_1 corresponding to a potential v_1, and also the time t_1.

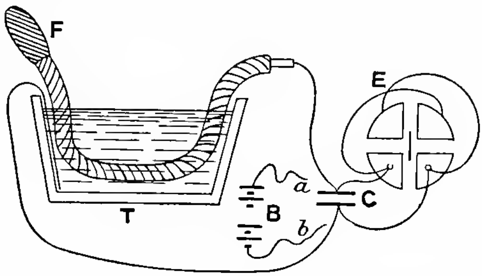


FIG. 24.

(3) When this deflection has fallen to about $\frac{1}{2}d_1$, note its exact value d_2 corresponding to a potential v_2, and also the time t_2.

(4) Calculate the high or insulation resistance R from the relation—

$$R = \frac{0{\cdot}4343 \cdot t}{C \log_{10} \frac{v_1}{v_2}} = \frac{0{\cdot}4343 \cdot t}{C \log \frac{d_1}{d_2}}$$

where $t = t_2 - t_1$ in seconds.

(5) Repeat (2)–(4) for about six different values of capacity, and tabulate thus—

Cable tested after minutes' electrification ; soaked for hours in water before test.

Length of cable in water.	Temperature of water.	Capacity, C microfarads.	Deflections		Times.		(t_2-t_1) in secs., t.	Insulation resistance, R megohms.
			d_1.	d_2.	t_1.	t_2.		

Inferences.—Prove the relation given in (4), and state any assumptions made in obtaining it.

NOTE.—If the insulation resistance R_c of the condenser is so low compared with the resistance R, through which it is discharged as to add materially to the rate of discharge, then its own

insulation resistance must be found by the following method, and from the law of combination of resistance in parallel we shall have—

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_c}$$

$$\text{whence } R_1 = \frac{R_c R}{R_c - R}$$

where R_1 = true insulation resistance of the cable under test, and R that found from the above experiment.

28. Insulation Resistance of Condensers (Loss of Charge Method).

Introduction.—The preceding method with the electrometer is one of the best for determining the insulation resistance of the condenser itself, for any ballistic method is likely to have its results vitiated by the effects of residual absorption, etc.

Apparatus.—Sensitive reflecting quadrant electrometer whose quadrants are very highly insulated; battery and condenser to be tested.

Observations.—(1) Place the electrometer across the terminals of the condenser, and then bring the flexible battery leads to the latter so as to charge it for a few seconds.

(2) When the deflection is quite steady, remove the battery leads, and take suitable time-readings of the fall of the deflection, until this reduces to about half its original value v_1; call the final value v_2.

Then the insulation resistance of the condenser—

$$R = \frac{t}{C \log_{10} \frac{v_1}{v_2}} \text{ megohms}$$

where t = number of seconds it takes for the deflection to fall from v_1 to v_2,

and C = capacity of the condenser in microfarads.

(3) Plot a curve having values of diminishing deflections as ordinates and times of noting these as abscissæ.

29. Measurement of the Insulation Resistance of a Leaky Condenser (Loss of Charge Method employing Shunts).

Introduction. — The following galvanometer method of measuring insulation resistance by "loss of potential" is suitable for leaky condensers whose capacity is not accurately known, and consists in altering the sensitiveness of the galvanometer as required by using a shunt, S, and applying the corrections for it as determined in this method.

Apparatus.—Battery; leaky condenser, C, to be tested; sensitive high-resistance reflecting galvanometer, G (p. 285); shunt, S; high-insulation two-way key, K; potentiometer arrangement, ACB, for getting two P.D.'s, v_1 and v_2, bearing a known ratio (viz. r_1 and r_2 respectively) to each other; a well-insulated condenser, C_1, of known capacity (p. 354).

Observations.—A. (1) Connect up as in Fig. 25, with C only in circuit at first, and adjust the galvanometer to zero.

(2) Fully discharge C, and with the galvanometer shunted by S, close K to *a*, so as to charge the condenser C to a potential v_1, and note the first throw D_1 on G.

(3) Allow C to *stand insulated* for *t* secs., and disconnect S by releasing the high-insulation key in its circuit.

(4) Close K to *a* again, and note the first throw D_2 on the unshunted galvanometer.

(5) Repeat (2)-(4) for three or four different values of S, r_1, and r_2.

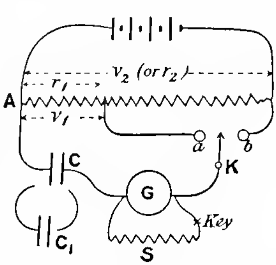


FIG. 25.

The value of C and the increased multiplying of the shunt S can be obtained as follows:—

B. (1) Insert the condenser of known capacity, C_1 (microfarads) in the place of C, and adjust the galvanometer to zero if necessary.

(2) Close K to *a*, so as to charge C_1 with the same potential v_1 and the galvanometer unshunted, and note the throw d_1 divisions on the galvanometer.

(3) Discharge C_1 completely, and shunt G with the same shunt as was used in Obs. A (2). Now close K to *b*, so as to charge C_1 with an increased potential v_2, and note the first throw d_2 on the galvanometer.

(4) Tabulate your results as follows :—

Capacity of well-insulated condenser, C_1, = mfds. ; galvanometer resistance, g, = ohms.

B.						A.				
v_1 (r_1).	v_2 (r_2).	d_1.	d_2.	Shunt, S.	Multiplying power of shunt, $M_s = \frac{S+g+g^1}{S} = \frac{v_2}{v_1} \cdot \frac{d_1}{d_2}$	D_1.	D_2.	Time in secs., t.	Capacity of leaky condenser, C mfds. $= M_s \cdot \frac{D_1}{d_1} \cdot C_1$	Insulation resistance, R in megohms, $= \frac{0{\cdot}4343t}{C \log_{10} K_1/K_2}$

NOTE.—If K_1 = number of coulombs in the leaky condenser C at first charge,

and K_2 = number of coulombs in the leaky condenser C at end of t secs.,

$$\text{Then } \frac{K_1}{K_1 - K_2} = \frac{D_1}{D_2} \cdot M_s$$

$$\text{and hence } \frac{K_1}{K_2} = \frac{D_1 M_s}{D_1 M_s - D_2}$$

30. Measurement of Low Resistance by the Potential Difference Method.

Introduction.—The Wheatstone bridge is inapplicable for measuring very low resistances, and even if such were just within its range, the measurement would not be accurate. The following method, which depends directly on the definition of resistance, can be used to accurately measure very low resistances, such as are met with in large electric light cables and the armatures of dynamos and motors. The P.D. at the terminals of each resistance can be measured relatively by a sensitive galvanometer, whose resistance is large compared with that between the two points to which it is applied. Under these conditions its insertion

will not lower the P.D. to be measured. If it is a reflecting instrument the scale deflections will be proportional to the P.D.

Apparatus.—Known standard low resistance, R (p. 312); low resistance, r, to be tested; Pohl's commutator, C; secondary cell, B; rheostat, Rh (p. 308); fairly high resistance galvanometer, G; reversing key, K (p. 329); switch, S.

Observations.—(1) Connect up as indicated in Fig. 26, and adjust the galvanometer needle to about zero.

(2) With Rh full in, close S, and adjust the current to give about quarter-scale deflection with the largest resistance of the two, for then the deflection with the other is bound to be on the scale; then note the galvanometer deflection on each side of zero by turning K, when G is across each resistance in turn.

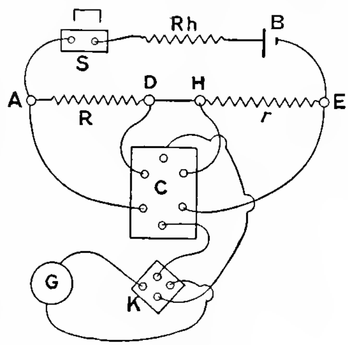


FIG. 26.

N.B.—The resistance Rh should be sufficiently high to prevent the current strength altering during any one pair of observations, and to prevent this current being strong enough to sensibly warm the resistances. The more sensitive the galvanometer the smaller this will be. After taking deflections with the second resistance, it is advisable to retake those with the first in case the current has altered. If they are not the same, take the mean of those on the respective sides of zero. For very accurate work a reversing key should be used with B to eliminate any thermo-current effects.

(3) Repeat (2) for half, three-quarter, and full scale deflections, and calculate the unknown resistance r from the formula—

$$R \div r = d_R \div d_r$$

Tabulate as follows:—

Low resistance tested $r=$; standard low resistance R= ohms.							
Deflection across R.			Deflection across r.			Ratio, $\frac{d_r}{d_R}$	Unknown, r ohms $= R\frac{d_r}{d_R}$
Right.	Left.	Mean, d_R.	Right.	Left.	Mean, d_r.		

Inferences.—Prove the formula given in (3), and state any assumptions made in deducing it. What sources of error is the method liable to? How can they be minimized?

31. Variation of Resistance with Length, Diameter, and Material.

Introduction.—The following constitutes an important and exceedingly instructive test, the results of which should be known and committed to memory in the early stages of experimental work. The coils to be experimented upon (*vide* p. 346) are contained in a box fitted with three rows of terminals, to which they are severally attached. The first row consists of coils of wire of the same material (platinoid) and diameter (0·013 inch), but of different lengths, viz. 5, 10, 20, 30, 40, and 50 feet respectively. The second row consists of coils of wire of the same length (20 feet) and diameter (0·012 inch), but of different materials, viz. copper, brass, lead, iron, German silver, platinoid, manganin, eureka. The third row consists of coils of wire of the same length (20 feet) and material (platinoid), but of different diameters, viz. 0·003, 0·0096, 0·013, 0·0155, 0·0184, 0·024 inch respectively.

Observations.—(1) Measure the resistance of each of the above coils by means of some convenient method, noting the temperature of the room at the time. Tabulate your results in the following form:—

Material.	Deflection (if any).	Resistance, R.	Length, L.	Diameter, *d*.	$\frac{1}{d^2}$	Sectional area, A.	ρ per cub. inch.	ρ per c.c.

(2) From the results of the experiments with wires of different lengths, plot a curve having resistance as ordinates and lengths as abscissæ.

(3) From those on wires of different diameters, plot a curve connecting resistances and values of $\frac{1}{d^2}$, where d = diameter of that wire.

Inferences.—Deduce the laws connecting (*a*) resistance with length; (*b*) resistance with diameter from the results of the above experiments. Combine these laws so as to apply to wires of the same material, in which both length and diameter may vary together. From results of the experiments on wires of different materials, calculate the "specific resistance" per inch cube and per centimetre cube, for each of the materials used, from the formula—

$$R = \frac{L}{A}\rho$$

where R = measured resistance in ohms,
L = length of wire,
ρ = specific resistance required,
A = sectional area of wire.

32. Laws of Combination of Resistances in Parallel.

Introduction.—The law or equation which gives the combined or *effective resistance* that two or more separate resistances in parallel introduce to the passage of an electric current in a circuit is a very important one. The following experiment is intended to prove the same, and it is assumed that the student is already familiar with the use of the "metre form" of Wheatstone bridge for measuring resistance.

Apparatus.—Metre bridge; sensitive galvanometer, G; standard adjustable known resistance, R_1; Leclanché cell, B; box containing four coils, A, B, C, and D, arranged so that they can be connected up singly or in all possible combinations of series and parallels, R_2 (*vide* p. 347).

NOTE.—In all cases measurements will be most accurate when the galvanometer slider key is in or near the centre of the bridge, *i.e.* when $R_1 = R_2$ approximately.

Observations.—(1) Connect up as indicated, and adjust the spot of light to zero (about).

(2) Measure the resistance of each of the four coils A, B, C, and D separately.

(3) Measure the resistance when they are two, three, and four in series.

(4) Measure the resistance of all possible combinations taken *two in parallel.*

(5) Measure the resistance of all possible combinations taken *three in parallel.*

(6) Measure the resistance when the four coils are in parallel, and calculate the combined resistance as given by the bridge from the formula $R_2 = \frac{l_2}{l_1} \times R_1$, and also as given by the formula to be proved, $\frac{1}{R_2} = \frac{1}{A} + \frac{1}{B} +$ etc., and tabulate as follows :—

Combinations of coils.	Position of slider.		Known resistance, R_1.	Unknown resistance.	
	l_1.	l_2.		Measured, $R_2 = \frac{l_2}{l_1} \times R_1$	Calculated, $\frac{1}{R_2} = \frac{1}{A} + \frac{1}{B} +$ etc.

Inferences.—Prove the formula given in the last column of the table.

33. Comparison of Resistances (Magneto-Inductor Method).

Introduction.—This method is based on a special application of Ohm's law, and depends on the principle that when a coil of wire forming part of a closed circuit suddenly cuts the lines of force due to a magnetic field, an E.M.F. is induced in the coil which sets up a transient current in the circuit. This current by Ohm's law will be inversely proportional to the total circuit resistance. It will thus be seen that the magneto-inductor constitutes the battery of the arrangement, with, however, the distinctive difference that its E.M.F. is only a transient one. The method is a very simple one, and has the advantage that the induced current is of too short a duration to alter the resistance of the circuit by heating it.

Apparatus.—Magneto-inductor, M (p. 358) ; suitable ballistic

galvanometer, G (p. 283); a three-way key, K; and the resistances, R_1, R_2, to be compared.

Observations.—(1) Connect up as in Fig. 27, and adjust the galvanometer to zero.

(2) Close K1, and with the galvanometer perfectly still, slip the inductor coil, and note the first throw d of the spot of light. Do this two or three times and take the mean.

(3) Repeat (2) with K2 closed, noting the mean first throw d_1.

(4) Repeat (2) with K3 closed, noting the mean first throw d_2.

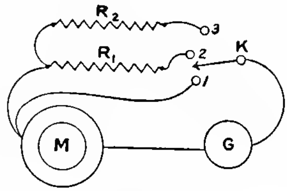


FIG. 27.

(5) Calculate the unknown resistance, if one of them is known, and compare them from the relation—

$$\frac{R_1}{R_2} = \frac{d - d_1}{d - d_2} \cdot \frac{d_2}{d_1}$$

(6) If R_1 (say) is an adjustable known resistance, repeat (3)–(5) for about six different values of it, and tabulate your results as follows:—

Galvanometer used ; resistance, G, = ohms ; resistance of M = ohms.

d.	d_1.	d_2.	R_1.	$\frac{R_1}{R_2}$	R_2.

N.B.—If the throws are too large to be readable with R_1 and R_2 alone, any resistance may be inserted in series with the galvanometer to reduce them.

Inferences.—Prove the relation given in (5), and state any assumptions made in obtaining it.

34. Determination of Specific Resistance.

Introduction.—The specific resistance of any substance is the resistance between opposite faces of either an inch cube or centimetre cube of the substance offered to the passage of a

current which is assumed to flow normally to those faces. It is usually denoted by the Greek letter ρ.

If d = diameter of a wire of circular cross-section, s and l its length—

$$\text{Then its resistance } R = \frac{l}{A}\rho = \frac{l\rho}{\frac{\pi d^2}{4}} = \frac{4l\rho}{\pi d^2}$$

$$\text{whence } \rho = \frac{R\pi d^2}{4l}$$

If R is in ohms, d in centimetres, and l in centimetres, then ρ will be the specific resistance per centimetre cube of the substance.

Apparatus.—A length of wire to be experimented upon, preferably not less than 2 metres long and about No. 22 B.W.G. gauge; Wheatstone bridge, complete with galvanometer and battery for measuring the resistance of the sample; finely divided scale for measuring the length and micrometer gauge for the diameter.

N.B.—Errors made in measuring l and R will be minimized when both these quantities are fairly large, but an error in the diameter is the most serious, and the most likely to occur from its being so small as a rule, and necessarily so if R is large. Any such error will magnify up from the fact that it is d^2 and not d which is used.

For great accuracy the micrometer gauge method of getting d may not be sufficient, and recourse may then be had to its calculation from the mass, length, and density of the wire by weighing in water.

Thus, if w = its true weight in grammes, and σ its density in grammes per cubic centimetre, the sectional area in square centimetres—

$$A = \frac{\text{vol.}}{l} = \frac{w}{\sigma l} = \frac{\pi d^2}{4}$$

Hence, substituting in the above equation, we get—

$$\rho = \frac{Rw}{\sigma l^2} \text{ [1]}$$

[1] We may obtain σ by suspending the specimen by a very fine wire from one arm of a delicate chemical balance, and finding its weight w_1 grms. in air and w_2 grms. in pure water, when σ will $= \frac{w_1}{w_1 - w_2}$.

Observations.—(1) Carefully measure the resistance R of the specimen by the bridge, using all possible combinations of the proportional coils, and take the mean of these determinations (*vide* p. 39).

(2) Very carefully bare the wire (if insulated) at some four or five places where it is perfectly straight, taking great care not to scratch or scrape it, and then measure the diameter at each by the gauge and take the *mean*.

(3) Very carefully measure the total length of the specimen two or three times and take the *mean*, and note the temperature of the room.

(4) Calculate the specific resistance of the material from the above relation, and record all your observations as follows :—

Materials tested ; temperature of room = ° C. = t° C.

Measurements of		Proportional arms of bridge.		Adjustable arm, r_2.	Resistance, $R = \frac{r_4}{r_3} r_2$	R corrected for temperature.	Specific resistance, ρ at t° C.
Length, l.	Diameter, d.	r_4.	r_3.				

N.B.—The relation between ρ and temperature can be obtained by immersing the specimen in paraffin oil, employing some convenient arrangement similar to that described on p. 350, and taking a series of measurements of resistance at different temperatures. These should be plotted in the form of a curve, with resistance as ordinates and temperature as abscissæ.

35. Relation between Current and Amount of Heat generated.

Introduction.—The relation mentioned above is one of the utmost importance in electrical work, and is embodied in the combination of two of the most important fundamental laws in current electricity. The object of the following experiment is to investigate this relation.

Apparatus.—Electro-calorimeter, E (p. 350); current measurer, A; variable resistance, Rh (p. 308); secondary battery, B (p. 338);

key, K; chronometer if available, or stop-watch beating half-seconds, etc.

Observations.—(1) Connect up as in Fig. 28, and adjust the pointer of A to zero if necessary.

(2) Fill the inner copper can with tap-water so that the resistance wire is completely immersed, and see that when in position this can is in the centre of and is separated from the outer tin can by the felt ring under its rim.

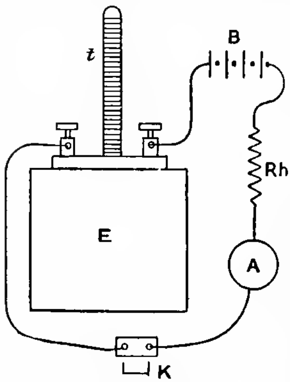


FIG. 28.

(3) When all is ready, close K and quickly adjust Rh to give a convenient current (say 2 amps.), which must be *kept constant*, and immediately begin to note the temperature of the calorimeter by means of the thermometer *t* every minute (say for 15 minutes), keeping the liquid continuously stirred all the time, so as to maintain a uniform temperature throughout the water; then open K.

(4) Now begin at once to note the temperature *t* as it falls every 5 or 10 minutes, from the moment of opening K down to almost the original starting temperature.

(5) Repeat (3) and (4) for two other current strengths (say 3 and 4 amps.) for 10 minutes, and tabulate all your results in a convenient manner.

(6) Plot a curve for each set of ascending and descending values of temperature for each current, on the same curve sheet having values of temperature as ordinates and time as abscissæ.

(7) Correct the heating curve by means of its corresponding cooling curve, so as to obtain a *third* curve for each current used, indicating what the rise of temperature would have been had there been no loss of heat by radiation and convection, etc., during the heating operation.

(8) Compare the "tangents" of the angles which each of these third curves make with the *axis of time* with the respective current strengths used, and see whether—

$$C_1^2 : C_2^2 : C_3^2 = \tan \alpha_1 : \tan \alpha_2 : \tan \alpha_3$$

NOTE.—To obtain this third corrected curve, proceed as follows: Divide the axis of time into minutes, or even half-minute intervals, then commencing at the beginning of the heating curve, add the *drop* of temperature during the given interval at the lower end of the cooling curve to the ordinate of the heating curve at the end of that same interval. Do this for a number of intervals throughout the range, and draw a *mean curve* through these new points, which should be a *straight line.*

Inferences.—State very clearly and concisely *all* the inferences you can draw from the results of the above experiment, and state what you consider to be the relations between current strength and (*a*) temperature, (*b*) amount of heat generated in each case.

36. Determination of the Mechanical Equivalent of Heat (Electrical Method).

Introduction.—The above-mentioned quantity, commonly termed "*Joule's equivalent,*" can be obtained approximately by means of an electro-calorimeter; but before proceeding with the method of performing the test, it may be advisable to deduce our expression for the "mechanical equivalent," or Joule's equivalent, which is usually denoted by the letter J. If a current A flows for a time t, the total quantity Q of electricity transferred in that time is $Q = At$; and if this is caused by a P.D. $= V$, then the work done, W, by the current in this time is—

$$W = AVt = A^2Rt = VQ$$

when $R =$ resistance through which the current A flows,

and $V = AR$ by Ohm's law.

Now, the amount of heat, H, produced is a measure of this work done when all the latter is converted into heat. If, then, J units of work are equivalent to one heat unit, the work required to produce H units of heat will $= JH$.

$$\text{Hence } JH = AVt = A^2Rt = W$$

$$\text{or } J = \frac{A^2Rt}{H}$$

This equation assumes that all the heat generated in the coil of the calorimeter whose resistance is R is transferred to the

water, and is operative in raising its temperature. This is not strictly true, for some still remains in the wire, some is conducted through the end connections of the coil, and some is lost by radiation. If, however, the calorimeter is suitably designed and the final temperature not allowed to rise more than a few degrees above that of the room, the errors introduced from these causes may be negligible for all but extremely accurate determinations, when of course the apparatus will require to be of a more elaborate form.

The amount of heat H is determined as follows:—

Let W = weight in grams of water in calorimeter,

W_1 = weight in grams of calorimeter alone, and S_1 the specific heat of its material,

W_2 = weight in grams of stirrer alone, and S_2 the specific heat of its material,

t_1°, t_2° = initial and final temperatures of the water in degrees Centigrade,

Then the total amount of heat generated, H, $= (W+W_1S_1 + W_2S_2)(t_2 - t_1)$, assuming no loss from radiation, conduction, or convection, and if A and R are in C.G.S. units and t in seconds, we have—

$$J = \frac{A^2Rt}{H} = \frac{A^2Rt}{(W + W_1S_1 + W_2S_2)(t_2 - t_1)} \text{ ergs per grm. degree Cent.}$$

This is commonly termed "*Joule's law*," and it states that the amount of heat required to raise the temperature of 1 grm. of water from 0° C. to 1° C. (which is the calorie) is the same as what would result from the work done in lifting this same weight of water a vertical height of 42,400 cms.

But to lift 1 grm. through 1 cm. requires the expenditure of 981 ergs. Hence the mechanical equivalent of heat, J, $= 42{,}400 \times 981 = 41{,}594{,}400$ ergs per gram degree Centigrade.

In electrical measure the joule is the amount of work done or turned into heat when 1 amp. flows through 1 ohm, and hence at the P.D. of 1 volt for 1 sec., or J = a volt-amp.-sec. $= 10^8 \times 10^{-1} = 10^7$ ergs or C.G.S. units. Hence, assuming that $J = 42 \times 10^6$ ergs in round numbers—

$$1 \text{ joule would raise } \frac{10^7}{4{\cdot}2 \times 10^7} = \frac{1}{4{\cdot}2} = 0{\cdot}238 \text{ grm. of water } 1^\circ \text{ C.}$$

Thus $H = 0{\cdot}238C^2Rt$ = number of heat units (calories) produced by C amperes flowing through R ohms for t secs.

Apparatus.—Precisely the same as that in the last experiment, and the thermometer should be capable of being read to at least $\frac{1}{50}^\circ$ C.

Observations.—(1) Connect up as in Fig. 28, and adjust the pointer of A to zero if it needs it.

(2) Wash the copper can with clean water, dry it thoroughly, and when quite cool, carefully weigh it accurately on a chemical balance.

(3) Nearly fill it with tap-water, and again accurately weigh it so as to obtain the total weight of water.

NOTE.—There must be no drops of water hanging to the outside or rim.

(4) Place it in position, and at a noted instant switch on quickly, adjusting the current to say 2 amps., and immediately note the reading on the thermometer.

(5) Keep the current *perfectly constant* by means of Rh, stirring gently and continuously all the time, and switch off when the temperature has risen to such a value that it is as much above the temperature of the room as it was below this when the test started. Note the temperature and time of switching off.

N.B.—This will practically correct for radiation, for it will receive as much heat from the room in the first part of the experiment as it gives out to the room in the last part.

(6) Repeat (3)–(5) for two other currents (say 3 and 4 amps.), and tabulate your results as follows :—

Weight of copper can, W_1 = grms. ; material ; specific heat, S_1 = } resistance of
,, stirrer W_2 = ,, ; ,, ; ,, ,, S_2 = } wire, R, = ohms.

Time of			Temperatures.			Weight in grms. of can + water, W_s.	Weight of water, $W = W_s - W_1$.	Current, A.	Mechanical equiv., J.
Start.	Finish.	Difference, t secs.	Initial, t_1° C.	Final, t_2° C.	Rise, $(t_2 - t_1)^\circ$ C.				

Inferences.—State clearly what precautions and corrections you would apply so as to obtain a very accurate result.

37. Relation between Resistance and Temperature for Different Metals.

Introduction.—The apparatus consists of a Post-office Wheatstone bridge with its battery and galvanometer (in this case a D'Arsonval) together with the coils to be tested (*vide* p. 350), composed of five different important metals wound on a hollow perforated bobbin. They are immersed in an inner vessel containing oil, and fitted with a thermometer and stirrer. This vessel is immersed in an outer one containing water, the temperature of which can be raised by a bunsen burner. The terminals of the various coils have the letters A to E between them as follows: A, platinoid; B, copper; C, manganin; D, German silver; E, soft iron. It is assumed that the manipulation of the Post-office Wheatstone bridge is already well known. *Uniformity in the temperature of the oil bath*, and therefore of every part of a coil, can only be ensured by continual stirring, and it is absolutely essential for a successful experiment.

Observations.—(1) Adjust the spot of light to about zero, and before applying any heat, measure the resistance of the coils selected at the temperature of the room, noting this on the thermometer immersed.

(2) *Slowly* raise the temperature about 8° C., keeping the oil well stirred, then remove the flame from underneath, and continue to stir for two or three minutes, so as to make the oil of uniform temperature throughout and that of the coils the same as the oil. Now measure the respective resistances and the corresponding temperature at the moment of balancing each.

(3) Repeat (2) about every 8° C. up to 80° C.

(4) Take a similar set of observations during cooling, and tabulate as follows:—

Material.	Resistance, R.		Temperature, T°.		a.
	Heating.	Cooling.	Heating.	Cooling.	

(5) Draw the heating and cooling curve for each coil tested, having R as ordinates and T° as abscissæ.

(6) Select, from the best curve for each coil, two pairs of points at different parts corresponding to resistances r_1, r_2 at temperatures t_1 and t_2 respectively, calculating for each pair of points the coefficient α of variation of resistance with temperature per degree Centigrade from the relation—

$$\frac{r_1}{r_2} = \frac{1 + \alpha t_1}{1 + \alpha t_2}$$

38. Measurement of Resistance and Temperature Coefficient (Carey-Foster Method).

Introduction.—It has already been mentioned, in connection with measurements in general by the Wheatstone bridge, that the best and most sensitive arrangement for a given galvanometer is when all the *four arms are equal*, or nearly so, for then the fall of potential down each arm is the same when balance is obtained, and any small deviation from this balance will cause a larger deflection with equal arms than when they are unequal. Or, again, if, as will generally be the case, it is impossible to make the arms equal to obtain balance, then to obtain the most sensitive arrangement, either the galvanometer or battery, which ever has the higher resistance, should be connected between the junctions of the two high and the two lower resistances forming the arms.

From these remarks it will be gathered that the accuracy of the metre bridge will be greatest when the resistances being compared in the two gaps are nearly equal, and therefore when the slider galvanometer key is at or near the middle of the stretched wire. It will, in addition, be obvious that as the resistance of single metre-bridge wires is usually from 1 ohm to 2 ohms only, the accuracy of the measurements becomes less as the resistances measured become greater. In other words, the form of bridge shown on p. 316 is only suitable for the measurement of low resistances of the order of, say, $\frac{1}{2}$ to 2 ohms. Its range and sensibility can, however, be increased by having more than one stretched wire, and putting them in series with each other and side by side, which arrangement has the further advantage that a given error in reading the position of the slider at a certain part

of the scale will cause a smaller error in the result as the length of the wire gets greater. This will be at once evident by considering the relation $r_1 = \frac{r_4}{r_3} . r_2$ for the metre bridge (p. 37), where r_3 and r_4 are the lengths or resistances of these either side of the slider key. If, now, an error of one division is made in reading the slider key, then—

$$r_1 = \frac{r_4 + 1}{r_3 - 1} . r_2$$

and it will be quite evident that $\frac{r_4 + 1}{r_3 - 1}$ will become more nearly $= \frac{r_4}{r_3}$ as r_3 and r_4 become greater; in other words, the error will become less and less as the length of the bridge wire gets greater.

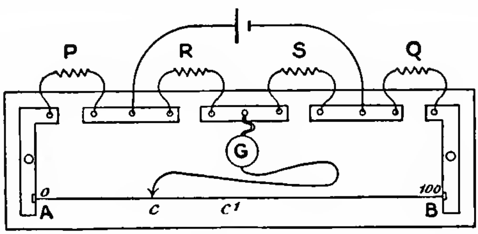


FIG. 29.

The metre bridge can, however, be modified slightly in construction to the form shown on p. 316, by giving it four gaps when it can be made available for comparing higher resistances. Thus suppose the resistances P and Q to be compared are inserted in the end gaps and are of about the same value, and that two other resistances, R and S, approximately equal to either P or Q, are placed in the other two gaps. Then, manifestly the four arms R, S, (P + AC), and (Q + CB) are about equal, and C will be in the middle portion of AB. Thus we have the conditions for maximum sensibility and accuracy, but at the same time the range of the bridge has been diminished. Now, considering the Carey-Foster method of comparing the two resistances P and Q, let r_1 = resistance of the end connection on the bridge between P and A, and r_2 that between Q and B; also let L = total length

of the wire AB, and ρ the resistance per unit length. Then, if balance is obtained at C, we have by the ordinary law of the bridge—

$$\frac{R}{S} = \frac{P + r_1 + \rho(AC)}{Q + r_2 + \rho(CB)}$$

If P and Q are interchanged and R, S kept as before, then—

$$\frac{R}{S} = \frac{Q + r_1 + \rho(AC')}{R + r_2 + \rho(BC')}$$

when C′ is the new point of balance.

From these two equations, by a well-known rule in proportion, we have—

$$\frac{R}{R + S} = \frac{P + r_1 + \rho(AC)}{P + Q + r_1 + r_2 + \rho(AB)}$$

$$\text{and } \frac{R}{R + S} = \frac{Q + r_1 + \rho(AC')}{P + Q + r_1 + r_2 + \rho(AB)}$$

$$\text{hence } P + r_1 + \rho(AC) = Q + r_1 + \rho(AC')$$

$$\therefore P - Q = \rho(AC' - AC)$$

In other words, the difference between the two resistances P and Q compared is equal to the resistance of the bridge wire between the points of balance C and C′, and the result is independent of the resistances between P and A and Q and B, also R, S, and AB. It is, however, necessary to know ρ, which can be found by the method given on p. 68 or 70, or as follows: Substitute for Q a thick copper connector across the right-hand gap, then Q = 0 very nearly, and if P has a value slightly smaller than the resistance of AB, then by the above method of interchange—

$$P = \rho(AC' - AC), \text{ or } \rho = \frac{P}{AC' - AC}$$

which assumes that the wire AB is of uniform cross-sectional area throughout. If this is not true, then ρ must be obtained from a calibration curve of the wire by the former method. It will now be seen that when an accurate standard resistance Q (p. 313) is available of the same magnitude as the unknown P, or nearly so, this latter is given by the relation—

$$P = Q + \rho(AC' - AC) \text{ ohms}$$

This method of Carey-Foster's constitutes one of the best and

most accurate for determining the "*temperature coefficient*" of variation of resistance, and the bridge described on p. 317 is one of the most convenient and accurate forms for carrying out the method, and for interchanging the coils P and Q. In the following remarks we shall assume that this form of bridge is the one used, but it should be remembered that they apply in general to any other convenient form of Foster bridge. Referring to Figs. 170, 171 (p. 317), the two ratio coils, r_1, r_2, corresponding to R and S in the preceding remarks, remain stationary throughout the tests, and for very accurate work should be placed in the same water-bath, and maintained at a uniform temperature by means of a constant stream of tap-water. The coils r_3 and r_4 (Fig. 171) correspond to P and Q, the coils to be compared, and should each have its own water-bath and thermometer. One of these must be a standard known resistance (say Q), and must be kept at a *constant temperature* by a stream of a tap-water passing through its bath; the other must be capable of being heated to 70° or 80° C., preferably by steam. Matters must be so arranged that r_1, r_2, r_3, and r_4 are all of about the same resistance, and that of the bridge wire *ww* (Fig. 171) to suit. If, then, R_1, R_2 are the resistances of the coil P to be tested at temperatures $t_1°$ and $t_2°$ C., we first find $R_1 - Q$ at temperature $t_1°$ C. and $R_2 - Q$ at $t_2°$ C. in the way indicated above, when we shall have the increase of resistance $(R_1 - Q) - (R_2 - Q) = R_1 - R_2$ corresponding to the rise in temperature $t_1 - t_2$, and if R_0 = resistance of P at the temperature 0° C., and a = coefficient of increase of resistance per degree Centigrade, then—

$$R_1 = R_0(1 + at_1)$$

$$\text{and } R_2 = R_0(1 + at_2)$$

$$\therefore a = \frac{R_1 - R_2}{R_0(t_1 - t_2)}$$

Since this is a small quantity, R_0 need only be approximately known without introducing appreciable error in the value of a so obtained.

Apparatus.—Foster bridge complete with ratio coils and standard known resistance; battery; sensitive galvanometer; reversing key; coil of material to be tested suitably mounted in the way mentioned.

Observations.—(1) Connect the battery through the reversing

key across terminals B_1 and B_2 (Fig. 171), and the galvanometer across G_1G_2, and adjust the galvanometer to zero. Turn the water on, and allow it to flow until the thermometers are quite steady.

(2) Balance the bridge by moving K until no deflection occurs on the galvanometer. Note the temperature t_1 of the coil tested and the reading C_1 of K.

(3) Now reverse the battery, and again quickly balance, noting the new position C_1' (if altered).

(4) Turn D so as to interchange the positions of r_3 and r_4, and again quickly balance, noting the new position C_2.

(5) Reverse the battery, and again balance, noting the position C_2' (if altered).

N.B.—By reversing the battery in this way any thermo-electric current effect due to unequal heating of the contacts of two dissimilar metals can be allowed for, otherwise an error might be introduced.

(6) Carefully warm up the bath containing the coil tested about 12° C., and allow it to fall gradually some 3° or 4° C., when the coil will presumably be at the same temperature as the bath. Note this temperature t_2, and very rapidly repeat (2)–(5).

N.B.—If the temperature falls appreciably, take the mean of the initial and final thermometer readings for tabulation.

(7) Repeat (2)–(6) for about seven or eight different temperatures of the coil to be tested up to about 70° or 80° C., the standard being maintained all the time at *constant temperature*, and tabulate as follows :—

Coil tested : material ; resistance, R_0, = ohms at 0° C. ; stand. resistance = ohms at ° C.
Ratio coils used : resis. = ohms ; bridge wire used ; ρ of bridge wire = ohms per div.

Reading of the bridge.						Temperature of coil tested.				
			Coils interchanged.							
C_1.	Current reversed, C_1'.	Mean, C_x.	C_2.	Current reversed, C_2'.	Mean, C_y.	t_1.	If altered, t_1'.	Mean, t.	$(r_3 - r_4) = \rho(C_x - C_y)$.	Temp. coeff., a.

(8) Plot a curve having values of temperature t as abscissæ, and corresponding resistances of the coil tested as ordinates.

Inferences.—State in words what α really is, and also any inferences that can be drawn from the test. For extreme accuracy are any other precautions necessary? and if so, state them.

39. Calibration of a Metre-bridge Wire (Carey-Foster's Method).

Introduction.—The following method is a very simple and convenient one for determining the resistance ρ *of unit length* of the stretched wire in different parts of the metre bridge, and hence the uniformity of this wire, which is the one condition to be fulfilled if the bridge is to measure resistances accurately. It will therefore be seen that the present test is a very important one. It can most conveniently be applied when a second metre bridge, *ab*, is available, when both bridges will be calibrated simultaneously. Fig. 30 shows the arrangement of apparatus where AB is really the bridge wire to be calibrated; G, a sensitive galvanometer; B, a battery; *c*, a thick copper bar of as *small* a resistance as possible, capable of closing either the gap P or Q; a piece of wire, W, soldered to two plates, which can be easily clamped in either pair of gap terminals. Its resistance may preferably be about one-fifteenth that of the wire AB. D and *d* are the two slider keys of the bridges.

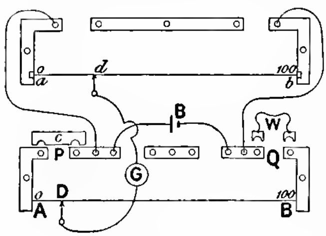


FIG. 30.

If a second bridge is not available, two resistance boxes joined in series may be used for *ad* and *db* respectively.

Observations.—(1) Place D at *o* and *c* and W in the gaps P and Q (Fig. 30); now move *d* so as to get no deflection on G, when both *d* and D are closed. Note these two positions.

(2) Interchange *c* and W; then, keeping *d* fixed and closed, move D until balance is again obtained on pressing it. Note these new positions.

(3) Interchange *c* and W back to their first positions; then,

with D now fixed and closed, move *d* to a new position of balance, and again note the positions.

(4) Repeat (2) and (3) until D gets to the other end, B, of the bridge wire.

NOTE.—AB and *ab* will now both have been divided into short lengths of equal resistance.

Tabulate all your results as follows :—

Resistance of bridge wire, AB, = R = ohms.			Resistance of bridge wire, *ab*, = *r* = ohms.		
Reading of bridge AB.	Successive differences.	ρ per division $= \frac{R}{N} \div a$, etc.	Reading of bridge *ab*.	Successive differences.	ρ per division $= \frac{r}{n} \div a'$, etc.
0 . . . 100	. *a* . *b* . *c* . .	. . . .			

(5) Draw a curve showing the uniformity of AB having distances along the bridge wire as abscissæ and ρ as ordinates.

N.B.—Evidently $R = (a + b + c + \ . \ . \ .)\rho$; and if N = the number of *steps* by which D has travelled from A to B, we see that—

$$\rho a = \rho b = \rho c = \ . \ . \ . = \frac{R}{N}$$

Hence, between *o* on the scale and the first position of D—

$$\rho = \frac{R}{N} \div a$$

and between the first and second position of D—

$$\rho = \frac{R}{N} \div b$$

and so on.

Since the total resistance in circuit with the bridge wire AB is constant in magnitude, we see that—

$$\rho a = \rho b = \rho c = \ . \ . \ . \ W - C$$

where W and C are the resistances of the wire and connector respectively.

If any thermo-electric current effects manifest themselves, the battery should be connected up *through* a reversing key (p. 329), and the *mean* of the two positions of balance taken for *each key*

which moves to points of balance, while the other remains stationary.

Inferences.—State whether you consider the method as it stands above possesses any disadvantages, and if so, show clearly how they could be minimized.

40. Calibration of a Metre-bridge Wire (Gray's Method).

Introduction.—The following is a modification of the Foster method, and is due to Mr. Thomas Gray. It has the great advantage that the contacts of importance are *permanent*, and only those of the battery B and sliding keys, which do not introduce any errors, are movable. The two bridge wires, AB and *ab*, are joined at their ends by two equal small resistances, *r*, *r*, which should be equal to about one-fifteenth of the resistance of AB. This latter will therefore be divided into about fifteen equal steps, the resistance of each of which will equal *r*.

Apparatus.—Metre bridge, whose wire, AB, has to be calibrated; a second metre-bridge wire, *ab*; galvanometer, G; two equal low resistances, *r*, *r*; Pohl's commutator, P (p. 330); battery, B.

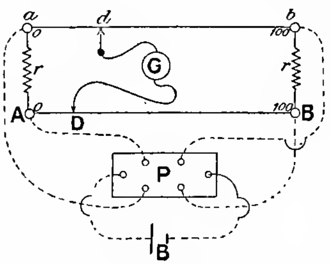


Fig. 31.

Observations. — (1) Connect up as in Fig. 31, and adjust the galvanometer to zero (about).

(2) Test whether the resistances *r*, *r* are equal by joining the battery across A and B, and finding any two points, *d* and D, that G does not deflect. Now place the battery across *a* and *b*, so and if $r = r$, no deflection will occur. This condition must be obtained.

(3) With the connections as in Fig. 31, turn P so as to put the battery across A and *b*, place D at *o*, and move *d* to such a position that when keys D and *d* are closed, no deflection occurs on G. Note the positions of the two keys.

(4) Turn P so as to put the battery across a and B; then, with d fixed, move D to a point of balance, and again note the positions.

(5) Turn P back again to A and b, and, with D fixed, move d to a point of balance, and note the positions of d and D.

(6) Repeat (3)-(5) until D gets to the other end, B, of the wire AB.

NOTE.—AB and ab will now both have been divided into short lengths of equal resistance, all of which has a resistance $= r$. Tabulate, calculate, and plot the results exactly as indicated in the Carey-Foster method.

If any thermo-electric current effects manifest themselves, the battery should be connected to the Pohl's commutator *through* a reversing key (p. 329), and the mean of the two positions of balance taken for *each key, which moves* to points of balance, while the other remains stationary.

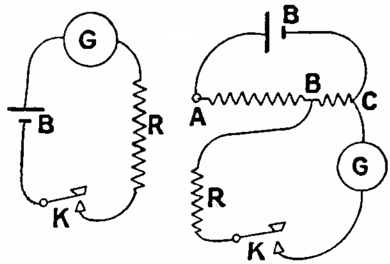
41. Measurement of Galvanometer Resistance (Half-deflection Method).

Introduction.—This method of obtaining the resistance of a galvanometer, although inferior to that of measuring it directly by a complete Wheatstone bridge (W.B.) set (the best way), can be used when a second galvanometer is not available for use with the W.B. It is not suitable for low-resistance galvanometers, as the battery resistance must be negligibly small compared with that of the galvanometer. It has the disadvantage that the relation between deflections, and currents producing them, must be known, as these currents must be comparable. The best result will be obtained when the first resistance used, *i.e.* the smaller of the two, is some fraction of the galvanometer resistance. With a tangent galvanometer the tangents of the deflections must be used, and the best result will be obtained when the deflections $= 55^\circ$ and $35{\cdot}5^\circ$.

Apparatus.—Suitable galvanometer, G; key, K; Daniell's cell, B; resistance box, R.

Observations.—(1) Connect up as indicated in Fig. 32 (*vide* note at end), and adjust the galvanometer to zero.

(2) Close K, and adjust R to get about a quarter-scale deflection d_1. Note this value of d_1 and R_1.



I. FIG. 32. II.

(3) Increase the resistance to such a value R_2 as will give a deflection $d_2 = \frac{1}{2}d_1$. Note the values of R_2 and d_2.

N.B.—If a tangent galvanometer is used, then $\tan d_2 = \frac{1}{2} \tan d_1$.

(4) Repeat (2) and (3) for about eight pairs of deflections, taken over the whole scale, and calculate the galvanometer resistance from the formula—

$$G = R_1 - 2R_1$$

Tabulate as follows :—

Galvanometer tested.	Deflections.		Currents.		Resistances.		G.	Mean, G.
	d_1.	d_2.	D_1.	D_2.	R_1.	R_2.		

NOTE.—With sensitive mirror galvanometers, it will be found best to use only a small fraction of the cell's E.M.F., as indicated in Fig. 32, II. This has the advantage that the battery internal resistance, which corresponds to that of BC, can be made very small compared with G, and also R of the same order as G. Another method of obtaining only a small E.M.F. is by the thermo-electric couple described on p. 339, which may replace the Daniell's cell direct in I., thus obviating the use of the arrangement in II.

Inferences.—Prove the formula given in (4), and state any assumptions made in obtaining it.

42. Measurement of Galvanometer Resistance (Equal-deflection Method).

Introduction.—This method of obtaining the resistance of a galvanometer, although inferior to that of measuring it directly

by a complete Wheatstone bridge (W.B.) set (the best way), can be used when a second galvanometer is not available for use with the W.B. It is not suitable for low-resistance galvanometers, as the battery resistance must be negligibly small compared with that of the galvanometer; but it has the advantage of being applicable to any type of galvanometer, since we have only to reproduce a given deflection, irrespective of whether the deflections are proportional to the currents producing them. For maximum accuracy in the galvanometer resistance obtained by this method, the resistance of the shunt should be as nearly as possible equal to that of the galvanometer.

Apparatus.—Galvanometer, G; resistance box, S, for use as a shunt to G; battery, B, of low internal resistance; keys, K_1, K_2; high-resistance box, R.

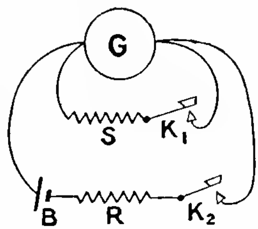


FIG. 33.

Observations.—(1) Connect up as indicated in Fig. 33, and adjust the galvanometer needle to zero.

(2) Adjust S to a suitable value, close K_1 and then K_2, adjusting R so as to get about a quarter-scale deflection *d*. Note this (for reference only), and the values of S and R_1.

(3) Open K_1, and, with K_2 closed, increase R so as to reproduce the deflection *d*. Note the value of R_2, and calculate the galvanometer resistance from the formula—

$$G = S\frac{R_2 - R_1}{R_1}$$

(4) Repeat (2) and (3) for about four different values of S with about the same deflection, by altering R.

(5) Repeat (2)-(4) for about half and three-quarter scale deflections, and tabulate as follows:—

Galvanometer tested.	Deflection, *d*.	Shunt, S.	Resistance.		G.	Mean, G.
			Shunted, R_1.	Unshunted, R_2.		

NOTE.—The infinity plug (if there is one in S) may be used as K_1.

Inferences.—Prove the formula given in (3), and state what assumptions are made in obtaining it.

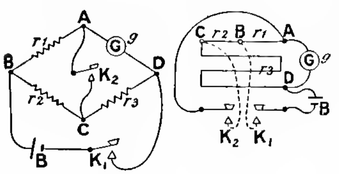
43. Measurement of Galvanometer Resistance (Thompson's Method).

Introduction.—The following method is a convenient one for determining the resistance of a galvanometer, when a second one is not available for use, with a Wheatstone bridge, which would otherwise be the best way to find its resistance.

The galvanometer is replaced by a key in its usual position, and the instrument itself is placed in one of the arms of a Wheatstone bridge. Then, manifestly, when the arms of the bridge satisfy the usual relation between them, the potential of the two points between which it is ordinarily connected will not be disturbed, nor the balance of the bridge, by joining the points through a key.

Apparatus.—P.O. Wheatstone bridge; cell, B, of fairly constant E.M.F.; galvanometer, G, to be tested.

Observations.—(1) Connect up as in Fig. 34, where diagram I. represents the arrangement symbolically, and II. the actual one in the test.



I. II.

Fig. 34.

(2) Make r_1 small and r_2 large, and close K_1 after unplugging something in r_3. Permanent currents will now flow through the four arms of the bridge, causing G to deflect (probably off the scale). Now adjust the position of the controlling magnet, and turn it so as to bring the spot of light or needle back to zero (about), which is the most sensitive position.

(3) Adjust r_3 so that on *making* and *breaking* at K_2, with K_1 still closed and the galvanometer about zero, no motion of the galvanometer is observed; then note the values of r_1, r_2, and r_3.

(4) Repeat (2) and (3) for different values of r_1, r_2, if possible,

and by adding a known resistance in series with G, which can be afterwards subtracted from the result.

(5) Calculate the galvanometer resistance g from the relation—

$$g = \frac{r_1}{r_2} . r_3 \text{ ohms}$$

and tabulate your results as follows :—

Galvanometer tested, ; No. ; temperature of room = ° C.

Known added resistance to galvanometer, R.	Resistances.			Resistance measured, $g = G + R$.	Corrected galvanometer resistance, G.	Mean, G ohms.
	r_1.	r_2.	r_3.			

Inferences.—Prove the relation given in (5), and state any assumptions made in obtaining it.

44. Measurement of the Resistance of Reflecting Galvanometers (Logarithmic-decrement Method).

Introduction.—This method, though of course not as accurate as that of measuring the resistance of the galvanometer directly by means of a complete Wheatstone bridge, is an excellent one in other respects, enabling a very clear conception to be formed as to the way in which the damping in a ballistic galvanometer is affected by different conditions of use. When there is damping in a galvanometer, the amplitudes of successive oscillations of the needle will gradually grow less and less (*vide* p. 134), and the *ratio* of the amplitude of any one oscillation to that of the succeeding one is called the "*decrement.*" This ratio is the same for any two successive oscillations. The Napierian logarithmic decrement λ is the logarithm of this ratio to the Napierian base ϵ, or—

$$\lambda = \log_\epsilon \text{ ratio} = \frac{\log_{10} \text{ ratio}}{\log_{10} \epsilon} = \frac{\log_{10} \text{ ratio}}{0·4343}$$

TABLE III.

Amplitudes to the	
Right.	Left.
130	
	120
105	
	97
85	
	74

As an example, suppose, on causing the needle of a galvanometer to oscillate to and fro past zero, we get the successive deflections each side of zero as indicated,

$$\text{Then } \lambda = \log_\epsilon \text{ ratio} = \log_\epsilon \frac{130 + 120}{120 + 105}$$
$$= \log_\epsilon \tfrac{250}{225} = \log_\epsilon \tfrac{225}{202} = \log_\epsilon 1{\cdot}114$$
$$= \frac{\log_{10} 1{\cdot}114}{0{\cdot}4343} = 0{\cdot}1080$$

If, however, the damping is very small, the ratio may be almost 1, and therefore difficult to obtain at all accurately. In such a case we may proceed thus—

Let A = amplitude of any one oscillation,

and A_n = amplitude of the nth oscillation after it.

$$\text{Then the “decrement”} = \left(\frac{A}{A_n}\right)^{\frac{1}{n-1}}$$

and hence we have—

$$\lambda = \log_\epsilon \left(\frac{A}{A_n}\right)^{\frac{1}{n-1}} = \frac{1}{n-1} \log_\epsilon \left(\frac{A}{A_n}\right) = \frac{1}{n-1}\left(\frac{\log_{10}\frac{A}{A_n}}{0{\cdot}4343}\right)$$

Taking the above case, therefore, we get—

$$\lambda = \frac{1}{6-1}\left(\frac{\log_{10}\frac{130}{74}}{0{\cdot}4343}\right) = \frac{1}{5} \cdot \frac{0{\cdot}2447}{0{\cdot}4343} = 0{\cdot}112$$

Though this is different from the other result, it is probably more accurate.

The present test consists in determining the damping under three different conditions, and calculating the galvanometer resistance therefrom.

Apparatus.—Suitable reflecting galvanometer (p. 285); box of known resistances.

Observations.—(1) Place the controlling magnet in the position of maximum control, *i.e.* as close to the needle as possible, and adjust the spot of light to zero on the scale by means of it.

(2) With the galvanometer terminals “free,” *i.e.* not connected to anything externally, deflect the needle by means of the damping coil (p. 349), and note the amplitudes of succeeding oscillations of the spot of light each side of zero. Call them positive when to

the *right* of zero, and negative when to the *left*, or *vice versâ*. From them calculate the log dec. λ_1, which will therefore give a measure of the damping due to the air friction and torsional resistance of the suspension.

(3) *Short-circuit* the terminals and repeat (2), calculating the log dec. λ_2 from these results. This will give a measure of the damping due to the two last-named causes, and in addition that due to the induced currents in the galvanometer coils.

(4) Connect the terminals through a suitable known resistance R and repeat (2), calculating the log dec. λ_3 from the results.

(5) Repeat (1)–(4) for three widely different values of R.

(6) Repeat (1)–(5) with the controlling magnet halfway along, and also at the extreme end of its guide or supporting-rod.

(7) Calculate the galvanometer resistance g from the relation—

$$g = \frac{\lambda_3 - \lambda_1}{\lambda_2 - \lambda_3} \,.\, \mathrm{R \ ohms}$$

and tabulate as follows :—

Galvanometer used, ; No. ; temperature of room = ° C.

Position of controlling magnet.	Terminals.									R.	Galvanometer resistance, g ohms.
	Disconnected.			Short circuited.			Connected by R.				
	Amplitude to		λ_1.	Amplitude to		λ_2.	Amplitude to		λ_3.		
	Right, $+d$.	Left, $-d$.		Right, $+d$.	Left, $-d$.		Right, $+d$.	Left, $-d$.			

Inferences.—Show how the relation given in (7) can be obtained, and state any assumptions made in obtaining it.

45. Measurement of the Internal Resistance of a Battery (Fall of Potential Method).

Introduction.—When any current generator of total E.M.F. E, and internal resistance B ohms, such as a primary cell, is connected to a galvanometer, the coil of which has a very high resistance, g, compared with B, the current C which it will send, by Ohm's law, is $C = E \div (B + g)$, and this will be therefore very

small; consequently the internal polarization, and therefore the fall of potential in the cell itself, will practically be *nil.* The galvanometer deflection will, however, form a measure of the E.M.F. E of the cell. Suppose, in addition, the cell is now made to send a current through a small resistance, R ohms, comparable with its own B, and which is placed as a shunt to its terminals. It will send a large current and polarize rapidly, producing a large fall of potential in itself. The diminished galvanometer deflection will now form a measure of the "terminal potential difference" V available for sending the current through R.

NOTE.—A current generator is said to be on "open circuit" when it is sending either *no* current or a *very small* one. An electrometer could be used to measure E and V. If either a mirror or tangent galvanometer is used, then the deflection simply, or the tangent of it respectively, will give E and V.

Apparatus.—High-resistance galvanometer, G (p. 267); cells to be tested, B; resistance box, R; reversing key, K_1 (p. 329), and ordinary key, K_2.

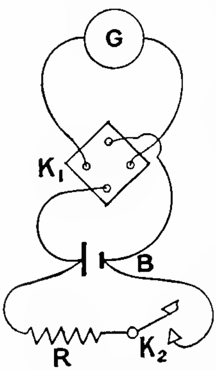


FIG. 35.

Observations.—(1) Connect up as indicated in Fig. 35, using a Leclanché cell, and adjust the pointer of G to zero.

(2) With K_2 open, take deflections each side of zero by turning K_1, and note the mean d_1.

(3) Close K_2, and adjust R (probably from 10 to 20 ohms), to give a sufficiently reduced deflection. Note the mean d_2 of the two sides as in (2), and the value of R.

(4) Open K_2, and repeat (2) and (3) for about six decreasing values of R.

(5) Repeat (2)–(4) with Daniell and bichromate cells, and tabulate as follows:—

Cell used.	Mean deflection.		R.	Battery resistance, $B = \frac{E - V}{V} \cdot R$	Mean, B ohms.
	d_1 or E.	d_2 or V.			

Inferences.—Prove the formula given in the table. On what does internal resistance of cells depend?

46. Internal Resistance of a Battery (Half-deflection Method).

Introduction.—The method is practically only available for testing the internal resistance of batteries having a fairly constant E.M.F. and considerable internal resistance, and it is merely a special application of Ohm's law. It is best to use a low-resistance galvanometer, which is very sensitive, and to shunt it so as to still further reduce the effective resistance between its terminals and also its sensitiveness.

Apparatus. —Sensitive low-resistance galvanometer, G; adjustable known resistance, R; key, K; and cells, B, to be tested. An adjustable known low resistance, S, to shunt G with if necessary.

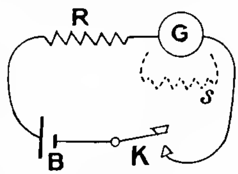


FIG. 36.

Observations.—(1) Connect up as in Fig. 36, and adjust the galvanometer needle to zero.

(2) Adjust R so that on pressing K a convenient steady deflection, d_1, is obtained up to the end of the scale. Note the values of R_1, d_1, and S (if used).

(3) Increase R to such a value, R_2, that the new deflection $d_2 = \frac{1}{2}d_1$.

(4) Repeat (2) and (3) for different pairs of deflections extending all down the scale.

(5) Repeat (2)–(4) for two other cells.

(6) Calculate the internal resistance of each cell from the relation—

$B = R_2 - 2R_1 - G$ when the galvanometer is unshunted,

$B = R_2 - 2R_1 - \dfrac{SG}{S+G}$ „ „ *is* shunted by shunt S

and tabulate your results as follows :—

Galvanometer used,		; resistance, G, =	ohms.		
Cell tested.	Resistances.		Shunt (if any), S.	B.	Mean, B ohms.
	R_1.	R_2.			

Inferences.—Prove the relation given in (6), and state any assumptions made in obtaining it.

47. Internal Resistance of a Battery (Equal-deflection Shunt Method).

Introduction.—The following, which is also known as Thomson's method of measuring the internal resistance of current generators, entails the use of a shunt either across the battery or galvanometer terminals, and is subject to the defects of current methods in general.

Apparatus.—Sensitive fairly low-resistance galvanometer, G; adjustable resistance, R, and shunt, S; spring tapping-keys, K_1 and K_2; cells, B, to be tested.

CASE I.—*Shunt used on Battery.*

Observations.—(1) Connect up as in Fig. 37, and adjust the galvanometer G to zero (about).

(2) Adjust S and R to some convenient values, so that on pressing K_1 and K_2 a suitable deflection, *d*, is obtained on G. Note the values of S and R.

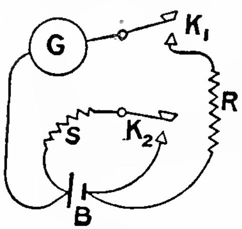


FIG. 37.

(3) Open K_2, and increase R to R_1, so as to again obtain the same deflection, *d*, with only K_1 closed, and note the value of R_1.

(4) Repeat (2) and (3) for about ten different deflections throughout, the scale differing by about equal amounts by altering the value of S.

(5) Repeat (2)–(4) with two other different types of cells.

(6) Calculate the internal resistance of the battery from the relation—

$$B = S\frac{R_1 - R}{R + G} \text{ ohms}$$

and tabulate as shown.

CASE II.—*Shunt used on Galvanometer.*

Observations.—(1) Connect up, placing S and K_2 across G instead of across B, as in Fig. 37, and adjust the galvanometer to zero (about).

(2) Adjust R and S to suitable values, so that on pressing K_1 and K_2 a convenient deflection, *d*, is obtained on G, and note the values of S and R.

(3) Open K_2, and increase R to R_1, so as to obtain the same deflection, *d*, again with only K_1 closed, and note the value of R_1.

(4) Repeat (2) and (3) for about ten values of *d*, differing by about equal amounts throughout the scale by altering S.

(5) Repeat (2)–(4) with two other different types of cells.

(6) Calculate the internal resistance of the battery from the relation—

$$B = S\frac{(R_1 - R)}{G} - R \text{ ohms}$$

and tabulate as follows :—

Galvanometer used, ; resistance, G, = ohms.

Cell used.	Resistances.		Shunt, S.	For reference, *d*.	B.	Mean, B.
	R.	R_1.				

Inferences.—Prove the formulæ given in (6), Cases I. and II., and state any assumptions made in obtaining them. State the merits and demerits of the two cases.

48. Measurement of the Internal Resistance of a Battery (Beetz's Method).

Introduction.—The following method has the advantage of not merely being a "zero" one, but of enabling the battery to be tested to be placed in circuit for an instant, thereby reducing to a minimum the tendency for the cell to polarize and cause irregular results.

Apparatus.—Sensitive low-resistance galvanometer, G ; battery, B, to be tested ; spring tapping-keys, K_1, K_2 ; two adjustable known resistances, r_1, r_2, or, in lieu of these, a low-resistance potentiometer, ABC ; cell, *b*, of fairly constant E.M.F., which is less than that of B.

Observations.—(1) Connect up as in Fig. 38, and adjust the galvanometer G to zero (about).

(2) Adjust r_1 and r_2 to some convenient values with, say, r_2 = about $\frac{2}{3}r_1$, depending on the ratio of the E.M.F.'s of b and B. Now close K_1, and the *next instant* tap K_2 for a moment. If G deflects, release both keys, and readjust r_1 and r_2 so that no deflection occurs on G, and note the values of r_1, r_2.

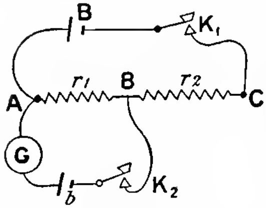


FIG. 38.

(3) Repeat (2) for about eight different values of r_1, r_2.

(4) Repeat (2) and (3) for two other cells inserted singly in place of B.

(5) Calculate the internal resistance of each cell from the relation—

$$B = \frac{r_1'(r_1 + r_2) - r_1(r_1' + r_2')}{r_1 - r_1'} \text{ ohms}$$

where $r_1 r_1'$ and $r_2 r_2'$ represent the values of AB and BC respectively in any two sets of balances obtained in (3) above.

Tabulate as follows:—

Cell tested.	r_1	r_2.	B.	Mean, B.

Inferences.—Prove the relation given in (5), and state any assumptions made in obtaining it.

49. Measurement of the Internal Resistance of a Battery (Mance's Method).

Introduction.—The method consists in measuring a resistance containing an E.M.F. by means of the Wheatstone bridge. Though a somewhat troublesome one to manipulate, it is nevertheless used to some extent in determining the resistance of cells, etc. It has the disadvantage that the effective E.M.F. of the cell varies, due to the alteration of current given by it when the

key is pressed. In one sense it has the advantage of being a zero method. An inconvenience inherent in the method arises from the spot of light in a sensitive reflecting galvanometer travelling about the scale as the galvanometer key is pressed or the arms of the bridge adjusted, and which arises from a redistribution of current in the arms and galvanometer branch. This frequently necessitates continually moving the controlling magnet in order to keep the spot of light about the middle of the scale, its most sensitive position. D'Umfreville has suggested replacing the galvanometer by the primary of two telescopic coils, and joining the galvanometer instead up in series with the secondary of the coils, thus causing it to indicate sudden changes in current, but making it unaffected by the actual magnitude of the current in the primary. Thus the spot always returns of its own accord to zero, and for the best results the secondary should have a resistance equal to that of the galvanometer. The present method cannot be applied to E.M.F.'s which polarize rapidly, but only to those which are fairly constant, owing to the impossibility of balancing quickly enough.

Apparatus.—Sensitive reflecting galvanometer, G; and either a P.O. Wheatstone bridge box or three adjustable resistance boxes, r_1, r_2, r_3; and a spring tapping-key, K.

Observations.—(1) Connect up as in Fig. 39, the right-hand diagram being the P.O. box arrangement, and the left-hand that of a symbolical Wheatstone bridge, with the three resistance boxes r_1, r_2, and r_3 forming three of the arms. Both diagrams are lettered similarly. Adjust the galvanometer to zero (about).

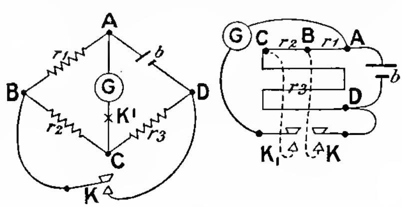


Fig. 39.

(2) Make r_1 small and r_2 large, and close K_1, adjusting the controlling magnet by placing it closer to the galvanometer needle, or turning it so as to bring the spot of light back to zero (about).

(3) Adjust r_3 so that on making and breaking with K (K_1 still being closed) no deflection of the spot of light is observed. Note the values of r_1, r_2, and r_3.

Note.—The control may have to be adjusted each time r_3 is

altered, so as to keep G at about zero, its most sensitive position. If on making and breaking at K, first a kick on one side of zero and then a creeping off to the other is observed, the former denotes "want of balance," the latter polarization of the E.M.F., due no doubt to the reduction of the resistance external to the E.M.F. on closing K. If the internal resistance is low, it will be best to "bank" it up by inserting a known resistance of 3 or 4 ohms in series with it, and then subtracting this from the final result. This will also have the good effect of tending to stop or reduce polarization.

(4) Repeat (2) and (3) with a different value for r_1, r_2, if possible.

(5) Repeat (2)–(4) with three or four different cells, and calculate the internal resistance from the relation—

$$b = \frac{r_1}{r_2} \cdot r_3 \text{ ohms}$$

and tabulate as follows :—

Galvanometer used, ; resistances = ohms.

Name of cell.	Resistances.			Resistances added (if any).	Internal resistance, b ohms.
	r_1.	r_2.	r_3.		

Inferences.—Prove the relation given in (5), and state any assumptions made in obtaining it.

50. Measurement of the Internal Resistance of a Thermo-electric Generator.

Introduction.—The electric generator to be tested is that designed by Mr. H. B. Cox for producing a difference of electric potential from difference of temperature maintained between certain parts of the generator. For fixed external conditions it furnishes a constant supply of electrical energy, and it has the advantage of having *no moving parts.* The maximum output of the generator under test is about 3½ amps. at 4·8 volts approximately.

Apparatus.—The generator, G (p. 341); ammeter, *a*; high-resistance voltmeter, *v*; carbon rheostat, R (p. 307); switch, S; key, K; voltmeter resistance, *r*.

Observations.—(1) Very slowly turn on the water until a gentle stream flows out at the waste pipe.

(2) Light the gas, and strictly adhere to the printed *directions for starting*, etc.

(3) Next couple up the above-mentioned apparatus as shown in Fig. 40, adjust the pointers of *v* and *a* to zero, and *r* to such a value that *v* will read about five volts for a full-scale deflection approximately.

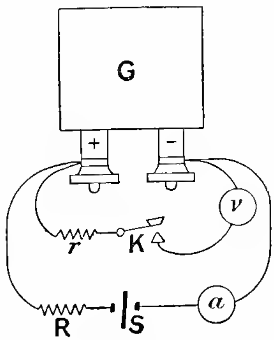


FIG. 40.

(4) Allow the gas to burn for about 12 minutes before starting the test, and increase R by unscrewing the carbon plates until they are quite loose.

(5) Close K, and note the reading E_1 of the voltmeter, which is therefore the E.M.F. of G on "open circuit."

(6) Next close S in addition to K, and adjust R so that *a* reads about 0·3 amp. Note its exact value A, and that of the voltmeter V_1, simultaneously; then open S.

(7) Repeat (5) and (6) for about ten equal increments of current up to the maximum, rising by about 0·3 at a time.

Tabulate as follows:—

$r =$ ohms; voltmeter resistance, v, $=$ ohms; $\frac{r+v}{v} =$

E_1.	E.M.F. of generator, $E = \frac{r+v}{v}E_1$	V_1.	P.D. at generator, $V = \frac{r+v}{v}V_1$	Current, A amps.	Internal resistance, $B = \frac{E-V}{A}$ ohms.

Inferences.—Explain how the formula for the internal resistance is arrived at.

51. Internal Resistance of a Battery (Condenser Method).

Introduction.—This method will be found convenient when the battery under test is liable to polarize rapidly. It has not, however, the advantages of a *null method*. There are two or three important points to note with regard to it. For accurate work the damping of the galvanometer should be small, and as it must be of the ballistic type, the discharge should be completed before the needle begins to move. In other words, the duration of the current must be very small compared with the period of vibration of the needle. Owing, however, to *residual absorption*, there is some uncertainty as to the time taken to discharge the condenser, and hence also as to whether this time is comparable with the period of the needle or not. Again, the polarization due to the flow of current into the condenser in charging may be sensible if the latter has a large capacity. It is therefore important to employ a condenser of as small a capacity as can be used to give a satisfactory deflection with the most delicate galvanometer available.

Apparatus.—Battery, B, to be tested; condenser, C (p. 354); charge and discharge key, K (p. 328); box of known resistance coils, S; ballistic mirror galvanometer, G (p. 285).

Observations.—(1) Connect up as shown in Fig. 41, and adjust the galvanometer needle to zero. Completely discharge the condenser by short-circuiting it.

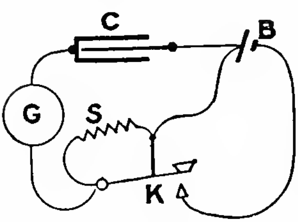


Fig. 41.

(2) Cut out S by taking out the infinity plug and thus breaking circuit; then, with the battery thus unshunted, note the first throw, d_1, of the spot of light on charging or discharging C, and completely discharge C.

(3) Insert the infinity plug and also a convenient resistance S ohms in the box. Note the throw d_2 on charging or discharging, and release K immediately. Completely discharge C.

(4) Repeat (2) and (3) for about six different values of S for each of the three cells tested, and calculate the internal resistance, B, of the cells from the formula—

$$B = S\frac{d_1 - d_2}{d_2} \text{ ohms}$$

(5) Repeat (2)–(4) with a larger capacity, and tabulate as follows:—

Name of cell.	Capacity used.	Throw, d_1.	Throw, d_2.	Shunt, S.	B ohms.	Mean, B.

NOTE.—The apparatus is connected up in such a way that the condenser and shunt are simultaneously disconnected from the battery, for with cells that polarize rapidly it is very important to shunt the battery *only at the moment of charging the condenser.* The best result will be obtained when S is not less than B or greater than 2B.

Inferences.—Prove the formula given in (4), and state what assumptions are made in deducing it.

52. Internal Resistance of a Battery (Electrometer Method).

Introduction.—This method is one of the best for determining the internal resistance of a current generator. Like the condenser method, the fall of potential in the battery is made to occur for a very short time, and thus errors due to polarization are reduced practically as low as they can be. In what follows it will be assumed that the operator is familiar with the construction and principle of the electrometer, almost any sensitive type of which will be suitable to use for the purpose (*vide* pp. 289–306).

The adjustments in general for any type will be obvious from a reference to those on p. 300 for the Thomson quadrant electrometer.

Apparatus.—Quadrant electrometer with its reversing key, K (p. 298), or, in lieu of this type, that described on p. 328; cells, B, to be tested; known resistance, R; electrophorus or Leyden jar

and Wimshurst machine for charging the needle of the electrometer.

Observations.—(1) Connect up either as in Fig. 42 or Fig. 50 (p. 100). In the latter case P can be dispensed with, and the cell connected direct to terminals *c* and *d*, with the resistance R across *a* and *b*.

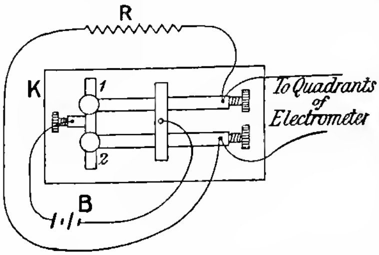


FIG. 42.

(2) With $R = \infty$, and the electrometer needle charged with a definite *indicated* potential, press K1, and note the steady deflection d_1. Then release K1 and press K2, noting the steady deflection d_1 on the opposite side of zero. These should be equal; if not, take the mean, calling it d_1.

N.B.—The potential of the needle must be kept constant.

(3) Insert a suitable resistance in R, and repeat (2), noting the value of R and the mean deflection d_2 on the electrometer.

(4) Repeat (2) and (3) for about eight different values of R.

(5) Repeat (2)–(4) for two other batteries.

(6) Calculate the internal resistance of each cell from the relation—

$$B = R\frac{d_1 - d_2}{d_2} \text{ ohms}$$

and tabulate as follows :—

Cell tested.	Mean deflections.		Shunt, R.	B.	Mean, B ohms.
	Unshunted, d_1.	Shunted, d_2.			

Inferences.—Prove the relation given in (6), and state any assumptions made in obtaining it.

53. Polarization of Cells (Experimental Investigation).

Introduction.—When a current generator, in the form of a primary cell or battery, is allowed to send a current, a phenomena known as "*polarization*" sets in; and if the resistance of the circuit through which the current flows remains constant, the current itself gradually falls in value, which fall is more rapid as the circuit resistance diminishes, or, in other words, as the current taken out of the cell increases. The effect is due to the decomposition (necessarily entailed) of the materials of the cell and the presence of free gases at the surface of the plates, thus setting up an influence of the nature of a recoil, or back E.M.F. as it is commonly termed. Some cells are particularly subject to this effect of polarization, and the object of this present investigation is to see the relative magnitudes of this effect in different types of cells.

Apparatus.—Low-resistance aperiodic galvanometer, G, or a tangent galvanometer will serve the purpose nearly as well; cells to be experimented upon; key, K.

Observations.—(1) Connect the above apparatus in simple series, using one of the cells, say the Daniell first, and adjust the galvanometer to zero.

(2) At a noted instant of time close K, and at once take the reading on G, which must be noted together with the time.

(3) Repeat (2) with K still closed, noting the deflection as it falls, at intervals of about one or two minutes, for about half an hour.

(4) Repeat (2) and (3) with a Leclanché and bichromate cell instead of the Daniell, and tabulate your results in a convenient form.

N.B.—The cells should all be good ones, and newly set up, in order to obtain a fair comparison. The intervals of time should be selected according to the conditions of the test and the cells in use, frequent readings being taken where the current varies rapidly, and so on. If a tangent galvanometer is used, the tangents of the deflections must be used.

(5) Plot curves for each cell on the same sheet of curve paper, having values of deflections as ordinates and time in minutes as abscissæ.

Inferences.—State clearly all the inferences which can be deduced from the results of your test.

54. Comparison of Electromotive Forces (Equal-resistance Method).

Introduction.—By means of this simple substitution method as it is sometimes called, E.M.F.'s may be compared by observing the relative currents they will send through a circuit of fixed resistance, which is very large compared with their own internal resistance. If the galvanometer used to indicate the currents is too sensitive, it may be shunted by any convenient resistance, providing the rest of the circuit has a high resistance compared with that of the cells. It is, however, preferable to make the resistance of the rest of the circuit high instead of using a shunt.

Apparatus.—Galvanometer, G; cells to be compared, E_1 and E_2; either a two- or three-way key, K; resistance box, R; resistance, S, to be used as a shunt to G if required.

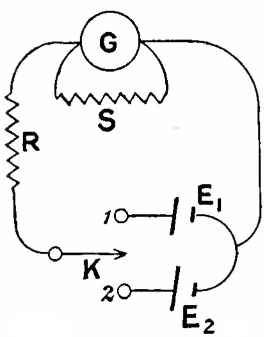


Fig. 43.

Observations.—(1) Connect up as indicated in Fig. 43, and adjust the galvanometer needle to zero.

(2) K being open, adjust R to a suitable high value; *the higher it is the more accurate will be the result.*

(3) Switch on to the cell of highest E.M.F. E_1, and adjust R, and if necessary S, so as to obtain about a quarter-scale deflection, d_1, on G. Note this, and, for reference only, the values of R and S (if any).

(4) With R and S as before, switch on to the next cell of E.M.F. E_2, and note the deflection d_2.

(5) Repeat (2)-(4) for a half, three-quarter, and full-scale deflection approximately.

(6) Compare the Daniell, Leclanché, and bichromate or other cells in this way, obtaining the results of (2)–(5), with each of the three possible combinations of the three cells taken, say two at a time.

(7) Compare the E.M.F.'s, and calculate one of them by assuming the other as a standard known E.M.F., from the relation—

$$E_1 : E_2 = d_1 : d_2$$

Note.—If a tangent galvanometer is used, tan $d_1°$ and tan $d_2°$ must be used in the above relation instead of d_1 and d_2 respectively, etc., and these latter should have equal values on either side of 45° for maximum accuracy.

Tabulate as follows :—

Galvanometer used, ; resistance = ohms ; known E.M.F., say E_1, = volts.

Names of cells of E.M.F.'s.		Deflections.		For reference only, R.	$\frac{E_1}{E_2} = \frac{d_1}{d_2}$	Unknown E.M.F., E_2 volts.
E_1.	E_2.	d_1.	d_2.			

Inferences.—Prove the relation given in (7), and state any assumptions made in obtaining it. Is the method open to any particular objections?

55. Comparison of Electromotive Forces (Equal-deflection Method).

Introduction.—By means of the following simple substitution method, E.M.F.'s may be compared by the resistances through which they will send equal currents, and such resistances must be very large compared with the galvanometer and battery resistances together. If the galvanometer is too sensitive, it may be shunted, as this will still further tend to diminish the effective resistance between its terminals.

Apparatus.—Identically the same as in the last experiment.

Observations.—(1) Connect up as in Fig. 44, and adjust the galvanometer to zero.

(2) K being open, adjust R to a suitable high value; *the higher it is the more accurate will be the result.*

(3) Switch on to the cell of highest E.M.F. E_1, and adjust R, and if necessary S, so as to obtain about a quarter-scale deflection d on G. Note the value of R_1 and of d and S (if any), for reference only.

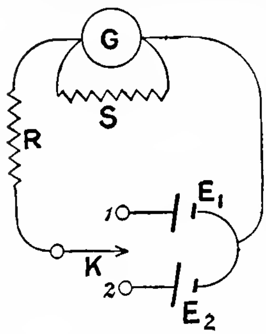


FIG. 44.

(4) With S unaltered, switch on to the next cell of E.M.F. E_2, and alter R so as to reproduce the deflection d again. Note the value of resistance R_2 to do this.

(5) Repeat (2)–(4) for a half, three-quarter, and full-scale deflection approximately.

(6) Compare the Daniell, Leclanché, and bichromate or other cells in this way, obtaining the results of (2)–(5), with each of the three possible combinations of the three cells taken, say two at a time.

(7) Compare the E.M.F.'s, and calculate one of them by assuming the other known from the relation—

$$E_1 : E_2 = R_1 : R_2$$

and tabulate as follows :—

Galvanometer used, ; resistance = ohms ; known E.M.F., say E_1, = volts.

Names of cells of E.M.F.'s.		Resistances.		For reference only, d.	$\frac{E_1}{E_2} = \frac{R_1}{R_2}$	Unknown E.M.F., E_2 volts.
E_1.	E_2.	R_1.	R_2.			

Inferences.—Prove the relation given in (7), and state any assumptions made in obtaining it. Is the method open to any particular objections? If so, state them.

56. Comparison of Electromotive Forces (Wiedemann's Method).

Introduction.—This is sometimes called the "sum and difference" method, and consists in determining the relative

strengths of currents sent through a fixed resistance, first, when the E.M.F.'s are in series assisting each other; and secondly, in series opposing one another.

Apparatus.—Sensitive fairly low-resistance galvanometer, G; high adjustable resistance, R; reversing key, K (p. 329); the two E.M.F.'s, E_1 and E_2, to be compared.

Observations.—(1) Connect up as in Fig. 45, and adjust the galvanometer to zero.

(2) With R at its highest, turn K so as to join the smaller E.M.F. E_2 in series with and *assisting* E_1; then reduce R to such a value as will give a full-scale deflection d_1 on G. Note this deflection d_1 and the value of R (for reference only).

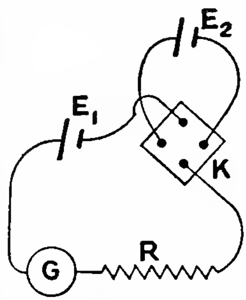


FIG. 45.

(3) With R as before, turn K through 90°, so as to place the smaller E.M.F. E_2 in series with but *opposing* E_1, and note the deflection d_2.

(4) Repeat (2) and (3) for three widely different and small deflections d_1 by suitably increasing R, but d_1 must not be reduced to such an extent that d_2 is too small to read accurately.

(5) Repeat (2)–(4) with the Daniell, Leclanché, and bichromate or other cells, taken two at a time for the three possible combinations of the three cells.

(6) Compare the E.M.F.'s, and calculate one of them by assuming the other known from the relation—

$$\frac{E_1}{E_2} = \frac{d_1 + d_2}{d_1 - d_2}$$

NOTE.—If a "tangent" or "sine" galvanometer is used, the tangent or sines of the deflections respectively must be used instead of simply d_1 and d_2.

Tabulate as follows:—

Galvanometer used, ; resistance = ohms; known E.M.F., say E_1, = volts.

Names of cells of E.M.F.'s.		Deflections.		For reference only, R.	$\frac{E_1}{E_2} = \frac{d_1 + d_2}{d_1 - d_2}$	Unknown E.M.F., E_2 volts.
E_1.	E_2.	Assisting, d_1.	Opposing, d_2.			

Inferences.—Prove the relation given in (6), and state any assumptions made in obtaining it. State the advantages and disadvantages of the method.

57. Comparison of Electromotive Forces (Wheatstone's Method).

Introduction.—This method consists in determining the resistances through which the cells will give the same pair of deflections. It is a simple and convenient one.

Apparatus.—Sensitive galvanometer, G; two-way key, K; an adjustable high resistance, R; the two E.M.F.'s, E_1 and E_2, to be compared.

Observations.—(1) Connect up as in Fig. 46, and adjust the galvanometer to zero (about).

(2) Adjust the resistance to such a value R_0 that a full-scale deflection d_0 is obtained on closing K1. Note the values of R_0 and d_0.

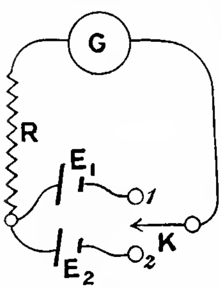


Fig. 46.

(3) Increase this resistance, say to double the amount, namely, R, and note the diminished deflection d and the value of R.

(4) Now close K2, adjusting the resistance to a value r_0, so as to reproduce the original deflection d_0.

(5) Increase this resistance to an amount r, so as to again reproduce the other deflection d.

(6) Repeat (2)–(5) for three widely different pairs of deflections by suitably adjusting the resistance.

(7) Repeat (2)–(6) for the Daniell, Leclanché, and bichromate or other cells, obtaining results with each of the three possible combinations of the three cells taken two at a time.

(8) Compare the E.M.F.'s, and calculate one of them, assuming the other known from the relation—

$$\frac{E_1}{E_2} = \frac{R - R_0}{r - r_0}$$

and tabulate as follows :—

Galvanometer used, ; resistance = ohms; known E.M.F., say E_1, = volts.

Names of cells of E.M.F.'s.		Deflections.		Resistances.		Resistances.		$\frac{E_1}{E_2} = \frac{R - R_0}{r - r_0}$	Unknown E.M.F., E_2 volts.
E_1.	E_2.	d_0.	d.	R.	R_0.	r.	r_0.		

Inferences.—Prove the relation given in (8), and state any assumptions made in obtaining it. What advantages or disadvantages does the method possess?

58. Comparison of Electromotive Forces (Lumsden's Method).

Introduction.—The method is variously known as Lumsden's, or Lacoine's, or Bosscha's, but the first name is applied here, as it is generally known by that in this country.

The method is a simple, convenient, and accurate one, and has the great advantage of being a "zero" method, *i.e.* one in which no deflection has to be noted. It is not, however, a "null" method in the sense of the word in which the author prefers to use the term, in that the *cells are sending a current* continuously even at the moment of balance, and hence there is the possibility of errors due to polarization unless the circuit resistance is high. The galvanometer and other resistances should be high, so as to make the internal resistance of the cells negligible compared with them, and the polarization effect practically *nil.*

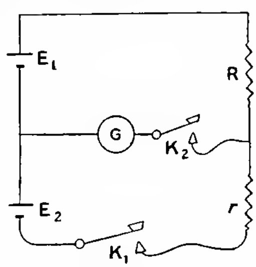


FIG. 47.

Apparatus.—Sensitive high-resistance galvanometer, G; cells of E.M.F.'s, E_1 and E_2, to be compared; two spring tapping-keys, K_1, K_2, and two high-resistance boxes, r and R; a single P.O. box can be made to do instead, if necessary.

Observations.—(1) Connect up as indicated in Fig. 47, the two cells being in series with the two resistances r and R, and *helping one another.*

(2) Adjust R to some suitable high value, and vary r so that on pressing K_1 first, and then K_2, no deflection occurs on G.

(3) Repeat (2) for about six or eight different values of r and R, and compare the E.M.F.'s, calculating one by assuming the other known from the relation—

$$E_1 : E_2 = R : r$$

(4) Repeat (2) and (3) for each pair or combination formed from the Daniell, Leclanché, and bichromate or other cells, and see whether the product of any two of the mean ratios equals the third ratio. Tabulate your results as follows :—

Galvanometer used, ; resistance = ohms ; known E.M.F., say E_1, = volts.

Names of cells of E.M.F.'s.		Resistances.		$\frac{E_1}{E_2} = \frac{R}{r}$	Unknown E.M.F., E_2 volts.
E_1.	E_2.	R.	r.		

Inferences.—Prove the relation given in (3), and state any assumptions made in deducing it.

59. Comparison of Electromotive Forces (Clark-Poggendorff Method).

Introduction.—The principle of this method was originally due to Poggendorff, who balanced the E.M.F.'s directly against one another by altering resistances. The present modification is due to Latimer Clark, who introduced a separate "*working*" battery against known fractions, of which the E.M.F.'s to be compared are balanced. This arrangement has been used by Lord Rayleigh for comparing the E.M.F.'s of standard cells. The method is a valuable one, and possesses the great advantage over almost all others in being, not only a "*zero*," but also a "*null*" method, *i.e.* while the E.M.F.'s are being compared neither can possibly polarize, as no current passes through them. This is indicated by a sensitive reflecting galvanometer, G, in series with them; hence the terminal P.D. of the cell will be its E.M.F. If two resistance boxes, AB and BC, are joined in series, and a constant P.D., E (greater than either of the E.M.F.'s E_1 and E_2 to be compared), maintained at the extremities A and C of

the combination by an independent battery, then ABC is merely a high-resistance potentiometer wire, and there will be a uniform fall of potential from A to C. Hence it must be possible to find a point B between A and C such that the P.D. between A and $B = E_1$ or E_2. Then, on pressing one side or other of the two-way key K, there will be no deflection on G, and if the total resistance $r_1 + r_2$ is kept constant, r_1 will be proportional to E_1. The method is quite independent of the internal resistance of the cells, and its sensitiveness is far greater than could be obtained with any electrometer or other method.

Apparatus.—High-resistance sensitive galvanometer, G; two-way spring tapping-key, K; working battery of two or more cells, E; adjustable high resistances, r_1 and r_2; E.M.F.'s, E_1 and E_2, to be compared.

Observations.—(1) Connect up as indicated in Fig. 48, joining all *like* poles (positive), say to A, and adjust the spot of light to about zero.

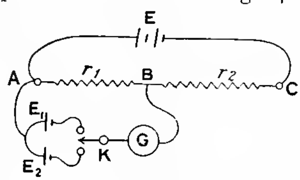


FIG. 48.

(2) Always maintain $r_1 + r_2$ a constant and = 10,000 ohms, say; then suitably adjust each so that on switching K to E_1 no deflection occurs. Note the values of r_1 and r_2. Repeat this for E_2, calling the new value of r_1, r_1'.

N.B.—To see whether the working E.M.F. has altered, repeat with the first cell again. If it has a new value of r_1 will be obtained, but the *mean* of this and the first value must be used.

(3) Repeat (2) for some six different values of $r_1 + r_2$, and compare the E.M.F.'s from the relation—

$$E_1 : E_2 = r_1 : r_1'$$

(4) Repeat (2) and (3), using the Daniell, Leclanché, and bichromate or other cells in pairs for E_1 and E_2, and tabulate as follows:—

Galvanometer used, ; resistance = ohms; known E.M.F., say E_1, = volts.

Names of cells of E.M.F.'s.		Resistances.			$\frac{E_1}{E_2} = \frac{r_1}{r_1'}$.	Unknown E.M.F., E_2 volts.
E_1.	E_2.	r_1.	r_1'	$r_1 + r_2$		

NOTE.—If AB and BC are two similar but separate resistance boxes, care should be taken not to mix the plugs, but to place them in the lids of their respective boxes.

Inferences.—Prove the relation given in (3), and state any assumptions made in obtaining it.

60. Comparison of Electromotive Forces (Condenser or Ballistic Method).

Introduction.—The ordinary galvanometric methods of comparing E.M.F.'s cannot be employed to obtain accurate results when the cells to be compared polarize at all rapidly. In such cases some *null* method, such as Clark's, is preferable. The following one (sometimes known as Law's method) can be conveniently employed when a condenser and ballistic galvanometer are available. The condenser should have as small a capacity as possible, but one that will give a satisfactory deflection with the most sensitive arrangement of the galvanometer available. This arises from the fact that if the capacity was larger, the larger current necessary to fully charge the condenser might result in an appreciable polarization. If one E.M.F., say E_1, is much larger than the other, it may be necessary to shunt the galvanometer G with a shunt S when this larger E.M.F. is being used (*vide* p. 113), so as to keep the respective throws much about the same in magnitude.

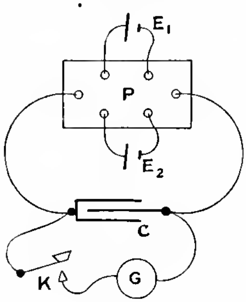


FIG. 49.

Apparatus.—Ballistic galvanometer, G (p. 285); condenser, C; Pohl's commutator, P (p. 330); batteries of E.M.F.'s, E_1 and E_2, to be compared; key, K.

Observations.—(1) Connect up as indicated in Fig. 49, and adjust spot of light to zero.

(2) With K *open*, free the condenser from any residual charge by short-circuiting its terminals; then turn P over to the *weakest* E.M.F., E_1, for a few seconds.

(3) Turn P so as to *open the battery circuit*, and quickly press K,

the spot of light being absolutely at rest. Note the first throw d_1 of the spot.

(4) Repeat (2) and (3) two or three times, and take the mean.

(5) With K *open*, free C from any residual charge, and turn P over to the other E.M.F., E_2, and repeat (3) and (4), noting the mean first throw d_2.

(6) Repeat (2)–(5) for about six different throws by altering C if possible, and compare the E.M.F.'s, calculating one of them by assuming the other known from the relation $E_1 : E_2 = d_1 : d_2$ for no shunt, or $E_1 : E_2 = \frac{S + G}{S} d_1 : d_2$ when a shunt is used as above.

(7) Repeat (2)–(6), using Daniell's, Leclanché, and bichromate or other cells in pairs, giving three possible combinations, and tabulate as follows [*Caution.—Never charge* C *with* K *depressed*]:—

Galvanometer used, ; resistance, G, = ohms; known E.M.F., say E_1, = volts.

Names of cells of E.M.F.'s.		Mean first throws.		Shunt, if used, S.	$\frac{E_1}{E_2}$	Unknown E.M.F., E_2 volts.
E_1.	E_2.	d_1.	d_2.			

Inferences.—Prove the relation given in (6), and state what assumptions are made in obtaining it. State why the weakest E.M.F. should be used first.

61. Comparison of Electromotive Forces (Electrometer Method).

Introduction.—This method necessitates the use of some form of electrometer, and with it the expenditure of much more time and care than in any of the preceding methods. Though a deflection method, it possesses the great advantage that *no current* is taken from the cells during the test, and consequently there can be no polarization whatever. In this particular instance, though almost any sensitive form of electrometer will do, we will assume the use of the Kelvin quadrant form (for a description of which see p. 290), and that it is charged ready for use in the manner described on p. 300.

Apparatus.—Kelvin quadrant electrometer, E, with its reversing key, K; high-insulation Pohl's commutator, P (p. 330); E.M.F.'s, E_1 and E_2, to be compared; either an electrophorus or Leyden jar and Wimshurst machine for charging the needle *n* of E.

Observations.—(1) Connect up as in Fig. 50, where *qq* are the terminals of the two pairs of quadrants; *n* that of the needle; *i* the induction-plate electrode; e_1 a terminal on the case of E, and therefore to earth; *r* the "replenisher."

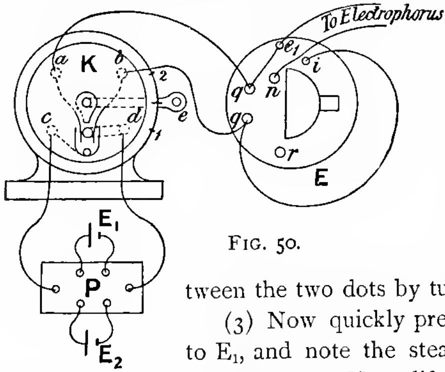


Fig. 50.

(2) With the lever *e* of K at O, obtain the final exact adjustment of the hair midway between the two dots by turning the replenisher *r*.

(3) Now quickly press *e* to 1, P being over to E_1, and note the steady deflection d_1 on the electrometer. Now lift *e* to 2, and note the deflection on the opposite side of zero. These should be equal; if not, take the mean and call it d_1, and leave *e* at the O position.

N.B.—The hair must be maintained midway between the dots.

(4) Repeat (2) and (3) for P over to E_2, obtaining a mean deflection d_2.

(5) Repeat (2)-(4) about four times with each pair of cells compared.

(6) Repeat (2)-(5) with the Daniell, bichromate, and Leclanché or other cells taken in pairs, giving the three possible combinations, with three cells taken two at a time.

(7) Compare the E.M.F.'s, and calculate one of them, assuming the other known, from the relation—

$$E_1 : E_2 = d_1 : d_2$$

and tabulate as follows:—

Electrometer used, ; known E.M.F., say E_1, = volts.

Names of cells of E.M.F.'s.		Deflections.			Deflections.			$\frac{E_1}{E_2} = \frac{d_1}{d_2}$	Unknown E.M.F., E_2 volts.
E_1.	E_2.	Right, d_1.	Left, d_1.	Mean, d_1.	Right d_2.	Left, d_2.	Mean, d_2.		

Inferences.—Prove the relation given in (7), and state any assumptions made in obtaining it.

62. Comparison of the E.M.F.'s of Standard Cells.

General Remarks.—In the case of very accurate standardizing work, it is desirable to employ more than one standard cell on the work, in order to obtain a complete check on the results obtained with a particular cell. In such cases two or more such cells are usually to hand, and it is advisable to compare their E.M.F.'s with one another in a separate test. On reviewing the previous methods already given for the comparison of E.M.F.'s, a little consideration will at once show that practically only those of "Clark-Poggendorff," "Beetz," the ballistic, and the electrometer methods are at all suitable for the comparison of the E.M.F.'s of standard cells when an accurate result is required. This will be evident when we remember that, in general, all standard cells are liable to very rapid polarization when anything but an extremely minute current is taken out of them; and nothing greater than this should ever be taken if it is desired to keep the cells as *accurate known* standards of E.M.F. In the next place, such cells have usually a very *high internal resistance*, and if this is not eliminated, it must be accurately known and allowed for, which again introduces difficulties. The question of internal resistance can only be eliminated when the cell is on either *open circuit* or an *extremely high external resistance*, when such internal resistance becomes entirely negligible compared with that of the rest of the circuit. Thus the Clark-Poggendorff and Beetz methods are by far the best for this purpose, for not only are they "*null*" methods in the sense that on "balance" absolutely no current flows through the cell, but they are also "*zero*" methods, *i.e.* ones in which *no deflection* on the galvanometer is the condition to be obtained, which allows any kind of galvanometer to be used as long as it is sensitive enough. The electrometer and ballistic methods entail the measurement of deflections, and are therefore not so convenient; and the last-named is open to a further objection, namely, that

unless the capacity of the condenser used is small, such a quantity of electricity may be required to charge the condenser as will suffice to appreciably polarize the cell. The electrometer method is better in this respect, as the cell is on *open circuit.*

63. Measurement of the E.M.F. of Standard Cells (Null-zero Method).

Introduction.—The method, which is a slight modification of the Clark-Poggendorff one for comparing E.M.F.'s, is a convenient and, with sufficient care, an accurate method of measuring the E.M.F.'s of standard cells such as Clark's. In general, this type of cell has a high internal resistance, and in addition polarizes very rapidly when allowed to send anything but an infinitesimally small current, and it should never be allowed to send more than this. Hence the present method has the all-important advantages that, being a "*null*" one, the cell is not required to send any current at all, thereby enabling its E.M.F. to be very accurately measured, whilst entirely eliminating polarization and the effect of high internal resistance. The method consists in measuring the P.D. at the terminals of a known standard low resistance, through which is passed a known measured steady current.

Apparatus.—Two high-resistance boxes, R_1R_2 and r_1r_2, or, in lieu of these, a fairly high-resistance potentiometer arrangement, or a Thomson-Varley slide resistance (p. 321); potentiometer "working battery," V, consisting of one secondary cell; sensitive high-resistance galvanometer, G; two-way spring tapping-key, K; Clark's standard cell, C, to be tested; known standard low resistance, R, capable of carrying 1 amp. without sensible heating (p. 311); main secondary battery, B (p. 338); adjustable resistance, *r* (p. 307); switch, S; Kelvin standard centi-ampere balance, A, or other standard current measurer (p. 274); mercury switch, *m*; complete "Wheatstone bridge set," WB, for measuring R accurately.

N.B.—Considerable care should be taken that *all contacts* are *clean* and *good*, and that the plugs in each of the resistance boxes belong to their respective boxes, and are *clean.* Also that the

connections of WB and m to R are of very low or negligible resistance compared with that of R.

Observations.—(1) Connect up as in Fig. 51, and adjust both G and A to zero, the former roughly, and the latter accurately, seeing that its constant is so arranged as to make it read 1 amp. for a full-scale deflection.

(2) Make PQ = QT = 5000 ohms, say, and R = 1 ohm, and the rheostat r a maximum.

(3) With K and S open, close m, and accurately measure the value of R, calling it R_0; then open m, leaving the WB arms unaltered.

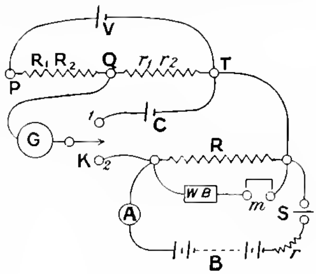


FIG. 51.

(4) With K and m *open*, close S, and adjust the current to 1 amp. by means of r, and great care must be taken to keep it constant.

(5) In manipulating K, *tap gently*, and *only for an instant* at first; then adjust PQ and QT, so that on closing K1, G does not deflect, PT still being maintained at 10,000 ohms. Note the value r_1 of QT.

(6) Adjust PQ and QT, so that on closing K2, G does not deflect, with PT still equalling 10,000 ohms, and note the value r_2 of QT.

(7) Repeat (4)-(6) for currents A = $\frac{3}{4}$, $\frac{1}{2}$, and $\frac{1}{4}$ amp. respectively.

(8) Calculate for each the E.M.F. of the standard cell C from the relation $E_c = \frac{r_1}{r_2} . AR$ volts, and tabulate as follows :—

Standard cell, ; No. ; (PQ + QT) = ohms ; E.M.F., V, = volts approximately.

Temperature of		Resistance of R when		Reading of A.		r_1.	r_2.	$E_c = \frac{r_1}{r_2} . AR$
Room.	Cell, C.	Cold, R_0.	Warm.	Deflection.	True amp.			

Inferences.—Prove the relation given in (8), and state any assumptions made in obtaining it.

Notes.—The following is the correction for variation of E.M.F. with temperature for any temperature $t°$ C. :—

For Board of Trade or ordinary Clark's cells, E.M.F. = $1·434\{1 - 0·00078(t° - 15°)\}$, legal volts.

For Carhart-Clark cells, E.M.F. = $1·434\{1 - 0·00038(t° - 15°)\}$, legal volts.

Corrections.—If R is subject to heating, its resistance when warm must be obtained and used in finding E_C. To do this, let the current used, A, flow for a sufficient time to allow R to gain a constant temperature, t_0; then suddenly open K and S, and close *in quickly*, balancing the WB. Note the resistance Rt_1 so obtained, and the number of seconds t_1 *after the "break,"* when this balance was accomplished. Repeat this for a longer and shorter time if possible. Then, if Rt_1, Rt_2, . . . are the resistances of R at the times t_1, t_2, . . . secs. after the break, the *resistance "warm"* at the instant of the break is found as indicated in Fig. 52, and is equal to the ordinate NQ.

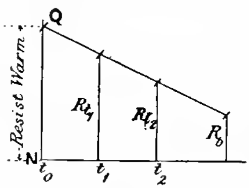


Fig. 52.

64. Effect of Size and Distance between the Plates of a Cell on its E.M.F. and Internal Resistance.

Introduction.—Relative measurements of different E.M.F.'s may be made by employing a galvanometer of high resistance compared with that of the source of E.M.F., and one in which the scale deflections are proportional to the current-strengths, as, for instance, in a tangent, D'Arsonval, or mirror galvanometer respectively. Internal resistance may be conveniently measured by what may be termed the "fall of potential" method (*vide* p. 77), using such a galvanometer as the above.

Apparatus.—Reversing switch, K_1; high-resistance galvanometer, G; variable known resistance, R; key, K_2; cell, B, to be tested, consisting of a Daniell's cell, so arranged that the plates

can be raised or lowered in the liquid, and also placed at various distances apart (*vide* p. 339).

Observations.—(1) Adjust the galvanometer needle to zero, and place the plates as near together and as deep in their liquids as possible. With K_2 unclosed and manipulating K_1, obtain deflections on both sides of zero. Denote the mean of these two by d_1.

(2) Repeat (1) with two-thirds and one-third of the plates immersed and the tips just in respectively.

(3) Separate the plates to one-third of the total length of cell, and repeat (1) and (2) for this new position.

(4) Repeat (3) with the plates two-thirds and the maximum distance apart.

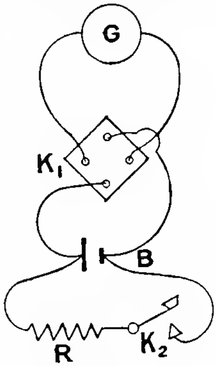


FIG. 53.

(5) Adjust R to such a value that on pressing K_2 an appreciably smaller deflection is obtained on G than hitherto. Now note the *mean* deflection d_2 on pressing K_2 for each of the positions of the plates mentioned in (1)–(4).

(6) Calculate the internal resistance of the cell for each position from the formula—

$$B = \frac{d_1 - d_2}{d_2} \cdot R \text{ ohms}$$

N.B.—If a tangent galvanometer is being used for G, and the deflections are in degrees, the tangents of these must be taken.

Tabulate as follows :—

Amount of plates immersed.	Distance between plates.	E.M.F. (mean), d_1.	Potential difference (mean), d_2	R ohms.	B ohms.

Inferences.—Write out carefully all the inferences which can be drawn from the above experimental results, and point out their bearing on the construction of cells intended for large currents. Prove the formula given in (6), and state what assumptions are made in deducing it.

65. Determination of the Relation between E.M.F. and Temperature of a Thermo-electric Element.

Introduction.—Whenever contact is established between two dissimilar metals, an electric P.D. is set up between their "free" extremities. Thus, if a rod of some metal, such as bismuth, for instance, has two copper wires soldered to its ends, which are at different temperatures, a P.D. is set up between the free ends of the copper wires, which will produce a current whenever they come in contact. Such an arrangement as this is commonly called a "*thermo-couple*," and a series of such, in which the two sets of alternate junctions are at different temperatures, constitutes a "*thermo-pile*." The magnitude of this effect depends on the difference of temperature of such junctions, and the present experiment is arranged with the object of investigating the relation between the two.

Apparatus.— Thermo-couple to be experimented upon (p. 339); sensitive galvanometer of fairly large resistance (p. 280); and a resistance box.

Observations.—(1) Connect this apparatus in simple series, and adjust the galvanometer to zero.

(2) With both copper cans nearly full of *cold* water, heat up the right-hand one to boiling point, as shown by its thermometer, the temperature of the other being kept at that of the cold water, and as constant as possible.

(3) Adjust the resistance so as to obtain a full-scale deflection on the galvanometer when the water in the right-hand can boils freely. Note this deflection d, which is proportional to the E.M.F. of the thermo-couple and the temperatures t_1 of the hot and t_2 of the cold water.

(4) Remove the flame, and take simultaneous readings of t_1, t_2, and d every 5° C. down to the lowest temperature obtainable, keeping the circuit resistance constant all the time, and tabulate as follows:—

Thermo-couple tested, ; resistance, b, = ohms ; galvanometer, g, = ohms ; box resistance, R, = ohms.

Total circuit resistance, $b + g + R$.	E.M.F. proportional to d.	t_1° C.	t_2° C	$(t_1 - t_2)$° C.	Values of $\frac{d}{(t_1 - t_2)}$	E.M.F. of couple in micro-volts.

NOTE.—The water in each can should be stirred frequently to ensure uniformity of temperature.

(5) From the total circuit resistance, deflections, and the "figure of merit" (p. 28) of the galvanometer, calculate the E.M.F. of the thermo-couple in micro-volts.

(6) Plot a curve having values of d or E.M.F. as ordinates, and values of $(t_1 - t_2)$ as abscissæ.

Inferences.—State clearly all that can be inferred from the results of your observations. Why should the circuit have a high resistance preferably?

66. Preparation of the Clark Standard Cell.

Definition of the Cell.—The cell consists of zinc and mercury in a saturated solution of zinc sulphate and mercurous sulphate in water, prepared with mercurous sulphate in excess, and is conveniently contained in a cylindrical glass vessel.

Preparation of Materials.—*The Mercury.*—To secure purity it should be first treated with acid in the usual way, and subsequently distilled in vacuum.

The Zinc.—Take a portion of a rod of pure zinc, and solder to one end a piece of copper wire. Clean the whole with glass paper, carefully removing any loose pieces of zinc. Just before making up the cell, dip the zinc into dilute sulphuric acid, wash with distilled water, and dry with a clean cloth or filter paper.

The Zinc Sulphate Solution.—Prepare a saturated solution of pure (recrystallized) zinc sulphate by mixing in a flask distilled water with nearly twice its weight of crystals of pure zinc sulphate, and adding a little zinc carbonate, in the proportion of about 2 per cent. by weight of zinc sulphate crystals, to neutralize any free

acid. The whole of the crystals should be dissolved with the aid of *gentle heat*, i.e. not greater than 30° C., and the solution filtered while still warm into a stock bottle. Crystals should form as it cools.

The Mercurous Sulphate.—Take mercurous sulphate sold as pure, which is white, and wash it thoroughly with cold distilled water by agitation in a flask; drain off the water, and repeat the process at least twice, but after the last washing, drain off as much water as possible. Mix the washed sulphate, in the proportion of about 12 per cent. by weight of $ZnSO_4$, crystals with the zinc sulphate solution, adding sufficient crystals of zinc sulphate from the *stock bottle* to ensure saturation, and a small quantity of pure mercury. Shake them well up together to form a paste of the consistency of cream. Heat the paste sufficiently to dissolve the crystals, but *not above* 30° C. Keep the paste for 1 hour at this temperature, agitating it from time to time, and then allow it to cool.

Crystals of zinc sulphate should then be distinctly visible throughout the mass. If this is not the case, add more crystals from the stock bottle, and repeat the process. This method ensures the formation of a saturated solution of zinc and mercurous sulphates in water. The presence of the free mercury throughout the paste preserves the basicity of the salt, and is of the *utmost importance*. Contact is made with the mercury by means of a platinum wire about No. 22 B.W.G., which is prevented making contact with the other materials of the cell by being sealed into a glass tube, the ends of the wire projecting beyond those of the tube. One end forms the terminal; the other end, and part of the glass tube, dip into the mercury.

To set up the Cell.—The cell may be conveniently set up in a small test tube of about 2 cms. in diameter and 6 or 7 cms. deep.

Place the mercury in the bottom of this tube, filling it to a depth of say 1·5 cms.

Cut a cork about 0·5 cm. thick to fit the tube. At one side of the cork bore a hole through which the zinc rod can pass tightly; at the other side bore another hole for the glass tube which covers the platinum. At the edge of the cork cut a nick through which the air can pass when the cork is pushed into the tube.

Pass the zinc rod about 1 cm. through the cork. Carefully clean the glass tube and platinum wire, then heat the exposed end of the wire red hot, and insert it in the mercury in the test tube, taking care that the whole of the exposed platinum is covered.

Shake up the paste, and introduce it without contact with the upper part of the sides of the test tube, filling the tube above the mercury to a depth of rather more than 2 cms.

Now insert the cork and zinc rod, allowing the glass tube to pass through the hole in the cork made for it.

Push the cork gently down until its lower surface is nearly in contact with the liquid. The air will thus be nearly all expelled, and the cell should be left in this condition for at least 24 hours before sealing, which should be done in the following way :—

Melt some marine glue until it is fluid enough to pour by its own weight into the test tube above the cork, using enough to cover completely the zinc and soldering. The glass tube should project above the top of the marine glue.

The cell thus set up may be mounted in any desirable way ; do it so that the cell is immersed in a water-bath up to the level say of the upper surface of the cork. Its temperature can then be determined more accurately than is possible when the cell is in air.

67. Action of Shunts on Galvanometer Deflections.

Introduction.—It is often the case that a certain galvanometer G, which is the most suitable one to use, happens to be too sensitive, and that the current which it is desired to measure produces a deflection quite off the scale. If the galvanometer is of the suspended needle type its sensibility might perhaps be sufficiently reduced by bringing the controlling magnet close to the needle. The best method, however, is to connect the galvanometer terminals by a suitable resistance, which allows a certain fraction only of the main current to go through G, the rest through the

"by-pass" or "shunt" circuit, as it is called. It is sometimes the case that the insertion of a shunt so reduces the resistance between the terminals of G, and therefore that of the circuit, that no effect is produced on the deflection owing to increase of total current. Thus the precise action of shunts on galvanometer deflections for different conditions of the circuit is a most important matter. The more elaborate forms of galvanometers are provided with shunt boxes fitted usually with three resistances respectively, $\frac{1}{9}$, $\frac{1}{99}$, and $\frac{1}{999}$, that of G thus making it possible to send only $\frac{1}{10}$, $\frac{1}{100}$, and $\frac{1}{1000}$ of the total current through G respectively. If $s =$ the resistance of the shunt, and g that of the galvanometer, then $\frac{s+g}{s}$ is called the "*multiplying power*" of the shunt, and is the amount by which the galvanometer current must be multiplied in order to obtain the total current in the main circuit.

Apparatus.—Galvanometer, G; resistance box, R; battery, B, of one or more cells; key, K; resistance box, S, for use as a shunt; plug or other similar key, K_1.

Observations.—(1) Connect up as indicated in Fig. 54, and adjust the galvanometer needle to zero, using one cell.

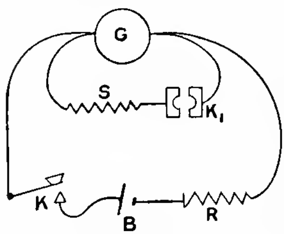


FIG. 54.

(2) With K_1 open, close K, and adjust R to as high a value as possible, preferably not less than say a hundred times the galvanometer resistance g, so as to get nearly a steady full-scale deflection d. Note this and the value of R.

(3) Close K and K_1, and adjust S, keeping R as before, so as to obtain about $\frac{1}{10}$ d less deflection than before. Note this d_1 and the value of S.

(4) Repeat (3) for about ten different deflections, decreasing by about equal amounts by altering S, keeping R constant, and show that the relation $\frac{d_1}{d} = \frac{s}{s+g}$ holds for each, $\frac{\tan d_1}{\tan d}$ being used in the case of a tangent galvanometer.

(5) Repeat (2)-(4) for two cells, reproducing the original deflection d, and tabulate as follows:—

Galvanometer resistance, g, = ohms.							
Number of cells used.	Resistance, R.	Total circuit resistance, $R + b + \frac{sg}{s+g}$	Shunt, s.	Deflections.		$\frac{d_1}{d}$	$\frac{s}{s+g}$
				Unshunted, d.	Shunted, d_1		

(6) Plot curves having values of s as abscissæ, and currents d_1 through G as ordinates.

Inferences.—Prove the relation given in (4), and state what assumptions are made in obtaining it. What inferences can you deduce from the results of your experiments? Prove that $\frac{sg}{s+g}$ = resistance of the shunted galvanometer.

68. Relation between the Current in a Shunted Galvanometer and the Total Current in the Main Circuit.

Introduction.—When the resistance of a galvanometer is small compared with that of the rest of the circuit, any alteration in its effective resistance will practically not alter appreciably the total circuit resistance, and consequently the current. Now, we have seen that the effective resistance between the galvanometer terminals is reduced from g to $\frac{sg}{s+g}$ on shunting it with a shunt s, and therefore if the diminution of this resistance, viz. $g - \frac{sg}{s+g}$, is at all comparable with the resistance of the rest of the circuit, the total circuit resistance, and hence the total current, will be considerably altered (increased). The object therefore of the following experiment is to see what effect such an increase of total current has on the galvanometer current, and the deflection it produces, and the conditions under which this effect takes place. It will be obvious at first sight that the question is one of great practical importance.

Apparatus.—Precisely that mentioned in the preceding test,

with the addition of another galvanometer, G_m, for the main circuit.

Observations.—(1) Connect up as in Fig. 54, placing G_m in the main circuit, and adjust both galvanometers to zero, using one or more cells of fairly constant E.M.F.

(2) With K_1 open, adjust the battery E.M.F. and R (which should preferably not be more than two or three times g) so as to obtain about a half-scale deflection, d_m, on the main galvanometer G_m. Note this, and also the value of R, and the deflection d on the other galvanometer G.

(3) Close K_1 and K, and, with R as before, adjust S to about double the resistance of G. Note the deflection d_1 of G and d_m of G_m and S.

(4) If the deflection d_m has altered, adjust R only, so as to reproduce the first deflection obtained in (2) above, and note the value R_1 required to do this. Then reduce the resistance again to its original value, as in (2) above.

(5) Repeat (3) and (4) for about ten different values of s, decreasing by about equal amounts to say one-tenth of the value of the galvanometer resistances G.

Tabulate your results as follows:—

Galvanometer resistance, g, = ohms.										
Resistances.				Shunt, s.	Deflections.			$\frac{d_1}{d}$	$\frac{s}{s+g}$	$\frac{s}{s+g} \cdot d_m$
R.	R_1.	$R_1 - R$.	Total R_T in circuit.		Unshunted, d.	Shunted, d_1.	Main, d_m.			

(6) Plot the following curves, one having values of s as abscissæ and currents d_1 through G as ordinates, the other with d as ordinates and $\frac{sd_m}{s+g}$ as abscissæ.

Inferences.—State concisely all that can be inferred from your experimental results.

69. Determination of the Apparent Increase of Resistance of a Ballistic Galvanometer when Shunted.

Introduction.—When a steady current flows through a galvanometer it obeys Ohm's law, or, in other words, is inversely proportional to the ohmic resistance G of the instrument. If, however, the current is varying from instant to instant, the above proportion no longer holds, for two reasons—(1) the coils of the galvanometer being necessarily wound inductively possess self-induction (*vide* p. 193), which, to a varying current, sets up a back E.M.F., opposing the passage of the current, and consequently introducing an *apparent* extra resistance in the galvanometer, in addition to that which the coils have for steady currents; (2) by Lenz's law the motion of the galvanometer needle will in itself produce a back E.M.F. in the coils, tending to stop the flow of current, and hence introducing an apparent extra resistance on this account. Thus, in the case of a varying current flowing through a galvanometer whose controlling magnet is fixed, the actual resistance offered by the instrument to the current is greater than it would be for a steady current by an amount due to the above two causes. We can therefore at once see that if the galvanometer is shunted by a shunt of resistance S, the fraction $\frac{S + G}{S}$, which *is* the "multiplying power" of the shunt when a steady current flows, *is not* the true "multiplying power" for a varying current such as would be obtained in a ballistic galvanometer when a discharge passes through it. The correction to be applied is the *apparent extra resistance*, which we may call K_R, due to both of the above causes, and which is added to the ohmic or ordinary galvanometer resistance. For any particular galvanometer, with its control definitely adjusted, the resistance in ohms of K_R has a definite value, and the "*true multiplying power*" of the shunt S used with the ballistic galvanometer $= \frac{S + G + K_R}{S}$. The object of the present test is to determine the value of K_R, which may easily be done experimentally.

Apparatus.—Galvanometer, G, to be tested; shunt resistances, S; condenser, C, of known variable capacity (p. 354); one two-way and one single-way spring tapping-key, K and K_1 respectively; battery, B, of fairly constant E.M.F. cells; two adjustable known resistances, r_1, r_2, or a potentiometer arrangement.

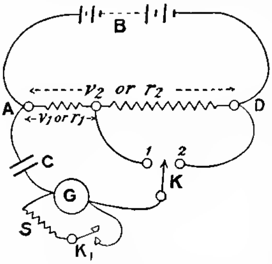


FIG. 55.

Observations.—(1) Connect up as in Fig. 55, and adjust the galvanometer G to zero.

(2) Adjust C and the total resistance AD $= r_2$ and the fractional resistance r_1 to some convenient values such that, on pressing K1 with K_1 open so as to charge C through the *unshunted* galvanometer, a convenient first throw d_1 is obtained on G. Note the values of C, r_1, and r_2.

(3) Completely discharge C by short-circuiting it, and adjust S to some suitable value such that, on pressing K2 with K_1 closed so as to charge C through the shunted galvanometer, a convenient first throw d_2 is obtained. Note the value of S (C, r_1, and r_2 remaining unaltered).

(4) Repeat (2) and (3) for about six different values of C, keeping the same values of S, r_1, and r_2.

(5) Repeat (2) and (3) for about six different values of S, keeping C, r_1, and r_2 unaltered.

NOTE.—C must carefully be discharged before each separate charge.

(6) Calculate the constant K_R as previously mentioned from the relation—

$$K_R = S\left(\frac{d_1 r_2}{d_2 r_1} - 1\right) - G$$

and tabulate your results as follows:—

Ballistic galvanometer tested, ; ohmic resistance, G, = ohms.

Capacity, C, for reference only.	Shunt, S	First throws.		Resistances.		K_R.	Mean, K_R.	$\frac{K_R}{G}$ 100
		d_1.	d_2.	r_1.	r_2.			

N.B.—The test will be most accurate when S is so adjusted as to make $d_2 = d_1$.

Inferences.—What can be inferred from your experimental results? Prove the relation given in (6), and state any assumptions made in obtaining it.

70. Determination of the "Constant" of a Tangent Galvanometer (by Calculation and Experiment).

Introduction.—The construction of two well-known types of tangent galvanometers is given on p. 267, *et seq.*, and we may here consider their theory more in detail with advantage. First take the case of a single coil or the ordinary form of this galvanometer. If the breadth and depth of the section of the coil be small compared with its diameter, a current passing through it produces a magnetic field, which is accurately calculable at the centre, and which is uniform throughout a considerable space in the neighbourhood.

If A = current in absolute C.G.S. units flowing through it, n = number of turns on the coil, and r its mean radius, then the field F at the centre acting perpendicular to the plane of the coil is $F = \frac{2\pi n A}{r}$.

But the needle will be deflected from its position of rest in the magnetic meridian, with its axis in the plane of the coil, through an angle $\theta°$ such that $F = H \tan\theta$ when it comes to a state of equilibrium.

$$\text{Hence } \frac{2\pi n A}{r} = H \tan\theta$$

$$\therefore\ A = \frac{H}{\frac{2\pi n}{r}} \tan\theta = \frac{H}{G} \tan\theta = K \tan\theta$$

G, which $= \frac{2\pi n}{r}$, is called the "*true constant*" of the galvanometer; and K, which $= \frac{H}{G}$, is called the "*working constant*" or

"*reduction factor*," H being equal to the horizontal intensity of the earth's magnetic field, and is a constant for a given locality.

In the above it is assumed that (1) the earth's field H is the only controlling force acting on the needle, and therefore that there is no friction in the pivots or suspension; (2) the needle is so short that its poles never move out of the uniform field at the centre of the coil.

Since 10 amps. = 1 absolute C.G.S. unit of current, we have from the above that $A = 10 \frac{H}{G} \tan \theta^{\circ}$ amps.

Now let us consider the double-coil form of tangent galvanometer known as the Helmholtz pattern, and described on p. 269. This is intended as a standard for measuring currents absolutely as well as relatively. Owing to the peculiar form of construction the field at and about the point O, at which the needle *ns* is pivoted, due to the current in the two parallel coils, is very uniform, enabling a longer needle to be used. Suppose the circles to represent turns of wire on the coil, part of one only of which is shown in Fig. 56. Consider

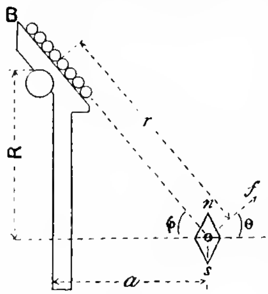


FIG. 56

first the action of a circular current on the needle or a pole at O in the axis of the coil, but not at its centre. An element *ds* of current of strength A produces at the point O a force $f = \frac{mA\,ds}{r^2}$ perpendicular to OB, where m = strength of the pole at O.

$$\left.\begin{array}{r}\text{The resolved component of}\\ \text{this force along the axis}\end{array}\right\} = \frac{mA\,ds}{r^2}\cos\theta = \frac{mA\,ds}{r^2}\sin\phi$$

$$= \frac{mA\,ds}{r^2} \cdot \frac{R}{r}$$

$$\therefore \left.\begin{array}{r}\text{the resolved component due}\\ \text{to a single current turn}\end{array}\right\} = \frac{mA2\pi R^2}{r^3}$$

And if n = number of turns on the two coils together, or $\frac{n}{2}$ on each, then this resolved force $= \frac{mnA2\pi R^2}{r^3}$.

But in a Helmholtz galvanometer $R = 2a$ and $r^2 = R^2 + a^2 = \frac{5}{4}R^2$; therefore the total force acting on the needle at O is—

$$F = \frac{mnA2\pi R^2 . 8}{5^{\frac{3}{2}}R^3} = \frac{16\pi}{5^{\frac{3}{2}}} . \frac{mn}{R} . A$$

Now, when the needle is deflected through an angle ϕ from its zero position in the magnetic meridian, and with its axis parallel to the planes of the two coils, this total force balances that due to the horizontal force H of the earth, or we have two balancing couples due to these two forces.

$$\text{Hence } mHl \sin\phi = Fl \cos\phi = \frac{16\pi}{5^{\frac{3}{2}}} . \frac{mn}{R} Al \cos\phi$$

$$\text{hence } A = \frac{H}{\dfrac{2\pi n 8}{5^{\frac{3}{2}}R}} \tan\phi = \frac{H}{G} \tan\phi = K \tan\phi$$

$$\text{where G now} = \frac{8}{5^{\frac{3}{2}}} . \frac{2\pi n}{R} = 0{\cdot}7135 \frac{2\pi n}{R}$$

In other words, the field intensity is only 0·7135 of what it would be if the needle were in the centre of the n turns, all on one bobbin of radius R.

71. Calibration of a Galvanometer by Comparison with a Mixed Gas Voltameter.

Introduction.—The following method is convenient for determining the relation between the current and corresponding amount of chemical decomposition produced per second, also the "constant" of a tangent galvanometer, which may be defined as the number by which to multiply the tangent of the angle of deflection, in order to obtain the current in amperes flowing through the instrument. One ampere flowing through a solution of sulphuric acid and water (1 to 10 say by volume) is found to liberate 0·1738 c.c. of mixed gas per second at 0° C. and 760 mm. pressure; hence the current that liberates any other volume per

second can at once be found. When the stop-cock of the voltameter is closed, the gas produced by the current cannot escape; the pressure inside therefore becomes greater than that of the atmosphere, and the liquid is forced up the other tube, to which a bulb and thistle funnel is attached. The rate at which the liquid rises in this tube is a measure of the amount of gas evolved per second. Opening the cock allows the gas to escape and the liquid to run out of the tube and bulb. Care must be taken that the liquid does not accumulate in the thistle funnel and overflow, also that the current in the connecting wires do not affect the galvanometer needle. The volume of the bulb and tube between the two scratches = 17·38 c.c.

Apparatus.—Tangent galvanometer, G; mixed gas voltameter, V (p. 332); switch, S; variable carbon resistance, R; battery of secondary cells, B (p. 338).

Observations.—(1) Connect up as indicated in Fig. 57, and adjust the galvanometer needle to zero by slightly turning the instrument.

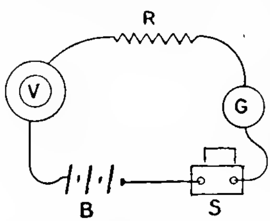


Fig. 57.

(2) Send a convenient current through the apparatus by closing S and suitably adjusting R. Open the stop-cock, and allow the liquid to become thoroughly saturated with gas.

(3) Now adjust R so as to get a deflection of 10° on the galvanometer scale, which must be kept constant; then close the cock, and note the exact number of seconds *t* taken by the liquid in rising from the bottom to the top scratch, also the deflection *d*° on the galvanometer.

(4) Repeat (3) about three times with the same deflection, and take the mean as being the more accurate result.

(5) Repeat (3) and (4) every 10° up to about 60°, and calculate for each current the value of the galvanometer "constant" K from the formula $A = K \tan d°$, where A = current in amperes producing the deflection *d*°, and from the formula $K = \frac{H}{G}$, where G is the "*true*" galvanometer constant, as mentioned previously, and H = the earth's horizontal intensity. Tabulate as follows:—

Galv. deflection, d°.	tan d°.	Time rise of liquid, t secs.	Vol. of gas per sec., $V = \frac{17 \cdot 38}{t}$	Current in amps., $A = \frac{V}{0 \cdot 1738}$	Galv. constant, $K = \frac{A}{\tan d^\circ}$	Galv. constant, $K = \frac{H}{G}$

(6) Plot two curves on the same curve sheet having values of V and A as abscissæ respectively, and tan d° as ordinates in each case.

Inferences.—State clearly all the inferences which can be drawn from the results of the above experiment, and also any corrections and precautions to be used, in order to determine the current in amperes accurately.

72. Standardization of Current Measuring Instruments (Silver Voltameter Method).

Introduction.—This method is one of the most accurate for calibrating or standardizing instruments for measuring current strength. It depends on the well-known principle that when a current of electricity flows through an electrolyte the amount of decomposition resulting in a given time T is directly proportional to the total quantity of electricity Q which has passed in that time, or if A = the current flowing during a very small interval of time dt, then $Q = \int_0^T A\,dt$; and if A is constant throughout, then—

$$Q = A\int_0^T dt = AT \text{ coulombs}$$

We therefore see that unit quantity (1 coulomb) of electricity is given by unit current (1 amp.) in unit time (1 sec.). Now, for any particular substance 1 coulomb will always decompose or liberate at the cathode the same fixed weight of the substance, which is usually termed its "*electro-chemical equivalent*," and denoted by Z.

In other words, Z = number of grams of the substance deposited by 1 coulomb. The value of Z in the case of silver has been very carefully determined to be 0·00111815, and in terms of this the value of Z for all other substances can be

calculated as follows: Let the atomic weight of the substance be A_W, and its valency be V.

$$\text{Then its "}\textit{chemical equivalent}\text{" } C_\epsilon = \frac{A_W}{V}$$

$$\text{For silver } C_\epsilon = \frac{108}{1} = 108$$

$$\text{Therefore for any other substance } Z = \frac{0{\cdot}001118{}15 A_W}{108 \,.\, V}$$

Now, a voltameter measures the quantity $\int_0^T A dt$, i.e. Q, by the amount of chemical decomposition which occurs. Hence, knowing this, and the time T in seconds for which the action lasts, we see that the current, if constant, is given by the relation—

$$A = \frac{Q}{T} = \frac{W}{TZ}$$

where W = number of grams deposited.

The silver voltameter, by means of which we can obtain W, and hence A, is shown in Fig. 195 (p. 333), where it is described. It may be repeated that the maximum current which may be used so as to get a good adherent and finely crystalline deposit in any platinum dish is approximate to 1 amp. per 6 sq. ins. of surface. Also with a bowl about 3 ins. in diameter and $1\frac{1}{2}$ ins. deep, a 15 per cent. pure silver nitrate solution may be used for 1 amp. for an hour, and a 30 per cent. solution for 2 amps. for 15 minutes. The density must be increased for increase of current.

Apparatus.—Silver voltameter, V; secondary battery, B (p. 338); carbon rheostat, R; current measuring instrument, A, to be standardized; two-way switch, S; drying cupboard or desiccator and spirit lamp.

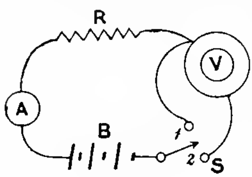


FIG. 58.

Observations.—(1) Connect up as in Fig. 58, and carefully clean the platinum dish by first treating with nitric acid to remove any silver, then wash with distilled water to remove acid, and either place in a drying cupboard or over a spirit flame. Lastly, allow it to cool in the desiccator. When cold, measure its weight W_1 grms. very accurately on a chemical balance.

(2) Place the bowl on its ring, and fix a good filter-paper round the silver disc so as to prevent any impurities being deposited on the bowl. Then clamp the disc in position so that its edges are equally distant from the sides and bottom of the bowl. Now pour in the neutral solution of pure silver nitrate to within about $\frac{3}{16}$ in. of the brim.

(3) Adjust A to zero, and switch S to 1, altering R so as to obtain the desired value of current.

(4) At a known noted instant switch S to 2, and very quickly readjust R so as to get the desired current. Keep this constant for 30 minutes, say, and switch off, having noted the temperature of the bath.

(5) Empty the solution from the dish into the stock bottle, and carefully wash the deposited silver with distilled water. Fill the dish with it, and allow it to stand about 15 minutes. Now rinse it out with the water, and lastly with alcohol, then dry over the spirit flame and cool in desiccator.

(6) When cold, accurately determine the weight W_2 of the dish, and calculate the current in amperes from the relation—

$$A = \frac{W_2 - W_1}{0{\cdot}00111815t}$$

where t = number of seconds for which the current flowed.

(7) Repeat (1)–(6) for another widely different current strength, and tabulate as follows :—

Area of cathode in sq. cms. per ampere.	Weight of bowl.		Deposit, $W_2 - W_1$ grms.	Time in secs., t.	Temperature of bath.	Reading on A.	True current, $A = \frac{W_2 - W_1}{Zt}$	Per cent. error of standard.
	Before, W_1 grms.	After, W_2 grms.						

73. Determination of the "Constant" of a Galvanometer or Ammeter by means of a Copper Voltameter.

Introduction.—From the results of a large number of tests it has been found that, using the necessary precautions, the constant of an electric current instrument can be obtained with certainty to

within $\frac{1}{10}$ per cent. of absolute accuracy by the electrolysis of copper. The voltameter V should consist of three or more pure copper plates dipping into a saturated solution of copper sulphate contained in a suitable glass or earthenware vessel, there being one more "anode" than "cathode," and the two sets arranged alternately with an anode at each end. The plates should be as square as possible, and placed from $\frac{1}{2}$ to $\frac{3}{4}$ in. apart; if too close, polarization will take place when strong currents are used, and the current density (reckoned in amperes per square centimetre) is too great. There should not be less than 30 sq. cms. per ampere; if there is, the plate surface will be too small, and the deposit on the cathode irregular, some of it falling to the bottom of the vessel. The resistance of V will also become high and variable, due to formation of copper oxide, and give trouble in keeping the current constant. The solution should be a saturated one (sp. gr. 1·211) of pure copper sulphate crystals and distilled water, with 1 per cent. by volume of strong sulphuric acid added, which is necessary to ensure success. The volume of solution should be about 1100 c.c. per ampere. The anodes may be made of about No. 18 S.W.G., and the cathodes or gain plates of No. 30 S.W.G. pure copper, all edges and corners being smooth and rounded. The electro-chemical equivalent Z of any substance, in this case copper, equals the number of grams deposited by 1 coulomb.

TABLE IV.

Cathode area in sq. cms. per ampere.	Values of Z for copper.		
	2° C.	12° C.	23° C.
50	0·0003288	0·0003287	0·0003286
100	88	84	83
150	87	81	80
200	85	79	77
250	83	78	75
300	82	78	72

Apparatus.—Voltameter, V (p. 333); rheostat, R (p. 308); switch, S; ammeter, A, to be standardized; secondary battery, B; drying cupboard, D; acid bath.

Observations.—(1) Connect up as indicated in Fig. 59, and adjust A to zero. Light the gas-jet under the steam boiler of D, after seeing that the latter contains enough water.

(2) Determine the necessary area of cathode, and hence the number of gain plates required for the current to be used, reckoning both sides in contact with liquid as the effective area of cathode.

(3) Carefully clean the cathodes all over with fine emery cloth until quite bright, then dust with a *dry clean cloth*, and *do not touch the part to be immersed with the fingers.* Clean the anodes if they look dirty.

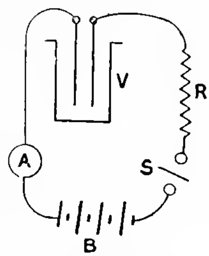


Fig. 59.

(4) Carefully weigh the gain plates on a chemical balance to 1 milligram, and note their weights W_1 grms.

(5) Insert the same area of trial plate to act as cathode, so as to adjust the current to the value required; then remove them, making sure that the positive of battery is joined to anode.

(6) Insert the weighed gain plates, and at a convenient and noted instant of time switch on, quickly adjusting the current to its proper value.

(7) Keep it flowing for at least 30 minutes, and maintain it constant all the time by altering R, if necessary.

Note.—1·177 grams of copper (cupric) are deposited per ampere-hour approximately.

(8) Note the exact instant of switching off. Very carefully remove the gain plates so as not to scratch them, rinse in acidulated water to prevent the nascent copper oxidizing, then in clean water, and place in D to dry.

(9) When dry and cool reweigh the gain plates, and note the weights W_2 grms.

(10) Repeat (2)–(9) for a different current strength, and tabulate as follows:—

Cathode area = sq. cms. per amp.; temperature of bath = ° C., Z, = .

Weight of plates in grams.		Deposit, $W = (\Sigma W_2 - \Sigma W_1)$.	Time in secs., t.	Reading of A.	True amperes, $\frac{W}{Zt}$	Per cent. error of A.
Before, ΣW_1.	After, ΣW_2.					

74. Conductivity and Specific Resistance of Electrolytes.

The *accurate* measurement of the resistance of electrolytic liquids is very troublesome and difficult owing to phenomena which become manifest immediately a current flows through the liquid, and which are usually designated by the term "*polarization.*" It consists, firstly, of a back E.M.F. set up by the decomposition of the electrolyte; and secondly, of what is called a *transition resistance* introduced at the surface of the electrodes in contact with the liquid, probably due to partly an air film and partly the separation or condition of the ions at these points. In the case of solutions of salts, these injurious effects can be to a great extent minimized by using electrodes composed of the same metal as in the salt. This is particularly the case with zinc and copper sulphates. Platinum may be used for other solutions. Numerous devices and methods have been suggested and used by various experimenters for eliminating or reducing the disturbing effects of polarization, and of these the following are the best.

75. Resistance of Electrolytes (Stroud-Henderson Direct-current Method).

Introduction.—There can be little doubt that direct-current methods possess an advantage over alternating-current ones in the matter of the current indicator, enabling a delicate reflecting galvanometer to be used instead of a telephone, the former being more reliable though less sensitive and far less dead-beat than the latter. The present method, though similar in some respects to continuous-current methods previously devised, differs from them in the form of electrolytic cell used, and in the use of high voltages and high resistances, so as to effectually drown any residual error due to differential polarization. It consists in using a *balancing cell*, *c*, having the same size of plates, etc., and in all respects similar and equal to the main electrolytic cell C, but having a much shorter length of liquid conductor. With such an

arrangement the disturbing effects of polarization can practically be eliminated, or, at all events, at least 99 per cent. of them can, though generally the *balance* is much more perfect than this. The construction and arrangement of main and balancing cells is described on p. 343. The measurement of the resistances are effected with a Wheatstone bridge arrangement, as in Fig. 60, from which it will at once be seen that if $r_1 = r_2$, then when balance is obtained in the usual way, equal currents will be traversing both cells, C and *c*; and therefore there should be equal and opposed polarizations, which, theoretically speaking, should neutralize each other. The best positions of G and B are as in the figure, and if G is a delicate D'Arsonval galvanometer of from 300 to 400 ohms

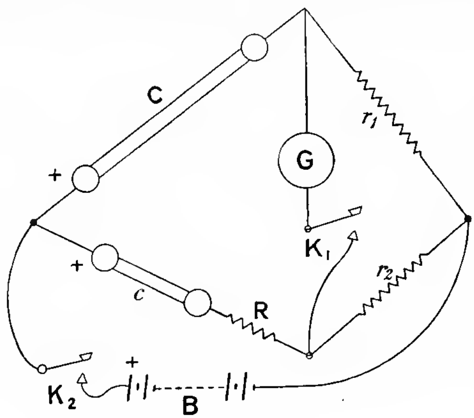


FIG. 60.

resistance, it is possible to easily detect differences of about 5 ohms in R, in say 30,000 ohms, when $r_1 = r_2 = 2000$ ohms, and the E.M.F. is 30 to 40 volts. A constant, K, can be obtained for a particular pair of tubes by calibrating them, or measuring their lengths, and weighing in a watch-glass the mercury required to fill them. In finding this weight, the results will be most concordant when the finely ground ends of the tube is stopped by means of a small piece of thin glass cover backed with cork about $\frac{1}{10}$ inch thick, the diameter of cover-glass and cork being about the same as the external diameter of the tube, so that one finger will keep the cover-glass firmly up against the end of the tube without fear of tilting one edge relatively to the end of the tube.

To find the *constant* K of the tubes—

Let L = the length in centimetres of the longer tube, and A the sectional area of its bore,

l = the length in centimetres of the shorter tube, and a the sectional area of its bore;

also let W = the weight in grams of mercury filling the long tube,

and w = " " " " short "

then if ρ = the specific resistance per centimetre cube of the liquid tested, and σ = the density of mercury in gram centimetre units, we have—

$$W = \sigma LA \quad \therefore A = \frac{W}{\sigma L}$$

$$\text{and } w = \sigma la \quad \therefore a = \frac{w}{\sigma l}$$

therefore the resistance giving balance, namely—

$$R = \rho\left(\frac{L}{A} - \frac{l}{a}\right) = \rho \cdot \sigma\left(\frac{L^2}{W} - \frac{l^2}{w}\right) = \rho \times K$$

$$\text{Hence the specific resistance } \rho = \frac{R}{K}$$

To find the *temperature coefficient* a of the electrolyte—

Let R_1 = the resistance of the liquid of temperature $t_1°$ C.

R_2 = " " " " $t_2°$ C.

R_0 = " " " " 0° C.

$$\text{Then } R_1 = R_0(1 + at_1)$$

$$\text{and } R_2 = R_0(1 + at_2)$$

$$\text{Hence } \frac{R_1}{R_2} = \frac{1 + at_1}{1 + at_2}$$

$$\therefore a = \frac{R_1 - R_2}{R_2 t_1 - R_1 t_2} \text{ per ohm per } 1° \text{ C.}$$

Apparatus.—Sensitive reflecting galvanometer, G; standard known resistances, r_1, r_2, each equal to about 2000 ohms; an adjustable high resistance, R, totalling up to about 100,000 ohms; a battery, B, to give 30–40 volts P.D.; electrolytic apparatus, the diameter of the bore of the tubes being so chosen as to give a convenient resistance with the liquid used; a delicate thermometer graduated, say in $\frac{1}{10}°$ C.; spring tapping-keys, K_1, K_2; a source of heat, if necessary, for heating the bath.

Vide remarks on p. 130 relating to certain precautions to be adopted.

Observations.—(1) Connect up as in Fig. 60, and adjust the galvanometer spot of light to zero (about). Make $r_1 = r_2 = 2000$ ohms.

(2) Close K_2, and quickly adjust R roughly, so that on pressing K_1 there is no deflection on G approximately; then open K_1, K_2.

(3) Keep the bath well stirred by raising the handle very gently and slowly up and down for about 4 minutes, so as to allow the electrolyte to attain the temperature of the bath t which is now noted, and K_1 closed, the bridge being very rapidly adjusted finally by R (accurately).

(4) Take out the electrodes, wash, then heat to redness, and replace them in the cells, and lastly, reverse the current.

(5) Repeat (2) and (3) with the current reversed.

(6) Repeat (4) first, and then (2) and (3) again some four times.

(7) Repeat (2)–(6) for about six different temperatures up to about 60° C.

(8) Repeat (2)–(6) for about four or five different densities of the same electrolyte, at as nearly as possible the same temperature, and tabulate as follows :—

Long tube, ; length, L, = cms. : weight of mercury, W, = grms. ; $\sigma =$ at 0° C.
Short ,, ; ,, l, = ,, : ,, ,, w, = ,, ; constant, K, =

Electrolyte used, and strength.	Direction of current.	Temperature of bath, t.	Density of electrolyte.	r_1.	r_2.	R.	Resistance of electrolyte reduced to 0° C.	Temperature coeff., a.	Specific resistance, $\rho = \frac{K}{K}$

(9) Plot two curves, one having temperatures of the electrolyte as ordinates and corresponding resistances at constant density as abscissæ, the other having densities as ordinates and the corresponding resistances (at constant temperature) as abscissæ.

Inferences.—Mention any sources of error which might be introduced in the method, and state clearly the inferences you can draw from the results of the experiment.

76. Resistance of Electrolytes (Kohlrausch Alternating-current Method).

Introduction.—In the following method, due to Professor Kohlrausch, the detrimental effects due to polarization, which tend to introduce errors in the measurement of electrolytic resistance, are almost eliminated. Such effects will be diminished still further by platinizing the electrodes (if these are not of platinum already), and by increasing their size. Although the method is superior to any at present in use in which direct currents are used, except the Stroud-Henderson one, there are difficulties arising from self-induction and capacity which are manifest when alternating currents are used, and for this reason—it is of the utmost importance that the bridge arms have no self-induction. To realize this condition more fully, a special form of bridge is usually used, of which Fig. 210, p. 345, shows one type. The electrolytic cell may be either of the form shown in Fig. 212, p. 346, or that used in the last method. In the present method we shall employ this form of balancing cell which has been found by Drs. Stroud and Henderson to be a distinct improvement for resistances not greater than 1000 ohms or thereabouts, enabling dead silence to be obtained on the telephone, while, without the balancing cell, there is usually always a feeble buzz.

Apparatus.—Telephone, G_T, for use as a galvanometer; two non-inductive resistances, r_1, r_2, of about 1000 or 2000 ohms each; one higher adjustable resistance, R; small induction coil, B, giving, say, $\frac{1}{4}$ in. spark for producing the alternating current, and a small battery, *b*, to excite it; electrolytic cell, C*c*, of such a size as to give not more than about 1000 ohms resistance with the solution used; key, K; delicate thermometer graduated in $\frac{1}{10}°$; delicate hydrometer or chemical balance.

N.B.—If an ordinary metre bridge is used, r_1, r_2 will form the stretched wire, but it is not suitable to use with the balancing cell. In such cases one similar to Fig. 169 should be used. If possible, the source of alternating current B should be removed to a distance from the operator, so that the noise of the "contact breaker" may not interfere with the hearing at the telephone. It will, for

this reason and that of obtaining greater sensitiveness, be an additional help if two telephones are used in parallel, one to each ear. *Vide* precautions (p. 130) to be adopted.

Observations.—(1) Connect up as in Fig. 61, or, if the special bridge (p. 345) is used, then connect as there described. If the present arrangement is used, make $r_1 = r_2$, and adjust R so that *no* sound or *minimum* sound is heard in G_T, and note the values of R and temperature and of r_1, r_2 for reference only.

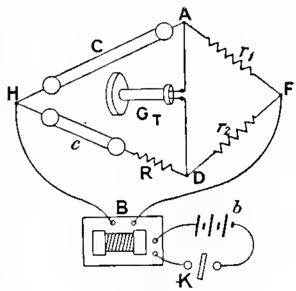


FIG. 61.

N.B.—The test will be most sensitive when r_1 and r_2 are only a little greater than the value of R required to balance, for then the arms of the bridge will be more nearly equal in resistance. Balance should then be disturbed by altering R, and re-obtained thus two or three times, and the mean value of R noted, which will thus be more accurately equal to the difference in the resistance of C and c.

(2) Repeat (1) for four or five different temperatures of the bath or solution.

(3) Repeat (1) for four or five different densities of the same solution at as nearly as possible the same temperature, and tabulate your results in exactly the same manner as in the last test.

(4) Plot two curves, one having temperatures of the electrolyte as ordinates and corresponding resistances (at constant density) as abscissæ, the other with densities as ordinates and resistances (at constant temperature) as abscissæ.

Inferences.—State clearly all the inferences deducible from your experimental results.

77. Resistance of Electrolytes (Secohmmeter Method).

Introduction.—This method may conveniently be used when an Ayrton and Perry secohmmeter is available, but not an induction coil. It differs slightly from either of the preceding

methods, owing to the employment of an alternating current in the arms of the bridge, produced by the secohmmeter interchanging the battery terminals while an ordinary aperiodic sensitive galvanometer is at the same time used, thus combining the advantages of the direct and alternating current methods. The precise action of the secohmmeter will be at once seen by reference to pp. 259 to 262. In the present case the commutators may be set in the midway positions, so that the galvanometer connections are reversed midway between two consecutive battery reversals. The higher the speed, the greater the sensitiveness.

Apparatus.—Secohmmeter complete, with means for driving it (Fig. 119, p. 261); battery, B, capable of giving 30–40 volts P.D.; sensitive aperiodic reflecting galvanometer, G; electrolytic cell (preferably the form shown on p. 343); known resistances, R, r_1, r_2; delicate thermometer and hydrometer or chemical weighing balance, for making up the test solution.

Observations.—(1) Referring to Fig. 61 for Kohlrausch's method, dispensing with B, K, and G_T, connect points A and D to the terminals marked "bridge" on the same side of the secohmmeter as those marked "galvanometer," and the points H and F to the remaining pair marked "bridge;" also join B and G to terminals marked "battery" and "galvanometer" respectively. Adjust the galvanometer to zero (about).

(2) Repeat *precisely* the observations in their order, as set forth in the Kohlrausch method, and answer the inferences there mentioned.

General Precautions to be used in the Preceding Methods.

If a metre bridge be used, the arms of the bridge should be made more equal by adding extra resistances at the ends of the stretched wire.

In all cases the utmost care should be taken in thoroughly cleaning the electrolytic cells, finally using distilled water, as mere traces of foreign substances sometimes serve to completely alter the resistance of the liquid tested.

The solutions for test must be made up with great care if the experimental results are to be compared with standard ones (*vide* p. 365).

78. The Ballistic Galvanometer.

In experiments on induction currents and capacity, "transient" or very short duration currents have to be measured. The ballistic galvanometer is a form of instrument specially adapted for the measurement of such, and its action depends upon the principle that, when the duration of the current is very short compared with the time of oscillation of its moving system (whether coil or magnetic needle), the total quantity Q of electricity transmitted by that current may be deduced from the first swing or "throw" of the needle, due to the magnetic impulse of the momentary current. The term "ballistic" is applied to such an instrument from its analogy to a ballistic pendulum, in that its moving system possesses a large moment of inertia. In fact, on this depends the whole principle of the galvanometer, namely, that the moment of inertia is so large that the whole quantity of electricity in the transient current has passed through the coils of the galvanometer before the needle begins to move. If a current A flows for a short interval of time dt, the latter quantity Q of electricity which passes is $Q = \int A dt$ taken between the correct limits, where A is variable as in an induced current.

This time-integral of the current has therefore to be measured by a ballistic galvanometer. Two or three forms of this type of instrument are described on pp. 283, 285, from which we see that its moving system may be either a needle or coil. But, in general, any galvanometer can be used as a ballistic one if damping is reduced to the utmost, and the moving system weighted in addition so as to give a large moment of inertia. We will now consider the principle of the instrument more in detail.

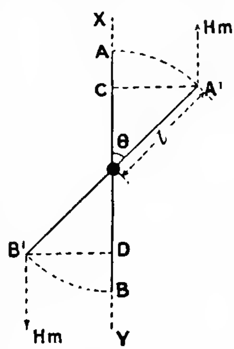


FIG. 62.

Let Fig. 62 represent an exaggerated plan of the galvanometer needle AB when freely suspended, and at rest in the magnetic meridian XY; let the strength of each of its poles be m, and their distance apart $2l$, and θ° = angle AOA′, where A′B′ is the instantaneous position of rest when

deflected. Draw A′C and B′D perpendicular to AB. Then, if O is the centre of rotation, and H = the horizontal intensity of the earth's force, the force acting on each end of the needle tending to bring it back to the position AOB = Hm. But Hm has acted through a distance AC one end, and BD the other,

$$\therefore \text{ the total work done against H} = \mathrm{H}m(\mathrm{AC} + \mathrm{DB})$$
$$\text{But AC} = \mathrm{DB} = \mathrm{OA} - \mathrm{OC} = l - l\cos\theta^\circ$$
$$= l(1 - \cos\theta^\circ)$$
$$\therefore \text{ total work done} = \mathrm{H}m(\mathrm{AC} + \mathrm{DB}) = \mathrm{H}m2l(1 - \cos\theta^\circ)$$
$$= \mathrm{HM}(1 - \cos\theta)$$

where M = $2lm$, the magnetic moment of the needle.

Again, if any body or mass is rotating round a fixed axis, the sum of all the products obtained by multiplying the mass of each particle m in the body by the square of its radius r from that axis, is termed the *moment of inertia* I about the axis, or $\mathrm{I} = \Sigma mr^2$.

If, now, G = the galvanometer constant, which depends on the form of its coils, etc., then the moment of the force on the needle produced by the current A is = MGA; and if the current flows for a short time dt,

$$\text{then moment of force} = \mathrm{MGA}dt = \mathrm{MG}dq$$

where dq = the small quantity which flows in the time dt;

$$\text{hence the total impulse on the needle} = \mathrm{MGQ}$$

where Q = the whole quantity that passed in the discharge.

But this impulse is equal to the moment of momentum Iω of the needle, where ω = the angular velocity of needle;

$$\therefore \mathrm{MGQ} = \mathrm{I}\omega, \text{ or } \omega = \frac{\mathrm{MGQ}}{\mathrm{I}}$$

Now, the kinetic energy of the needle at starting = $\frac{1}{2}\mathrm{I}\omega^2$, and this must equal the total work done, namely, HM$(1 - \cos\theta)$, which also = $2\mathrm{HM}\sin^2\frac{\theta}{2}$;

$$\therefore 2\mathrm{HM}\sin^2\frac{\theta}{2} = \tfrac{1}{2}\mathrm{I}\omega^2 = \tfrac{1}{2}\mathrm{I}\frac{(\mathrm{MGQ})^2}{\mathrm{I}^2} = \tfrac{1}{2}\frac{(\mathrm{MGQ})^2}{\mathrm{I}}$$

$$\therefore \mathrm{M}^2\mathrm{G}^2\mathrm{Q}^2 = 4\mathrm{HMI}\sin^2\frac{\theta}{2}$$

$$\text{Hence Q} = 2\sqrt{\frac{\mathrm{HI}}{\mathrm{MG}^2}}\sin\frac{\theta}{2}$$

We therefore see that, on the assumption that the discharge

through the galvanometer is completed before the needle moves, the total quantity of electricity passing is $\propto \sin\frac{\theta}{2}$, where θ = first throw of the needle. The above equation for Q can be still further reduced, for if T = periodic time of oscillation of the needle,

$$\text{Then } T = 2\pi\sqrt{\frac{I}{MH}}, \text{ or } \frac{I}{M} = \frac{T^2H}{4\pi^2}$$

and substituting this value of $\frac{I}{M}$ in the value for Q, we have—

$$Q = 2\sqrt{\frac{T^2H^2}{4\pi^2G^2}}\sin\frac{\theta}{2} = \frac{TH}{\pi G}\sin\tfrac{1}{2}\theta$$

This relation assumes that the earth's field H is the only controlling force tending to damp the motion of the needle, or that the whole kinetic energy of the needle, after the flow has ceased, is used in overcoming the retarding forces. Other damping may, however, arise from the resistance of the air, torsion of the suspension, and any induced currents which may be generated in surrounding metal-work, if there is any. To correct for this, let λ = the *logarithmic decrement*. Then the observed $\sin\frac{1}{2}\theta$ must be multiplied by the factor $(1 + \frac{1}{2}\lambda)$ to obtain the true value, which $\sin\frac{\theta}{2}$ would have if no damping existed.

$$\therefore Q = \frac{TH}{\pi G}\sin\tfrac{1}{2}\theta(1 + \tfrac{1}{2}\lambda)$$

It should here be noted that the magnetic moment enters into the previous value of Q, and indirectly in the last formula; hence the needle should be a magnet which is least liable to change in strength, either by the passage of too strong a discharge or otherwise.

Periodic Time of Oscillation.—The value of this will depend on the moment of inertia and magnetic moment of the needle and on H. The latter two may vary, the magnetic moment from the cause just mentioned, and H through altering the controlling magnet. Therefore T should be obtained afresh at each experiment, and may be found thus: Set the needle swinging, and note the number of seconds it takes for the spot of light to pass zero, moving in the same direction on two succeeding occasions.

This should be repeated about six times, and the mean result noted as the value of T.

Napierian Logarithmic Decrement.—It has already been pointed out that, owing to the throw of the needle of a ballistic galvanometer being damped through various causes there mentioned, it should be multiplied by a coefficient greater than one, in order to give the throw which would have resulted had there been no damping.

If $A_1, A_2, A_3, A_4, \ldots A_n$ are the amplitudes of successive oscillations of the needle, then $\frac{A_1}{A_2} = \frac{A_2}{A_3} = \frac{A_3}{A_4} = \ldots \frac{A_n}{A_{(n+1)}}$ is called the *decrement*, and $\log_\epsilon \frac{A_n}{A_{(n+1)}}$ is termed the *Napierian logarithmic decrement* λ, where ϵ is the base of the Napierian logarithms, and $= 2{\cdot}71828$.

$$\text{Then } \log_\epsilon \frac{A_n}{A_{(n+1)}} = \frac{\log_{10} \frac{A_n}{A_{n+1}}}{\log_{10} \epsilon} = \frac{\log_{10} \frac{A_n}{A_{n+1}}}{0{\cdot}4343} = \lambda$$

$$\text{Any error will be a minimum when } \frac{A_n}{A_{n+1}} = \epsilon = 2{\cdot}718.$$

$$\lambda \text{ also } = \log_\epsilon \frac{A_1 + A_2}{A_2 + A_3}, \text{ etc.}$$

If the damping is very small, A_n and A_{n+1} will be too nearly equal to allow of λ being obtained accurately. In such a case we can proceed thus:

Let A = amplitude of any one oscillation, and

A_n = „ the nth oscillation after it.

$$\text{Then the decrement} = \left(\frac{A}{A_n}\right)^{\frac{1}{n-1}}$$

$$\therefore \lambda = \log_\epsilon \left(\frac{A}{A_n}\right)^{\frac{1}{n-1}} = \frac{1}{n-1} \log_\epsilon \left(\frac{A}{A_n}\right)$$

$$= \frac{1}{n-1}\left(\frac{\log_{10} \frac{A}{A_n}}{0{\cdot}4343}\right)$$

79. Calibration of a Ballistic Galvanometer (Vibration and Deflection Method).

Introduction.—Referring to the formula deduced for a ballistic galvanometer a page or two back, we found that the whole quantity of electricity Q in the transient current producing a first throw θ° on the galvanometer was—

$$Q = \frac{T}{\pi} \cdot \frac{H}{G} \sin \tfrac{1}{2}\theta(1 + \tfrac{1}{2}\lambda) = K \sin \tfrac{1}{2}\theta$$

where $K = \frac{T}{\pi} \cdot \frac{H}{G}(1 + \frac{1}{2}\lambda)$, and is the ballistic galvanometer constant.

The present method consists in experimentally determining the periodic time T of vibration of the needle in seconds and the logarithmic decrement λ in the manner just shown; and, lastly, the value of $\frac{H}{G}$ by passing a steady current C through the galvanometer, and noting the steady deflection ϕ, whence $\frac{H}{G} = \frac{C}{\tan \phi}$, though for small deflection $\frac{H}{G} = \frac{C}{\phi}$ approximately.

Apparatus.—Stop-watch; current ammeter; resistance; battery; and key.

Observations.—(1) Connect up the ammeter, resistance, key, and battery in series, and adjust the galvanometer needle to zero.

(2) Determine T and λ in the way indicated on pp. 33, 34.

(3) Send about six different currents C through the ballistic galvanometer by varying the resistance, and note the corresponding steady deflection ϕ° with each.

(4) Tabulate your results as follows:—

Mean periodic time, T, = secs.; log dec. λ = ; distance between galvanometer and scale = scale-divisions.

Steady current through galvanometer, C.	Steady deflection on galvanometer, d scale-divisions.	Value of $\tan \phi^\circ = \frac{d}{2L}$	Galvanometer constant, $K = \frac{T}{\pi} \cdot \frac{C}{\tan \phi} \cdot (1 + \frac{1}{2}\lambda)$	Mean constant, K.

80. Calibration of a Ballistic Galvanometer (Earth-inductor Method).

Introduction.—This method is applicable in cases where either the vertical or horizontal components of the earth's magnetic field is accurately known, and it then constitutes the simplest method of calibration. The principle of it is as follows: If a conductor, as, for example, a coil of wire of one or more turns N and mean area A sq. cms., placed in a magnetic field of intensity F, and connected to the galvanometer, be suddenly turned so as to cut the lines of force of that field, an E.M.F. will be induced which will cause a certain quantity Q of electricity to pass through the galvanometer, depending on the total resistance R of the circuit. A "throw" of the spot of light on the scale will result, which is a measure of the number of lines of force cut by the coil.

This latter may conveniently take the form shown on p. 360, and if it is suitably placed and suddenly turned through 180°, so as to cut one of the components of strength F, then the whole quantity Q of electricity set up in the transient current produced will be $Q = \frac{2NAF}{R}$, and we have $Q = K \sin \frac{\theta}{2}$, where K = the galvanometer constant and θ = angle of first throw (in degrees) caused by the passage of Q. If d = deflection of spot of light (in scale-divisions) and L = length between scale and galvanometer mirror (also in scale-divisions), then $d = L \tan 2\theta$, or if the angular displacement is small, as it always is in mirror galvanometers, then $\sin \frac{\theta}{2} = \frac{d}{4L}$ very approximately.

There are three ways of using the "earth inductor" in connection with cutting the earth's field, as follows:—

(*a*) *Horizontal Component.*—To cut this, place the inductor stand so that the *axis* of turning is *vertical*, and the *plane* of the coil *perpendicular to the magnetic meridian.*

(β) *Vertical Component.*—To cut this, place the *axis* of turning *horizontal* and in a line with the magnetic meridian, and the plane of the coil horizontal also.

(γ) *Total Field.*—To cut the whole of the earth's field, place the axis of turning horizontal and at right angles to the magnetic meridian, and the plane of the coil perpendicular to the direction of "dip."

Then, in turning the coil sharply through 180° from either of these three positions, it will cut the particular field in question. It will, however, be found most convenient to employ (α) or (β). The direction of the magnetic meridian can be found in the way indicated on pp. 13, 14.

Apparatus.—Earth inductor, E; sensitive ballistic mirror galvanometer, G; box of known resistances, r; either a short-circuit key, K, or damping coil to bring the spot of light quickly to rest and save time.

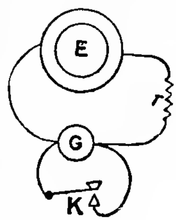


Fig. 63.

Observations.—(1) Connect up the above apparatus, and adjust the spot of light to zero on the scale.

(2) Make the box resistance as small as possible, and place the earth conductor in position (α), so that when turned as "rapidly" as possible through 180°, it cuts the horizontal component of the earth's magnetic force F.

(3) With the spot of light *absolutely at rest*, rotate the inductor rapidly through 180°, and note the first throw d scale-divisions of the spot.

N.B.—Do this two or three times, and take the mean as being more accurate.

(4) Repeat (3) for about ten different and increasing values of box resistance to the highest convenient.

(5) Repeat (2)–(4), cutting the earth's other component, and record your results in the following manner:—

Earth inductor, ; turns, N, = ; mean area, A, = sq. cms. ; resistance, r_E, = ohms ; galvanometer resistance, G, = ohms ; distance between scale and mirror, L, = scale-divisions.

Earth's field cut, F.	Mean first throw of spot, d.	Value of $\sin\frac{\theta}{2} = \frac{d}{4L}$ very approximately.	Box resistance, r ohms.	Total resistance, $r + r_E + G = R$.	Constant of galvanometer, $K = \frac{2NAF}{10^9 R . \sin\frac{\theta}{2}}$	Corrected mean galvanometer constant.

Inferences.—Show how the formula for the constant K can be deduced, and state the assumptions made, if any. Will any corrections be necessary to obtain an accurate result? State in words what the above constant really means.

81. Calibration of a Ballistic Galvanometer (Standard Solenoid Method).

Introduction.—As was stated in connection with the earth-coil method, the earth's force must be accurately known in order that that method shall be accurate; but as the earth's field is liable to serious alteration in a room if iron girders or pipes are in the vicinity, the following method is often a much better one to employ, and is quite independent of the earth's field, but entails the accurate measurement of a current. For it we require a magnetizing solenoid 3 or 4 feet long, of wire uniformly wound on a non-magnetic core, and having a mean sectional area A sq. cms. and number of turns T, both accurately known. A short search-coil of N turns is wound over the centre of this coil and joined to the galvanometer. If, now, C = current in amperes sent through the magnetizing coil, the magnetizing or magnetic force H at and near its centre is $H = \frac{4\pi CT}{10l}$ C.G.S. units, where l = length of solenoid in centimetres between extreme end turns; and since the core is non-magnetic, this H = B, the magnetic induction in lines per square centimetre; therefore the total number of lines enclosed by search-coil $= \frac{4\pi CTA}{10l}$. Hence, if R = total resistance of the search-coil circuit, we have that the total quantity of electricity in the transient current set up by either making or breaking the magnetizing current C is $Q = \frac{4\pi CTAN}{10^9 10l R}$ C.G.S. units, corresponding to an angular throw $\theta°$ of the galvanometer needle.

Apparatus.—Standard solenoid, S; battery, B; switch, K; rheostat, R_1; sensitive ballistic mirror galvanometer, G; box of known resistances, r; either a short-circuit key, K_1, or damping coil (p. 349), for bringing the galvanometer needle quickly to rest,

thus saving time; delicate ammeter, *a*, for reading the current accurately.

Observations.—(1) Connect up as in Fig. 64, and adjust G and *a* to zero.

(2) Make *r* as small as possible; close K and adjust R_1, so as to give a small current on *a*.

(3) With G *absolutely at rest*, open K, and note the first throw or deflection on G, namely, $\theta°$, and the current C amps. on *a*.

(4) Damp G quickly to rest, and make the same current C at K. Repeat (3) and (4) about three times, and take the mean of all the values of θ and of C respectively on the make and break.

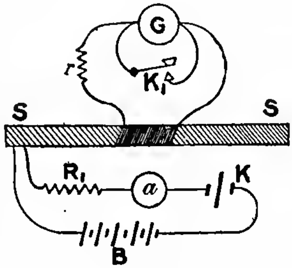


Fig. 64.

(5) Repeat (2)–(4) for about six different and increasing values of box resistance *r*, noting its value at each.

(6) Repeat (2)–(5) for about three different current strengths C, and tabulate your results as follows:—

Standard solenoid, ; magnetizing turns, T, = ; total length, *l*, = cms. ; mean area, A, = sq. cms.
Search coil, ; number of turns, N, = ; resistance, r_E, = ohms.
Galvanometer resistance, G, = ohms ; distance between scale and mirror, L, = scale-divisions.

Current in amperes, C.	Mean first throw, *d* scale-divisions.	Value of $\sin \frac{1}{2}\theta° = \frac{d}{4\,L}$ approximately.	Box resistance, *r* ohms.	Total resistance, $R = r + r_E + G$.	Galvanometer constant, $K = \frac{Q}{\sin \frac{1}{2}\theta°}$	Corrected mean galvanometer constant.

Inferences.—What does the constant K represent, and ought any corrections to be applied to the equation to obtain an accurate result?

82. Calibration of a Ballistic Galvanometer (Standard Condenser Method).

Introduction.—The following "capacity method," for calibrating or determining the *constant* K of a ballistic galvanometer, consists in charging a condenser of (C) M.F.D.S. capacity

to a known potential of V volts by means of a cell of known E.M.F., and then discharging the quantity Q of CV micro-coulombs through the galvanometer, noting the "throw" produced. The galvanometer so calibrated can now be used, not only for measuring quantities of electricity, but also for measuring magnetic fields as follows: If a coil of N turns, joined to the galvanometer, cuts a field of induction, it will be linked with the latter N times, and the throw produced will mean so much quantity in micro-coulombs passing through the galvanometer. In other words, induction × linkages = quantity × total circuit resistance. If the induction is measured in microwebers (1 weber = 10^8 C.G.S. lines of induction), then microwebers × linkages = micro-coulombs × ohms. The galvanometer should have a moderately small logarithmic decrement λ, say not greater than 10 per cent.

Apparatus.—Sensitive mirror ballistic galvanometer, *g*; standard condenser, C, of variable known capacity; high insulation charge and discharge key, K; two or three constant cells, *b*, of known E.M.F., V, or an accurate voltmeter to measure the P.D.'s.

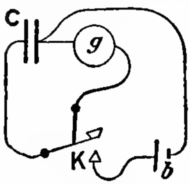


FIG. 65.

Observations.—(1) Connect up the above apparatus as in Fig. 65, and adjust the spot of light to zero on the scale.

(2) Insert a known capacity C in the condenser box, and charge it by a known P.D. V volts by pressing the key K.

(3) With the spot of light absolutely at rest, release K, thereby discharging the quantity of electricity Q = CV in the condenser through *g*. Note the first *throw d* divisions of the spot of light. Repeat two or three times, and record the mean throw.

(4) Repeat (2) and (3) for ten different known capacities C, and tabulate your results as follows:—

Distance between scale and mirror, L, = scale-divisions.

Throw of spot of light, *d*.	Value of $\sin\frac{\theta}{2}=\frac{d}{4\,L}$ very approximately.	Capacity used, C.	P.D. used to charge, V.	Constant of galvanometer, $K=\frac{CV}{\sin\frac{\theta}{2}}$.	Corrected mean galvanometer constant.

Inferences.—State clearly the assumptions that are made in

the above experiment, and also what corrections, if any, would have to be applied to obtain an accurate result. Has the method any particular advantages or disadvantages? State in words what the above galvanometer constant really means.

83. Calibration of a Ballistic Galvanometer (Standard Magneto-inductor Method).

Introduction.—The method is a very convenient one to employ when some standard inductor, such as Hibbert's (p. 358), is available. It, moreover, has the advantage of being independent of the earth's field, which, as we have seen, is in many cases an advantage, especially where this field is liable to variation.

Apparatus.—Standard inductor, I; ballistic galvanometer, G; box of known resistances, r; and either a short-circuit key or damping coil (p. 349), with its battery and key, to bring the galvanometer needle quickly to rest.

Observations.—(1) Connect up I, G, and R in simple series with each other, and the key (if one is used) across the terminals of G. Adjust the galvanometer needle to zero.

(2) With r small enough, and the needle *at rest*, slip I, and note the first throw d to the end of the scale, also the value of r.

(3) Repeat (2), increasing r so as to get about ten different values of d, decreasing by about equal amounts to O.

N.B.—Two or three throws should be taken for each value of r, and the mean noted as being more accurate.

(4) Calculate the galvanometer constant K from the relation—

$$K = \frac{NF}{10^9 R \sin \frac{1}{2}\theta}$$

and tabulate your results as follows:—

Standard inductor, ; turns, N, = ; resistance, r_E, = ohms; total flux cut, F, = C.G.S. lines.
Galv. resistance, G, = ohms; distance between scale and mirror, L, = scale-divisions.

Mean first throw, d.	Value of $\sin\frac{\theta}{2} = \frac{d}{4L}$ approximately.	Box resistance, r ohms.	Total resistance, $R = r + r_E + G$.	Galvanometer constant, $K = \frac{NF}{10^9 R \sin\frac{\theta}{2}}$	Corrected mean galvanometer constant.

Inferences.—State in words what the above constant really means. Are any corrections necessary in the formula to make it correct?

84. Standardization of Standard Magneto-inductor (Capacity Method).

Introduction.—It will be evident, on considering the preceding methods, that a standard inductor can be calibrated by comparison either with a standard solenoid, condenser, or earth coil. The present method is chosen as an example of the mode of procedure, because it is easily manipulated and independent of the earth's field.

Apparatus.—Standard inductor to be tested, I; sensitive ballistic galvanometer, G; standard condenser, C (p. 354); box of known resistances, r; battery, B, of known E.M.F., or, if this is unknown, a standard voltmeter to measure the P.D.; two-way spring tapping-key, K (p. 327); ordinary spring tapping-key, K_3; damping coil (p. 349) with its cell and key.

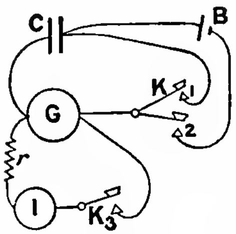


Fig. 66.

Observations.—(1) Connect up as in Fig. 66, and adjust the galvanometer needle to zero.

(2) K1 and K2 being open, adjust r to a low value, such that pressing K_3 nearly a full-scale throw d_I is obtained on slipping on I. Note this value of d_I and the box resistance r ohms.

(3) K_3 being open, adjust the capacity C to such a value that on closing K2 for 2 or 3 seconds, then opening it, and immediately closing K1, a first throw d_C is obtained on discharging C, as nearly as possible equal to the former.

N.B.—Two or three throws should be taken in both (2) and (3), and the means noted as being more accurate.

(4) Repeat (2) and (3) for about ten different decreasing pairs of throws by altering r and C.

(5) Calculate the total magnetic flux in the air-gap of the inductor from the relation—

$$F = \frac{100CVR}{N} \cdot \frac{d_I}{d_C} \text{ C.G.S. lines}$$

Tabulate as follows :—

Standard inductor, ; turns, N, = ; resistance, r_E, = ohms.
Galvanometer resistance, G, = ohms ; P.D. used, V, = volts.

Mean first throws.		Resistance in ohms.		Capacity in micro-farads, C.	P.D. if variable, V.	Total flux, F.	Mean flux.
d_I.	d_C.	In box, r.	Total, $R = r + r_E + G$.				

Inferences.—Show how the relation given in (5) can be obtained, and state any assumptions made in obtaining it. Is any correction required for high accuracy in the relation for F ?

85. Determination of the Relative Inductivities of Materials (Induction Balance Method).

Introduction.—The form of induction balance which will be used in the following tests is the one introduced by Hughes, a description of which will be found on p. 356. The extreme sensitiveness of the induction balance to minute differences of electric conductivity and magnetic permeability enable a very interesting and instructive series of observations to be carried out. The relative effects and inductivities will be determined by what we may term the "zero" telephone method, *i.e.* by balancing each material in the same way and against the same arrangement, so that *no sound* occurs in the telephones.

Apparatus.—Hughes' induction balance complete, with two telephones, battery, interrupter, and materials to be experimented upon.

Observations.—(1) Connect the two telephones in parallel, and across the two terminals of the secondary coils marked S.

Join the two terminals of the primary coils marked P in simple series with the battery and interrupter, which should be placed as far away from the balance as possible, so as not to interfere with the hearing at the telephones. A long length of flexible twin electric lighting lead should be used for the purpose. Wind the interrupter up *if necessary*, but *not to the full extent*, so as *not to overwind it.*

(2) Start the interrupter, by turning the lever controlling its clockwork, place a telephone to each ear, and with *no* metal in or near the coils of the balance, adjust the small ivory set screw which raises or lowers the top right-hand coil until *no sound* is heard in the telephones. The balance is now adjusted to "*zero*."

(3) Place the metal wedge with its attached millimetre scale between the guides of the left-hand top coil and a wooden cup in the other top coil. Insert one of the discs, all of which are of the same diameter and thickness, in the cup. Now slowly slide the metal wedge along the guides until just no sound is heard in the telephones. Note the scale-reading d_1, and then slide the wedge further on, and gradually bring it back until again no sound is heard; note the scale-reading d_2 and the *metal* used.

(4) Repeat (3) for each of the metals provided, noting whether the same kind of sound occurs in each case, and tabulate as follows:—

Metal tested.	d_1.	d_2.	Mean, $\frac{1}{2}(d_1 + d_2)$.

N.B.—If the arrangement is not sufficiently sensitive, increase the battery power. It will be observed that each metal disc being balanced against the same zinc wedge, the scale-readings will give a measure of the disturbing effects of these materials.

(5) Replace the wooden cup by the coil of insulated wire and remove the wedge, and note the effect, (*a*) when the ends of the coil are "*free*," (*b*) when they are joined together.

(6) Again insert the cup, and note the effect, (*c*) when a disc of non-magnetic material, (*d*) when one of magnetic material, is inserted with its plane perpendicular to that of the coils.

(7) Insert the two plugs, containing iron wires, in the two cups

placed in the respective secondaries, and move the one which has no handle attached to it up and down until no sound is heard in the telephones. Note the effect of slightly twisting the other wire by means of the handle.

Inferences.—State very clearly all the inferences which you can deduce from the results of the preceding experiments, suggesting the causes of the various phenomena observed.

86. Magnetic Permeability and Hysteresis (Absolute and Comparative Measurements).

Introductory.—Before proceeding to the actual methods of measuring the various magnetic properties of materials, it may be advisable to first give a brief *résumé* on some of the principles underlying such determinations.

Every magnet is surrounded by or sets up a magnetic field composed of lines of magnetic force, and the intensity of the magnetic field at any point can be measured by the number of lines of force passing through a square centimetre of surface placed across the field. If a magnetic pole is surrounded by a sphere of unit radius, and therefore containing 4π sq. cms. of surface, there will be 4π lines of force emanating from the pole if of *unit* strength. In other words, there will be one line of force per square centimetre, which represents unit field.

Thus a magnet of pole strength m has $4\pi n$ lines of force emanating from it. A uniformly wound solenoid of n turns per unit length and carrying a current A, will therefore exert a magnetic force $= 4\pi(\mathrm{A}n)$. Suppose now we have a straight bar of length l cms. and section s sq. cms., uniformly magnetized and possessing a pole strength m. Then $\mathrm{M} = ml$ is called its magnetic moment; and if V = its volume, then $\frac{\mathrm{M}}{\mathrm{V}} = \frac{ml}{sl} = \frac{m}{s} = \mathrm{I}$ is the intensity of magnetization of the bar. And further, if this is produced by a magnetic field or magnetizing force of intensity H, then $\frac{\mathrm{I}}{\mathrm{H}}$ is termed the susceptibility K or coefficient of magnetization. Again, let B stand for the magnetic induction or *number*

of lines of force per square centimetre *in the bar*. Then $B = 4\pi I + H$, and the ratio between the magnetizing force H and the internal induction which it produces in the bar is called the magnetic permeability μ of the material. Thus $\mu = \frac{B}{H}$.

The permeability of any magnetic material is therefore a property which it possesses, in virtue of which a given magnetizing force is able to produce a certain magnetic induction in the material. A certain magnetizing force is unable to produce the same magnitude of induction in different materials having the same size and form. This is owing to one material being more *permeable*, or offering greater facilities to the passage of magnetic lines of force through it, than the other, consequently, although the same number of lines are generated by the force in each case, they will gather up in and flow through that material which has the greater permeability, in larger numbers than in the other. For cores of air or non-magnetic materials, $B \propto H$ directly, μ being $= 1$ approximately. The dimensions of either H, B, or I are of the order $\frac{\text{force}}{\text{pole strength}}$, or $\frac{(\text{mass})^{\frac{1}{2}}}{(\text{length})^{\frac{1}{2}} \times (\text{time})}$. The methods for measuring permeability may be classified as follows :—

A. Magnetometric or steady deflection methods.

B. Balance or null deflection methods, applicable to comparative tests of two or more samples of material.

C. Ballistic or inductive methods.

D. Traction methods.

E. Optical methods, which are suited for measuring the permeability in intense magnetic fields.

In the following pages we shall only consider methods A, B, and C, and these we may now proceed with.

87. Measurement of Magnetic Permeability (Magnetometer Method).

Introduction.—This method, which is very generally associated now with the name of Professor Ewing, who has employed it in a large amount of research work in magnetism undertaken by him,

is one in which the magnetic qualities to be measured are deduced from the steady deflections of a suitable magnetometer needle, affected by the specimen to be tested. It is only applicable to long straight rods of the material for the following reasons. When a rod of magnetic material is uniformly wound with insulated wire and magnetized by a current sent through the coil, it develops *free magnetism* and definite polarity at its ends, which exerts a demagnetizing influence or force, opposing the main magnetizing force due to the coil. This effect becomes less marked as the rod is longer, owing to the ends, in which the free magnetism chiefly resides, being too far away from the middle regions to affect the magnetic force there. If the rod is very long (300 to 400 diameters), the magnetization will be practically uniform throughout a considerable portion of its middle region, though falling off towards the ends. In such cases no correction, *i.e.* deduction, need be applied to the magnetizing force, as calculated in the ordinary way from the current-turns, in order to obtain the actual magnetizing force producing a given induction. Hence the importance of using long straight rods. We must now see what the numerical value is of the correction to be applied when the rod is not long enough to neglect the demagnetizing action of its ends. The correction may be obtained approximately by treating the long cylindrical rod as a very long ellipsoid. Thus, if n is a number depending on the relation of the length of the ellipsoid to its transverse dimensions, and if H = the *effective* magnetizing force producing an induction B, and H_1 = the total magnetizing force as would be obtained from the ampere turns in the usual way, and I = intensity of magnetization—

$$H = H_1 - nI = H_1 - n\left(\frac{B - H}{4\pi}\right)$$

where $I = \frac{B - H}{4\pi}$, and if the material is very permeable to magnetic lines of force, B will be large compared with H.

$$\text{Hence I will} = \frac{B}{4\pi} \text{ very approximately}$$

$$\text{and therefore } H = H_1 - \frac{n}{4\pi} . B \text{ very approximately}$$

The following table gives a few values of n and $\frac{n}{4\pi}$, worked

out by Professor Ewing, for different ratios of length to breadth in ellipsoids :—

TABLE V.

$\frac{\text{Length}}{\text{Breadth}}$	n.	$\frac{n}{4\pi}$
50	0·01817	0·001446
100	0·00540	0·000430
200	0·00157	0·000125
300	0·00075	0·000060
400	0·00045	0·000037
500	0·00030	0·000024

Hence, if our long straight rod of circular section was 300 diameters long, the effective magnetizing force $H = H_1 - 0{\cdot}00006 \,.\, B$ approximately. We therefore see that even if B were carried up to 20,000 C.G.S. lines, the above deduction would be only 1·2 when the rod was 300 diameters long.

Another interesting phenomenon is that in connection with unclosed cores or rods of magnetic material. When a magnetizing force is removed, the residual magnetism still left in is partly or wholly annihilated by some force at or near the ends of the specimen. Long rods show this in a much more marked degree than short ones, and, if long enough, completely demagnetize themselves.

Another effect, known as "magnetic lag," or "creeping," is particularly noticeable in some specimens. It consists in a gradual creeping up of the magnetization after the particular magnetizing force has been applied. This is more apparent as the rod or specimen is thicker, and is especially so in the case of unannealed and hard materials, when the full induction may not be obtained for several seconds, and even minutes, after application of the magnetizing force. In such cases the magnetometer method is the only available one for use in order to obtain accurate results.

There are three modes of procedure available in performing this method, depending respectively on the relative positions in which the specimen, with its magnetizing coil, is placed with regard to the magnetometer needle when unaffected by external magnetism, and therefore at rest in the magnetic meridian. These we shall now consider, and call A, B, and C.

A. Position of Specimen.—This is shown in plan symbolically in Fig. 67, and is known commonly as the "*A. position of Gauss.*" RR is the magnetic meridian of the earth, XY the magnetometer bench, NS the specimen and its solenoid, *ns* the magnetometer needle. Thus it is seen that the magnetic axis of the solenoid NS is perpendicular to the magnetic meridian, passes through the centre of the needle *ns*, and lies in a horizontal plane.

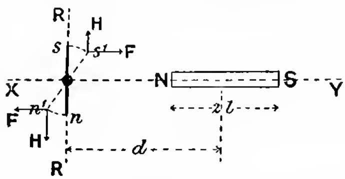


Fig. 67.

If V = the volume in cubic centimetres of the specimen,
H_1 = magnetizing force of the solenoid,
H_E = the horizontal component of the earth's magnetic force at the needle,
and θ° = the steady angular deflection of the needle for the position shown and some particular magnetizing force H_1, then—

$$\text{The permeability of the specimen } \mu = \frac{4\pi(d^2 - l^2)H_E}{V2d} \cdot \frac{\tan\theta^\circ}{H} + 1$$

B. Position of Specimen.—This is shown in plan symbolically in Fig. 68, and is known as the "*B. position of Gauss.*" The lettering corresponds with that of the various details of the previous figure, and need not again be explained.

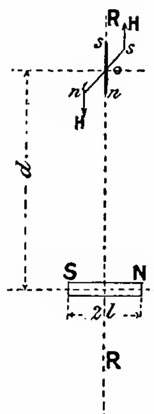


Fig. 68.

Here it will be observed that the magnetic axis of the solenoid NS is still perpendicular to the magnetic meridian RR, and lies in a horizontal plane passing through the needle, but the axis does not pass through the needle. We now have—

$$\text{The permeability of the specimen } \mu = \frac{4\pi(d^2 + l^2)^{\frac{3}{2}}H_E \tan\theta^\circ}{VH} + 1$$

C. Position of Specimen.—This is shown partly in plan and partly in perspective symbolically in Fig. 69, and is known as the "*one-pole position.*" It will be seen that now the magnetic axis of the solenoid NS is vertical, but does not cut the needle, also that the line joining the needle-centre and upper pole of NS is perpendicular to the magnetic meridian RR.

If D = distance between needle-centre and lower pole,
d = „ „ „ upper pole, then—

The permeability of the specimen $\mu = \dfrac{4\pi H_E \tan\theta^\circ}{S\left(\dfrac{1}{d^2} - \dfrac{d}{D^3}\right)(H \pm V_E)}$

where S = cross-sectional area of the rod in square centimetres,

and V_E = vertical component of the earth's magnetic force at the place where the test is formed, and = $H_E \tan\phi$, where ϕ = "angle of dip" at that place.

We thus see that in this magnetometer method either H_E or V_E, or both, must be accurately known, and the test ought to be performed in a room in which there are no iron pipes or girders, etc., as such may seriously affect the values of those earth components, the first of which directs or controls the needle, and the latter the value of the effective magnetizing force. Hence for accurate work it is important to determine these components just before an experiment at the place where the magnetometer is used. This can be most easily accomplished as follows: Place a small magnetic needle in the stirrup of the instrument shown on p. 335, which must be placed where the magnetometer is to stand. Now give the needle a motion of rotation (about 10° or 15° of arc) by bringing a magnet near it, which must then be taken right away. Count and note the time T taken for, say, twenty complete oscillations, each being counted from one middle position of a swing to the next but one, when the needle is again moving in the same direction. Thus the periodic time of oscillation will be $\frac{T}{20}$ seconds = t, say. Next, take the instrument to a part where the earth's field H_E' cannot be affected, taking care not to jar the needle in removing, again note the periodic time t_E in a similar manner as before. Then, since the respective field intensities are inversely proportional to the square of the periodic times

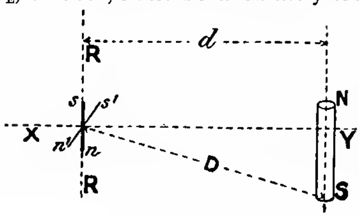


FIG. 69.

for the same needle, we have, if H_E = field at the magnetometer—

$$H_E = \frac{t_E^2}{t^2} H_E',$$

the value to use in the equations.

In the preceding formulæ the angular deflection θ° is assumed to be that due to the specimen alone. In order that this may be the case, a compensating coil, consisting of a few turns of wire on a wooden bobbin, is connected in series with the main solenoid, and is so placed that when a fairly strong current is sent through both, *no deflection* of the magnetometer needle occurs. This is the best way, but if no compensating coil is used, the effect of the solenoid alone can be allowed for by noting the deflection $\theta^{\circ\prime}$ produced by it for some given value of current A′. Then the deflection θ_1° for any other current A would be $\left(\frac{A}{A'}\theta^{\circ\prime}\right)$. Hence in the formulæ we should use $\left(\tan \theta^\circ - \tan \frac{A}{A'}\theta'\right)$ instead of $\tan \theta^\circ$ simply.

Since the angular motion of the needle is small we have $\tan \theta \propto \theta$ simply $\propto \frac{a}{2L}$, where a = scale-deflection and L = distance from mirror to scale in such divisions. When the method employs position C. of the solenoid, the vertical component of the earth's field will affect the total effective value of the magnetizing force on the specimen. This can be got over by sending a constant current from a separate source through a small coil wound over the solenoid, of such a strength as will neutralize V_E. This should be done after the specimen is completely demagnetized and in position inside the solenoid, the neutralization being effected when no deflection of the needle out of the meridian occurs. The demagnetization of the specimen can be effected by alternating a direct current from its maximum value to O, either by means of a suitable commutator or one of those on an Ayrton and Perry secohmmeter.

An ordinary alternating current of fairly high frequency will, when reduced to O, do equally as well.

From the preceding we therefore see that, though the magnetometer method is capable of giving very accurate results,

considerable care and some corrections must be applied to obtain such.

If T = the number of turns on the magnetizing coil,
l = its total length between the extreme ends,
and A = current in amperes flowing through it, then—

$$\text{The magnetizing force} = \frac{4\pi AT}{10l} = H_1 = M_f A$$

where $M_f = \frac{4\pi T}{10l}$ = a constant; and circumstances, as mentioned above, must decide whether this is the effective value.

Apparatus.—Delicate magnetometer, bench, and clamp, or sliding-table to hold the specimen and solenoid (p. 337); reversing key, K; suitable secondary battery, B, to supply the magnetizing current (p. 337); one Daniell's cell, *b*, or other constant E.M.F., together with a resistance box, *r*, for maintaining a constant current through the compensating coil, C; sensitive ammeter, A, for measuring the magnetizing current; and a rheostat, R, for controlling the latter (p. 310); the specimen, SS, either wound with turns of insulated wire from end to end or contained in the centre of a long solenoid, whose length is about a third again as great as the specimen, which latter may be 300 to 400 diameters long; this arrangement will ensure the material being very uniformly magnetized. We shall here choose the C. position of solenoid as less liable to error from the shifting of the poles P slightly for various magnetizations. This usually occurs, but it would alter tan θ infinitely less than with positions A and B.

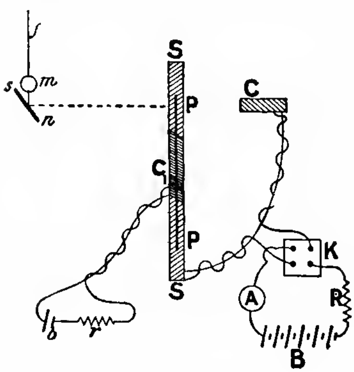


FIG. 70.

Observations.—(1) Connect up as in Fig. 70, adjust the magnetometer and ammeter A to zero, and use either twin flexible wire for connections as indicated, or run the leads in pairs close together so that they may not affect the magnetometer *ns*.

(2) Make a preliminary test to ensure only a full-scale

deflection with the maximum magnetization to be used, by setting SS at some convenient distance from *ns*; then, without any specimen inside, send a fairly strong current through SS and C, altering the distance of C to give no deflection. If now, on sending the maximum current with the specimen in position, the deflection is off the scale, move SS further away, and repeat until the spot of light keeps on the scale.

(3) With this current still flowing, and the centres of SS and specimen coinciding, raise and lower SS until the deflection is a maximum, and then fix SS in this position. The upper pole P will now be level with *ns*.

(4) Completely demagnetize the specimen in position.

(5) Carefully balance the vertical component by h, r, and coil C_1.

(6) With K open, make R a maximum, and close K, adjusting the current to its lowest value. Note the current A amps. and scale-deflection a.

(7) Raise the current step by step so as to obtain about fifteen different deflections, rising by about equal increments after the first two or three, which should be small, to the maximum, and note the value of A and a at each.

NOTE.—Sufficient time must be allowed the magnetization to assume or creep up to its final steady value, corresponding to A at each step prior to reading A and a.

(8) Calculate the values of B, H, and μ from the preceding formulæ, and tabulate your results as follows:—

Material, tested, ; length = cms. ; diameter = cm.; $\frac{\text{length}}{\text{diameter}} =$
Solenoid, ; length, l, = cms.; $M_f =$; sectional area of specimen, S, = cms.
D = ; $d =$ cms.; $H_E =$; $V_E =$
Distance, L, = scale-divisions; magnetizing turns, T, =

Sign of current.	Amperes, A.	$H_1 = M_f A$.	Effective value, H.	Scale-deflection, a.	$\tan \theta^\circ = \frac{a}{2L}$	B.	$\mu = \frac{B}{H}$	$I = \frac{B-H}{4\pi}$

(9) Plot two curves, one with B as ordinates and H as abscissæ, the other with μ as ordinates and B as abscissæ.

Inferences.—Prove the formulæ for μ given in the introduction, and state any assumptions made in obtaining them. State clearly all the inferences which you can draw from the results of your test.

88. Measurement of Magnetic Permeability (Ring Ballistic Method).

Introduction.—It has been pointed out in connection with the magnetometer method that prejudicial effects, due to the demagnetizing action of the ends of a straight bar on the magnetizing force at its central region, are introduced, and must be corrected for unless the bar is very long compared with its diameter. This action ceases to exist when the bar takes the form of a circular continuous or "endless" ring, whose actions therefore in this respect approximate to those of a long straight bar. Such a ring possesses no free magnetism—in other words, is non-polar—and if completely overwound uniformly with a magnetizing coil, allows none of the magnetic lines to leave the iron. The magnetic force, however, is not quite uniform throughout, being greatest at the inner periphery of the ring, and decreasing with increase of diameter, *i.e.* length of periphery. This is due to two causes: first, the propensity which every magnetic field has of finding the shortest path of least resistance; and secondly, owing to the magnetizing force proportional to the number of turns per centimetre length being greater at the inner periphery than the outer, for manifestly if r_1 and r_2 be the radii of these from the centre of the ring, n the total number of magnetizing turns, and A the current flowing through them, the magnetic force will vary from $\frac{4\pi An}{2\pi r_1}$ to $\frac{4\pi An}{2\pi r_2}$, which will be a minimum when r_1 practically $= r_2$. Thus the iron should have a small radial thickness compared with r_1 and r_2, and for this reason may preferably be of the form of a fairly thin short cylinder rather than having either a circular or square cross-section. This method is the only one by which we are able to test the magnetic quality of a ring, and when applied to such, it serves to measure sudden changes of magnetism only, due to sudden alterations in the strength of the magnetizing current.

The principle of the method consists in determining any sudden change in magnetic induction by measuring the quantity of electricity in the transient current induced in a *search-coil*

wound over the usual magnetizing coil of the specimen to be tested. The *throw* produced on the galvanometer by any sudden change in magnetism, due either to making or breaking the current, increasing or decreasing it by steps, or reversing it, will be proportional to the whole quantity of electricity which passes in the transient current, and hence to the change of magnetic induction within the coil. If the search-coil of N_2 turns encloses a total number of lines N of induction and δN = any sudden change which this undergoes, then, if R_2 = total resistance in ohms of the circuit in which the search-coil is placed, the galvanometer *throw* d_2 scale-divisions = whole quantity of electricity in the transient current $= \frac{N_2 \delta N}{R_2}$. The value of this can be found by observing the throw produced by a known change of induction due to an earth inductor of N_1 turns, area A_1 sq. cms., cutting a field of strength F. If R_1 now = total resistance in ohms of the whole circuit (inductor included), the whole quantity of transient current $= \frac{2 N_1 A_1 F}{R_1}$, corresponding to a throw d_1 scale-divisions

$$\text{Hence } \frac{N_2 \delta N}{R_2} : d_2 = \frac{2 N_1 A_1 F}{R_1} : d_1$$

If the earth inductor is kept continuously in circuit, the resistance of the rest of which remains unaltered,

$$\text{then } R_1 = R_2 \text{ and } \delta N = \frac{2 N_1 A_1 F}{N_2} \cdot \frac{d_2}{d_1}$$

If n = number of magnetizing turns, r = mean radius of ring, then mean value of the magnetizing force per ampere of current $M_f = \frac{4\pi n}{10 \times 2\pi r}$ in C.G.S. units. Again, if A = sectional area of the iron ring in square centimetres, and the resistance of the secondary circuit is constant all through the experiment, also the corrections for air-space within the coils negligible, then the change of induction per square centimetre in the iron corresponding to one scale-division of ballistic throw $= \frac{2 N_1 A_1 F}{d_1 N_2 A_2} = B_1$, say. To prove that the damping effect is not large enough to prevent the throws being proportional to the changes of induction, send

successively increasing currents through the magnetizing coils, and see whether the throws vary directly with the current when the circuit is broken.

In all cases the search-coil should have a small axial length, and, generally speaking, should be placed where the magnetization is most uniform, though in rings it may practically be at any part. It should be wound as close to the iron as possible, and may preferably be *next* to it when the ring is built up for permanent use (p. 349). If, however, the search-coil is wound outside the magnetizing coil, and its area includes any sensible air-space between the iron and its turns, or, which is equivalent, copper space, the following correction must be applied. Let A_3 = mean area of the magnetizing coil and H = magnetizing force. Then $(A_3 - A_2)H$ lines of force are enclosed in this intermediate equivalent air-space.

Hence we shall now have—

$$B_1 = \left\{ \frac{2N_1A_1F}{d_1N_2} - (A_3 - A_2)H \right\} \div A_2$$

This ballistic method can be carried out in one of three ways.

A. *Step by step*, consisting in increasing the magnetizing current progressively in steps from zero to the maximum without ever breaking it; then the throw produced by suddenly increasing the current by a step from, say, A_0 to A_1, or A_1 to A_2, etc., will be proportional to the change of induction within the coil. The harder the material the smaller should the steps be, especially just at first, in order to nullify as much as possible any magnetic creeping in the specimen which tends to diminish the throws, which ought really to represent the true integral change of induction for that particular step in the current. Since the total effect is given by the cumulative effects of the several throws, any error made in one of these will be carried forward to the end; hence care should be taken to ensure each being correct.

B. *Make and Break.* Here the current is increased or diminished by suitable degrees, but the throw is taken at each both by *making* and *breaking*, and the mean throw recorded. It will thus be evident that this mode of procedure would give considerably less accurate results with harder specimens than the last.

C. *Reversals*, in which the current is increased or diminished by suitable amounts at a time, and the throw taken on reversing the current at each, the actual magnetization being then proportional to *half the throw* approximately, the *residual magnetism* at each stage of the process being $\propto \frac{1}{2}$ effect of reversal − ballistic effect of breaking the current.

Apparatus.—Earth inductor, E; sensitive ballistic mirror galvanometer, G, having a periodic time of oscillation of at least 3 secs.; resistance box, r; damping coil or short-circuit key, K; dead-beat accurate ammeter, A; battery, b (p. 337); continuously variable rheostat, R; key, K_1; reversing switch, S; ring of magnetic material, M, to be tested, wound evenly all over with magnetizing coils, m, and a narrow search-coil, C (p. 349).

C. Reversals.

Observations.—(1) Connect up as shown in Fig. 71, and adjust the spot of light to zero. Completely demagnetize the specimen ring, either by a gradually diminishing ordinary alternating current of fairly high frequency or by a rapidly reversed direct current obtained through using a suitable commutator. This should be done after first adjusting r, so that when the maximum current is reversed, only a full-scale throw is produced.

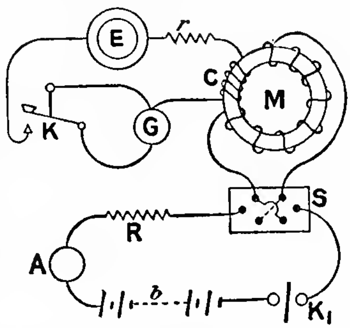


Fig. 71.

(2) Adjust R so as to get a very small current A. If the galvanometer needle is absolutely at rest, close K, and then suddenly *reverse* S, noting the throw d scale-divisions. Release K.

(3) Note the throw d_2 scale-divisions on *breaking* circuit at K_1, with the same current strength as was reversed in (2).

(4) Repeat (2) and (3) for about fifteen continuously increasing values of A up to that necessary for maximum saturation.

N.B.—At least two throws should be taken at each current, and the *mean* noted in each case.

Tabulate your results as follows:—

$N_1 =$; $A_1 =$ sq. cms. ; F = C.G.S. units ; mean earth inductor throw, d_1, =
$N_2 =$; $A_2 =$,, ; turns, n, = ; $r =$ cms.

Amps., A.	$H = AM_f$.	d.	$\frac{1}{2}l$.	d_2.	B_1.	$B = \frac{1}{2}dB_1$.	$(\frac{1}{2}d - d_2) = B_r$	R_1.	R_2.	$\mu = \frac{B}{H}$	$I = \frac{B-H}{4\pi}$

(5) Plot two curves having H as abscissæ and B and B_r as ordinates, also one with μ as ordinates and B as abscissæ.

Inferences.—State clearly all the inferences which you can draw from the results of the above experiment. What corrections ought to be applied to H and B to obtain an accurate result?

89. Measurement of Magnetic Permeability (Rod and Yoke Ballistic Method).

Introduction.—The method, which was devised by Dr. Hopkinson, is a convenient and fairly accurate one for testing the magnetic properties of tolerably short straight bars. It enables the condition of endlessness to be approximated to, thereby avoiding the prejudicial effects of the ends of the specimen.

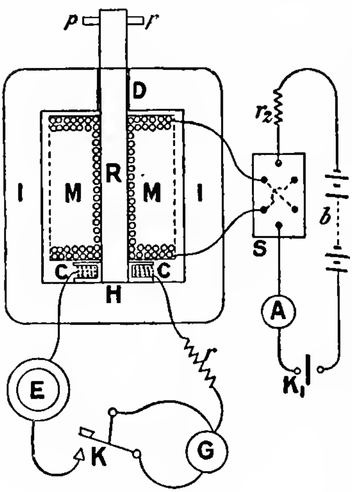


FIG. 72.

The rod R to be tested just passes freely through one end D of a massive yoke or block of soft annealed wrought iron I, the cross-section of which is many times greater than that of R. In the present case, the lower end of R butts against that of I, and the two surfaces thus in contact are extremely carefully faced so as to be quite plane, and make the best contact possible. R may, however, pass through the lower end of I as at D. R slips through a flat search-coil CC capable of sliding between guides

in a very small frame fixed to the block at H, and also through the magnetizing coil M, which practically extends the whole length of I inside. Thus the conductivity of I being so much better than R, keeps the magnetic field solely to the iron path, with the result that there is practically no free magnetism at D or H, and therefore no demagnetizing action due to such. Strip springs fixed to I and attached loosely to CC; spring the latter out of the influence of M when R is raised sufficiently high. The arrangement is shown in Fig. 227, p. 357. It may be noted that there is a thin fibre of air-gap at H and D; and owing to the effective area of the junction being smaller than that of the yoke, small errors may arise from this cause. This method is not very suitable for determining the hysteresis of the specimen R.

A steel pin, *pp*, passes through the upper end of the rod R for the purpose of obtaining a hold so as to withdraw it when any particular magnetizing force is applied. The search-coil need not be more than half a centimetre thick; hence, if magnetizing coil M was 20 cms. long, the uniformity of R's magnetization need not be seriously affected. In fact, the contact at H, however good, is far more likely to affect this. It will be necessary to allow for the effect due to the difference between the mean area of CC and the sectional area of the rod R, in the manner indicated in the preceding "ring method."

If l = length of magnetizing coil of n turns, then the magnetizing force at the centre will be $= \frac{4\pi n}{10 l}$ C.G.S. units per ampere of current.

Again, if B = the magnetic induction in the rod in C.G.S. units, and A_2 = the cross-sectional area of the rod, then BA_2 = the total induction or magnetic flux in the rod; hence, if R_2 = the total resistance of the search-coil circuit, and N_2 = the number of turns on the coil, then, on lifting R up so as to let CC fly out, the throw d_2 on the galvanometer will be proportional to the whole quantity of electricity in the transient current, and will $= \frac{BA_2N_2}{R_2}$, and from the ring method we see that the throw d_1 from the earth-coil will $\propto \frac{2N_1A_1F}{R_1}$. Hence, if A_3 = mean area of CC, we have, as before,

the induction per square centimetre in the iron per scale-division—

$$B_1 = \left\{ \frac{2N_1A_1F}{d_1N_2} - (A_3 - A_2)H \right\} \div A_2$$

The method can be applied in four ways: A, "Step by step;" B, "make and break;" C, reversals; D, detaching R and allowing the coil CC to spring out, the magnetizing current at the moment now being steady.

Fig. 72 shows a sketch of the connections and apparatus, which is exactly similar and lettered the same as that of the preceding ring method. The student should now carry out a test on a specimen precisely as indicated in this ring method, tabulating results as shown there.

90. Determination of the Relative Magnetic Qualities of Different Samples of Magnetic Material.

Introduction.—The tests may be conveniently and rapidly made by means of Hughes' magnetometer balance, which consists of a magnetizing solenoid S_m and its compensating solenoid S_c placed, one on each side of a magnetic needle *ns*, and with their axes in one horizontal line with the needle. S_c, which is connected up in *opposing* series with S_m, can be moved towards or away from *ns*, and clamped in such a position that its magnetic effect on *ns* exactly neutralizes that of S_m alone without any core. This being done for one current, it will be true for any other current strength. If a current is flowing through S_m and S_c, and a magnetic sample is inserted in the former, the needle will be deflected, but can be brought back to its zero position again by turning the controlling magnet C_m. If the distance between the centres of C_m and $ns = 2{\cdot}3$ times the length of C_m (about), then the magnetic effect on the needle is proportional to the angle through which C_m has been turned from the vertical in order to bring *ns* back to zero, and also to the magnetic force exerted by the specimen. This is true for angles not exceeding 60°. The method we therefore see consists in balancing each specimen against the same

permanent magnet, and being a "zero" one, enables very fair results to be obtained. Two important tests can be made on specimens, viz. that of finding—(α) the relative amounts of magnetism induced for equal currents; (β) the relative loss in different samples due to magnetic hysteresis on taking them through a complete cycle of magnetization.

Apparatus.—The magnetic balance (p. 355); two or three secondary cells, B (p. 337); ammeter, A; carbon rheostat, R (p. 307); reversing switch, S (p. 330).

Observations α.—(1) Carefully level the balance so that *ns* swings quite freely. Connect up as shown in Fig. 73, adjust the pointer of A to zero, and unscrew R.

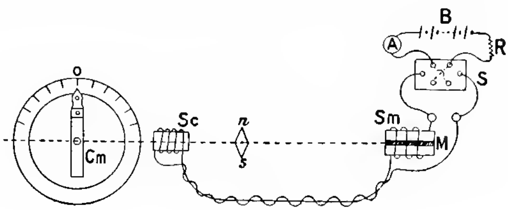


FIG. 73.

(2) Close S, and adjust R so as to pass a convenient current (say 1 amp.) through S_m and S_c; then, with C_m at O, move S_c so as to make the images of the fixed object-pin in the fixed and movable mirrors coincide, clamp S_c, and *ns* is then in its zero position.

(3) Adjust the current to 0·1 amp. Take a specimen, and demagnetize it as much as possible; then insert in S_m.

(4) Turn C_m so as to bring *ns* back to zero position. Note the deflection D of C_m and the current A amps.

(5) Carefully increase A to just 0·2 amp. without overstepping it, and repeat (4).

(6) Repeat (4) and (5) for currents up to about 1·5 or 2·0 amps., rising by 0·1 at a time.

(7) Repeat (3)-(6) for two or three different specimens, and tabulate as shown.

(8) Plot a curve for each specimen to the same axes, having D as ordinates and A as abscissæ. This will show how the induction varies with the magnetizing force for the specimen.

Observations β.—(1) With C_m and *ns* at zero, and the

same connections as above, adjust the current to 2 amps., say, or to a value sufficient to saturate the specimen. Then, by varying R, decrease the current to O, reverse by S, and increase to the negative maximum. Then decrease to O again, reverse, and lastly, increase to the same positive maximum as at first.

(2) Repeat this operation or cycle of magnetization two or three times, finally noting the deflection D of C_m to bring *ns* to zero and the current A.

(3) Decrease A by about 0·2 amp. at a time to O, noting D for each current; then reverse S.

(4) Increase A by about 0·2 amp. at a time to negative maximum, noting D for each current.

(5) Decrease A by about 0·2 amp. at a time to O, noting D for each current; then reverse S.

(6) Increase A by about 0·2 amp. at a time to positive maximum, noting D for each current.

(7) If necessary, repeat (1)–(6) for different specimens, and tabulate as follows:—

α.			β.		
Specimen.	Deflection, D.	Current, A amps.	Specimen.	Deflection, D.	Current, A amps. with right sign.

(8) Plot the relative hysteresis curves for each specimen, having D as ordinates and A as abscissæ, with due regard to the sign of D and A.

NOTE.—The area of the looped curve so formed is proportional to the work done, and therefore to the energy wasted in taking the specimen through that complete cycle of magnetization.

91. Measurement of Magnetic Hysteresis (Ballistically).

Introduction.—When a magnetic substance is subjected to different magnetizing forces, it exhibits a tendency to persist in any magnetic state which it may have acquired. This is illustrated

by the residual magnetism in the material after the magnetizing force has been wholly withdrawn. If it is gradually made to decrease from a positive maximum to zero, then reversed and increased to a negative maximum, now decreased to zero, again reversed, and lastly, increased to the positive maximum from which it started, the curves between induction B and magnetizing force H will not be coincident, but will form a loop which represents a complete *cycle* of magnetization. Its area is a measure of the work wasted or transformed into useless heat in carrying the iron through the cycle. With hard iron, and particularly steel, the loop has a large area, indicating a large waste of energy. The tendency of the changes of magnetization to lag behind the changes of magnetizing force is called *magnetic hysteresis*, and it may be regarded as a kind of internal or molecular magnetic friction, by reason of which alternate magnetizations cause the iron to grow hot. The effect of hysteresis is to prevent any simple relation existing between H and B or H and I; hence, to define μ, the way in which the iron has been treated must be known. The area of the loop, viz. $\int H \, . \, dB$ approximately = that of a rectangle, the height of which is double the *remanence* and the breadth double the *coercive force*. The curves forming the loop, which is usually called the B – H curve of hysteresis, can be conveniently determined by the step-by-step ballistic method.

Apparatus.—The same as that for the Permeability Test for Rings (Ballistically).

Observations.—(1) Connect up the apparatus as shown in Fig. 71, and adjust the galvanometer needle to zero. With K open and K_1 closed, take the specimen through two or three complete cycles of magnetization in the way indicated above, stopping at the positive maximum, but still allowing this positive maximum current to continue to flow.

(2) With S in its last position in (1), and K, K_1 closed, quickly increase r_2 so as to suddenly reduce the current by a suitable amount to a lower value A_1 amps. Note this and the throw d_1 divisions.

(3) From this new current value A_1 take the specimen through one complete cycle, ending at the same value A_1.

(4) Repeat (2) and (3) alternately for about five equal decrements of current down to zero.

(5) Reverse the direction of magnetizing current by means

of S, and with K, K_1 closed, obtain throws for about ten sudden equal increments of current, with one cycle after each throw, up to the negative maximum, as indicated in (3).

(6) Repeat (2)–(4), with S in position used in (5).

(7) Reverse the current, and repeat (5) up to the positive maximum.

NOTE.—Care must be taken that no error is made in measuring the throw at any step, as it will cause an error in all future readings. To avoid this, *three throws* should be taken at each current, and the mean noted. The amount of magnetism at any stage is given by the *algebraical sum* of the throws up to that point; hence care must be taken to affix the right sign.

Tabulate as follows:—

$N_1 =$; $A_1 =$ sq. cms.; F = C.G.S. units.
$N_2 =$; $A_2 =$,, ; turns, n, = ; $r =$ cms.

Amperes, A.	$H = AM_r$	Throw, d_1.	B_1.	Throws with proper sign, $D_1, D_2, \ldots$	Algebraical sum, ϵD.	Induction, $B = B_1 \epsilon D$.	R_1.	R_2.

For the various symbols, see experiment on Permeability (Ring Method).

(8) Plot the B – H curve, reckoning axes north and east positive and south and west negative, and having B on the ordinates.

Inferences.—State clearly the inferences you can draw from the results of the above experiment. Find the residual magnetism, as a percentage of the induced, also the watts wasted in the specimen, if the work done per cubic centimetre of the metal

$$W = \frac{1}{4\pi}\int_{H_1}^{H_2} H.dB = \frac{\text{area of loop}}{4\pi} \text{ per cycle.}$$

If H and B are in C.G.S. units, W will be in ergs per cubic centimetre.

If N = number of cycles per second, then watts wasted $= \dfrac{WN}{10^7}$.

The actual method of plotting a B – H curve of magnetic hysteresis from the experimental figures obtained may not perhaps be at first sight perfectly obvious to every student. The method of procedure, therefore, is as follows:—

Take any point A as a temporary origin, from which draw the

two rectangular axes, AC and AD. Arrange so that the ordinates AC represent the figures in the column headed B of the table, and the abscissæ AD those in the column under H. Now, on plotting in the usual way the first half of the two columns up to the negative maximum for H, we obtain the curve APS_1A_1; next plotting the return set of figures, or last half of these columns, we get the curve A_1P_1SA. These two curves will probably not coincide at A. However, together they form a complete *cycle* of magnetization. Next halve AC, the vertical distance between AA_1, drawing the horizontal line XOX_1. Similarly bisect AD, the horizontal distance between AA_1, drawing the vertical line YOY_1, intersecting XOX_1 in the point O, which is the true origin of the cyclic curve. Now renumber the abscissæ XX_1 either way from O with the same scale as AD; also renumber the ordinates YY_1 each way from O with the same scale as AD. Then OX and OX_1 will represent $+H$ and $-H$ respectively, while OY and OY_1 will represent $+B$ and $-B$ respectively.

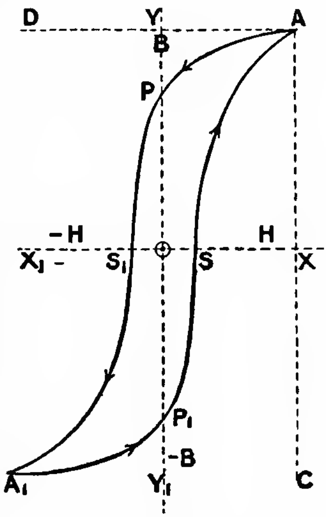


FIG. 74.

OP, the induction which remains after the magnetizing force H is gradually reduced from its maximum to O, is called the *remanence*, or *retentiveness*, and the magnetizing force OS_1 required to annul the induction is called the *coercive force* of the material.

92. Measurement of Magnetic Hysteresis (Magnetometer Method).

Introduction.—This method only differs from the corresponding one for measuring magnetic permeability solely in the manner in which the magnetizing current is varied. It forms a

convenient and accurate means of measuring the magnetic hysteresis of any material, and is eminently suited for testing those materials which possess considerable magnetic lag and creeping. The remarks made in connection with the permeability test by the magnetometer method (pp. 146–153) apply in the present instance precisely, and therefore the reader is referred to that test for all details of the method.

Apparatus.—Same as in the above-named method.

Observations.—(1) Carry out Obs. (1)–(8) of that test exactly as there indicated, but do not break the magnetizing circuit when the maximum (which will be termed the positive) has been reached.

(2) Reduce this current in about three or four steps to O, noting its value A and the steady scale-deflection a at each.

N.B.—The deflection a, when A = O, gives the *remanence*, or residual magnetism, in the specimen.

(3) Reverse at K and close it, repeating Obs. (7) up to the negative maximum.

(4) Repeat (2) above, down to zero.

(5) Reverse at K, and close it, repeating Obs. (7) again up to the positive maximum, thereby completing one *cycle* of magnetization.

(6) Calculate the values of B and H, and tabulate as shown.

(7) Plot the B – H curve of magnetic hysteresis, having B as ordinates and H as abscissæ, paying every regard to "sign," and reckoning axes north and east as positive and south and west as negative.

Inferences.—State any inference you can draw from the above test. Find the watts wasted in the specimen and the residual magnetism as a percentage of the induced.

93. Capacity (Absolute and Comparative Determinations).

Introductory.—Without in any way considering the actual practical methods of constructing condensers, in connection with which we are most frequently accustomed to speak of capacity, the

following remarks, which apply generally to all such apparatus, are perhaps necessary to impress upon the reader the various terms, phrases, and phenomena met with in this class of work.

A condenser may be defined as an arrangement of two conductors separated by an insulator; the former are called the coatings, and the latter the dielectric, of the condenser. If the poles of a source of E.M.F. be connected to the respective coatings, a P.D., V, will be acting across them, and a quantity, Q, of electricity will flow into the condenser. It will then be found that, if there is no leakage, $Q \propto V$, or $\frac{Q}{V} = C$ (a constant), which is called the *capacity* of the condenser. Hence the capacity of any condenser is the quantity of electricity required to be given to either coating in order to produce unit potential difference between the coatings. The unit of capacity is called a farad, and $= 10^{-9}$ C.G.S. absolute units of capacity. A condenser has a capacity of 1 farad when a P.D. of 1 volt between its coatings charges each of them with 1 coulomb.

Since, also—

$$C = \frac{Q}{V} = \frac{\text{quantity}}{\text{potential}} = \frac{\text{current} \times \text{time}}{\text{potential}} = \frac{\text{time}}{\text{resistance}}$$

we see that the dimensions of capacity are of the order of $\frac{\text{time}}{\text{resistance}}$, *i.e.* $\frac{\text{time}}{\text{velocity}}$, or $\frac{T}{\frac{L}{T}} = \frac{T^2}{L}$, *i.e.* $\frac{\text{time}}{\text{length}}$ in the electro-magnetic system of units. This "absolute unit" is far too large to deal with, and even the farad, which, as we see, is only $\frac{1}{10^9}$ of this absolute unit, is a great deal too large for practical purposes, so that a smaller unit = one-millionth part of a farad, and called, consequently, a microfarad, is universally adopted. Hence 1 microfarad $= 10^{-15}$ C.G.S. unit. When a condenser is charged, some time elapses before its terminal P.D. rises to its maximum value, or, in other words, before the whole charge is absorbed or gets in, which does not occur immediately. This "soaking in," so to speak, is due to surface action in the dielectric. Similarly, on discharging, the whole of the previous charge is not removed immediately; some has *soaked* in giving rise to what is called a *residual charge*, part of which can be obtained on again

discharging, and a condenser exhibiting such properties is said to possess *residual absorption.* Again, if the condenser is not well insulated, the charge will slowly leak away, independent of any proper discharge. We thus see that the *actual capacity* of a condenser is a more indefinite quantity than we are accustomed to regard it, and this is due to absorption, leakage, residual charge, etc. In fact, the capacity of any condenser depends on those quantities, together with the time for which it is charged or discharged, and the manner of doing this. Hence the difficulty of obtaining a very accurate measure of the capacity of condensers by any of the following methods will be apparent. Possibly the best definition of the capacity of a condenser is that it is the instantaneous quantity or charge required to produce unit P.D. between the coatings.

The method to be adopted in any particular instance will depend on the needs of the case, the apparatus at hand, and the type of capacity to be tested. For instance, much residual absorption in the "direct deflection" and "bridge" methods leads to inaccurate results, whereas the "method of mixtures" is not so much affected by it. Thus it will be seen that one method may give accurate results when one particular phenomenon is present, whereas another method would be quite inaccurate. In most cases null methods are to be preferred, and invariably high insulation is absolutely necessary. It is important to note that the quantity of electricity required to charge a condenser is quite independent of the resistance of the circuit, but the *time* required to charge increases as the resistance increases. If $C =$ capacity of the condenser in *microfarads* and R the resistance in megohms through which it is charged, then CR is called the "time constant" of the condenser and will be in seconds; and practically a condenser becomes fully charged in a time = 8CR or 10CR secs. In deflection methods, a sensitive ballistic galvanometer will be required, having a periodic time of oscillation not less than about 2 or 3 secs. The damping will not be very important, as all throws are affected in the same manner, though it should be small. A galvanometer resistance of from 5000 to 10,000 ohms is common in very sensitive instruments.

The standard condenser (such as that illustrated on p. 354), which is a most convenient one for use both in comparative

determinations and those requiring only one condenser of known capacity, should possess the two all-essential properties of high insulation and accuracy.

If the former is low in any part, it will not hold its charge for any appreciable time, and cannot therefore be used with any degree of success.

The insulation resistance, which may amount to many thousands of megohms, if high enough may enable the condenser to retain its charge for an hour or two without appreciable loss.

A high potential difference applied to a condenser for a considerable period may so strain the dielectric that it might take days to discharge and attain its original neutral condition.

94. Leakage, Absorption, and Residual Charge in Condensers (Experimental Determinations).

Introduction. — In a perfect condenser it has just been remarked that the above phenomena are absent; in an air condenser they are usually present to a slight extent only; but in the ordinary condenser they are in force to a much greater extent. The following observations are arranged with a view to obtaining a concrete idea of the extent to which they are present in a given condenser.

LEAKAGE.—The amount of this will depend on the insulation resistance between the two coatings and on that between either of these and earth, and also on the material forming the dielectric. An idea can be formed of the magnitude of the leakage in any condenser during a given time as follows:—

(1) First see that the condenser is free from dust and not dirty. Then charge it by a suitable E.M.F., and immediately discharge it through a suitable ballistic galvanometer, noting the first throw d_1 produced.

(2) Again charge the condenser for the same length of time and with the same E.M.F., and let it stand with its terminals quite free in air for, say, 15 minutes, and then at once discharge it through the galvanometer, noting the throw d_2 produced.

Now, evidently the amount of leakage is proportional to $d_1 - d_2$, and is $\frac{d_1 - d_2}{d_1}$ of the original charge in the 15 minutes.

$$\therefore \text{ it amounts to } \frac{d_1 - d_2}{d_1} \times \tfrac{100}{15} \text{ per cent. per minute}$$

For mica as the dielectric, this might be only about $\frac{1}{2}$ to 1 per cent., but for paraffin paper it would probably be something like ten times as great. In this way, by charging several kinds of condensers by the same E.M.F., a comparison can be obtained as to which is the best dielectric, providing of course they are built up with the same care in an exactly similar way.

ABSORPTION AND RESIDUAL CHARGE.—These phenomena, occurring as they do together, will be considered in conjunction with each other, for the latter is a result of the former.

They occur to a greater or less extent in all condensers having either a solid or fluid dielectric, and, as will be seen later, tend to vitiate the results of comparisons as well as absolute determinates of capacity. Electric absorption is the name given to the phenomenon whereby a condenser, charged with a long contact with the battery, takes or absorbs a larger quantity of electricity than with a short contact with the battery. Thus a portion of this charge is gradually absorbed. Again, if such a condenser is discharged, after a few minutes' rest another smaller discharge can be taken from it, and again another after a further rest, and so on, before it is ultimately completely discharged. Thus a "residual charge" seems to be left behind after each discharge, which phenomenon is known as "residual absorption."

By means of the following method of procedure we can determine how the absorption or soaking in of the charge, so to speak, varies with the time of charge.

ELECTRIC ABSORPTION.

Apparatus.—Sensitive ballistic galvanometer, G (p. 285); two-way spring tapping-key, K, of high insulation (p. 327), and a suitable battery, B, of fairly constant E.M.F.; condenser, C, to be tested.

Observations.—(1) Connect up as in Fig. 75, and adjust the spot of light to zero.

(2) After completely discharging C by a prolonged short circuit, press K1 for an *instant*, thereby charging C with a P.D. = V,

and immediately discharge it through G by pressing K2 with a short contact. Note the first throw d_0 on discharge.

(3) Press K2 so as to discharge C in, say, half-minute intervals, noting the respective first throws d_1, d_2, d_3, . . . up to about five in number.

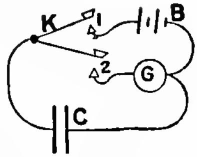


FIG. 75.

(4) Now charge C with the same P.D., V, for some *definite time* by keeping K1 pressed for, say, 2 minutes, and then immediately discharge it through G, noting the first throw d_0'.

(5) Repeat (3), getting throws d_1', d_2', d_3', d_4', and d_5'.

(6) Repeat (4) and (5), allowing 2 minutes longer for charging each time up to about six charges, lasting respectively t_0, t_1, t_2, t_3, . . . minutes, and employing half-minute intervals between all discharges, and tabulate as follows :—

Time taken in charging, t.	d_0.	Residual charge proportional to first throws.				
		d_1.	d_2.	d_3.	d_4.	d_5.

(7) Plot curves between values of t in minutes as abscissæ, and the respective sets of values of d_1, d_2, d_3, . . . as ordinates on the same curve-sheet.

Inferences.—State clearly all the inferences which you can make on the results of your observations.

95. Relation between "Time Constant" and Current in a Condenser Discharge (Siemens's Deflection Method).

Introduction.—The present test will at once be seen to have an important bearing in all capacity work by reference to the remarks on p. 168, from which it will be gathered that the *time* in which a condenser takes to properly charge or discharge depends on the resistance of the circuit in which it is placed, but

that the actual quantity of electricity in either case is independent of this resistance.

Apparatus.—Sensitive ballistic galvanometer, G (p. 285); adjustable very high known resistance, r; charge and discharge key, K (p. 328); fairly constant battery of about ten cells, B; condenser, C.

Observations.—(1) Connect up as in Fig. 76, and adjust the spot of light to one end of the scale as a zero.

(2) With $r = 0$, alter the number of cells in B so as to get a first throw of the spot of light to the other end of the scale on charging C by closing K. Note this throw D_0 scale-divisions.

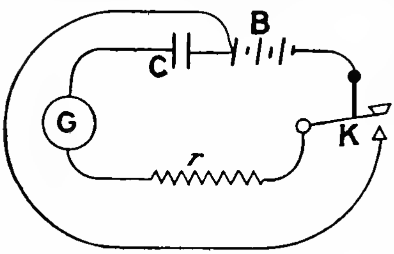


FIG. 76.

(3) Adjust r to such a value that, on pressing K, the resulting first throw = about $\frac{1}{10}D_0$. Note this throw D and the value of r.

(4) Repeat (3) for about ten values of D, rising by about equal increments to the maximum. The throws D are proportional to the current flowing during discharge, and hence to the potential of the condenser.

(5) To correct an error due to the battery E.M.F. having altered, repeat (2) again, and if D_0 is now less than before, take the mean of the two values.

(6) Determine the periodic time of oscillation T of the galvanometer needle in seconds, and tabulate your results as follows :—

Capacity, C, = microfarads ; galvanometer resistance, g, = ohms ; throw, D_0, = ; T = secs.

Resistance, r ohms.	Total resistance, $R = \frac{(r+g)}{10^6}$ megohms.	Mean throw, D	$\frac{D}{D_0}$	Time constant, CR secs.	$\frac{CR}{T}$

(7) Plot curves on the same curve-sheet between D and $\frac{D}{D_0}$ as ordinates and CR and $\frac{CR}{T}$ respectively as abscissæ in each case.

Inferences.—State clearly all you can infer from the results of your observations.

96. Comparison of Capacities Ballistically (Direct Deflection Method).

Introduction.—This direct discharge or charge method is one of the simplest for finding the ratio of the electrical capacities of two condensers, or of finding the capacity of an unknown condenser in terms of a known standard capacity. It, however, possesses the objection common to all such methods, namely, that a deflection is being measured. The rationale and whole essence of the method consists in charging the condensers separately with the *same* or *equal* P.D.'s, and noting the respective *first* throws of the needle or spot of light of a ballistic galvanometer on charging or discharging them separately through it, which are directly proportional to the respective capacities. Should the two condensers differ very much in capacity, so that when, say, the larger gives a convenient deflection the other gives too small a one, or *vice versâ*, then the deflections must be made more nearly equal, either by shunting the galvanometer and so reducing its sensibility with the larger one, or using different P.D.'s bearing a known ratio to one another, to charge the condensers. This latter method is the more preferable one of the two, for, as was first pointed out by Mr. Latimer Clark, the use of a shunt with ballistic galvanometers introduces a certain vagueness in connection with "transient" or short-duration currents, which are dealt with in all ballistic work, due to the self-induction of the shunt and galvanometer being probably very different, which would seriously affect the proportion in which the current is usually divided between these two branches in the case of steady current work. For a more detailed discussion on this matter, *vide* p. 113. If, therefore, the galvanometer of resistance g is shunted with a shunt of resistance s when used with the larger capacity C_1, then the relation is $C_1 : C_2 = \frac{s+g}{s} . d_1 : d_2$ approximately, where d_1 and d_2 are the respective first throws of the spot of light in scale-divisions, when C_1 and C_2 are charged or discharged, from which it will be at once seen that, if the throws d_1 and d_2 are equal, the approximate multiplying power $\frac{s+g}{s}$ of the shunt gives the ratio

of the two capacities directly, and s could be adjusted to get this. Owing to the resistance of an ordinary condenser being so enormous compared with that of any ballistic galvanometer, the shunting of this latter with any resistance does not appreciably alter the battery current from the resulting diminution of the resistance of the rest of the circuit. If, instead of employing a shunt to reduce the sensitiveness of the galvanometer, the two throws are made more nearly equal by using two different P.D.'s, the smaller, V_1, of which is used to charge the larger capacity C_1, and the larger, V_2, to charge the smaller capacity C_2, then we have the relation—

$$\frac{C_1}{C_2} = \frac{V_2}{V_1} \cdot \frac{d_1}{d_2}$$

where d_1 and d_2 are the first throws as before.

Since merely the ratio of the two P.D.'s is required, and not their actual value in volts, the best way to obtain them is shown in Fig. 77, in which a potentiometer box, AC, of total resistance r_2, is placed across a battery, B; then, if the two leads l, l going to the charging circuit pick off potential respectively from the whole resistance r_2, and then from a known fraction AD $= r_1$, we have $V_1 : V_2 = r_1 : r_2$, which at once gives the required ratio of P.D.'s.

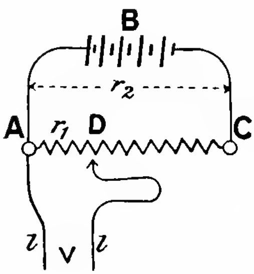


FIG. 77.

In this test, and, in fact, with all condenser work especially, great care should be taken to have the *whole circuit*, except the battery, *well insulated*, and consequently the number of keys and all apparatus, not absolutely necessary, should be reduced to a minimum, so as to prevent leakage from the condenser, when charged, taking place, thereby causing an apparent diminution of the capacity.

Apparatus.—Delicate mirror ballistic galvanometer, G, of high resistance (p. 285); condensers, C_1, C_2, to be compared; high-insulated Pohl's commutator, P (p. 330), and charge and discharge spring-key, K (p. 328); suitable battery, B, of one or more cells; a suitable damping coil (p. 349), with its extra cell and key.

Observations.—(1) Connect up the above apparatus as in

Fig. 78. See that P and K are quite clean and free from dust, and adjust the spot of light to zero. Slide out the little ebonite plugs on the terminal rods of G, and of C_1, C_2, if they have them.

(2) With P over to C_1 and the spot of light absolutely at rest, press K, thus charging C_1 through G, and note the first throw on G.

(3) With K still down, bring the spot of light to zero quickly by either a magnet or damping coil (p. 349); then release K, thus discharging C_1 through G, and note the first throw on G.

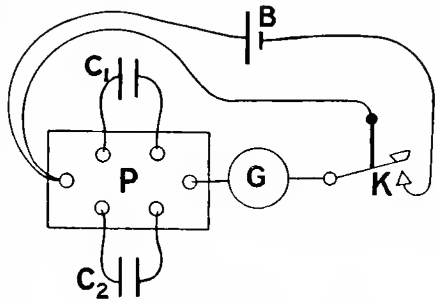


FIG. 78.

(4) Repeat (2) and (3) about four times, and take the mean d_1 of all the throws.

(5) Turn P over to C_2, and repeat (2)–(4) with condenser C_2, obtaining a similar mean d_2.

(6) Repeat (2)–(5) with about three different values of the standard capacity, if this be an adjustable one such as that on p. 354.

(7) If it is not adjustable, then repeat (2)–(5) with about three different magnitudes of scale-deflections by varying the number of cells in B.

NOTE.—To see whether the battery P.D. has altered after doing (5), repeat (2) and (3) again with condenser C_1. If, then, d_1 has altered, take the mean of the first and last values of d_1 for use in the formula, thus eliminating the variation when small.

(8) If no shunt across the galvanometer and also none but the same P.D. has been used, compare the two capacities, and calculate the value of the unknown from the relation—

$$C_1 : C_2 = d_1 : d_2 \text{ approximately}$$

and tabulate your results in the following form :—

Galvanometer resistance = ohms at ° C.

Condensers.		Shunt, resistance (if any), s.	$\frac{s+g}{s}$	P.D.'s used (if different).			First throws.		Capacity.	
No. of unknown.	Value of standard.			V_1.	V_2.	Ratio, $\frac{V_2}{V_1}$	d_1.	d_2.	Ratio, $\frac{C_1}{C_2}$	Unknown.

Inferences.—Prove the relation given in (8), and state any assumptions made in obtaining it. Why is the relation only approximate, and how can it be made a perfectly true one? What sources of error (if any) is this method liable to?

97. Comparison of Capacities (Wheatstone Bridge Method).

Introduction.—This method, due to De Santy, resembles closely the ordinary Wheatstone bridge method of measuring resistance, and has the great advantage of being a zero or null one. It is applicable to ordinary condensers and short lengths of cable, though not for long lengths, owing to the effects of inductive retardation. It is most easily applied in the case of similar condensers having the same material as dielectric. In other cases, the effects of electric absorption make accurate balancing a difficulty. These can be minimized by "making" and "breaking" the battery circuit very quickly. The sources of error and necessary corrections to be applied in deflection methods are non-existent here, which makes it one of the best-known zero methods. In Fig. 79, (*a*) and (*b*) are similarly lettered, the former showing the arrangement symbolically, the latter the actual one for experiment.

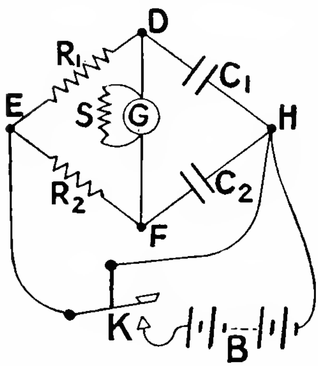
The effect of a difference in the electric absorption of the two condensers C_1 and C_2 is that when the bridge is balanced for, say, the charge, it will not be so for the discharge. If, however, either the terminals of each condenser be short-circuited or R_1, R_2 made $= 0$ for a moment before charging by the key K, then, on pressing K, the deflection will more accurately represent the state of

balance, or otherwise, when K is pressed up and down rapidly, the spot of light should not gradually *creep* away from zero when balance is obtained.

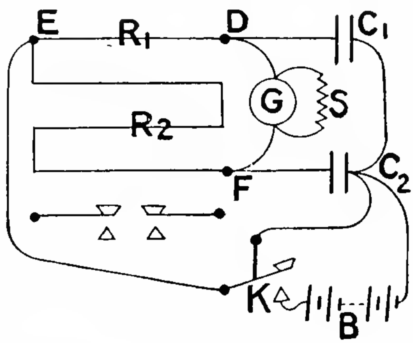
Apparatus.—Sensitive high-resistance mirror galvanometer, G (p. 284), and shunt, S (p. 323); charge and discharge key, K; P.O. bridge, or other adjustable resistance boxes, to form the arm DE and EF; condensers, C_1, C_2, to be compared; battery, B, of about ten or more Leclanché cells.

Observations.—(1) As a precautionary measure against damaging G, short-circuit it by means of S, and connect up as indicated in Fig. 79 (*b*), and adjust the spot of light to about zero.

(2) The resistances R_1, R_2 should be high for maximum



(*a*)



(*b*)

FIG. 79.

accuracy, and if they are obtained from a P.O. bridge, first set R_1 to 2220 ohms by taking all the six plugs out; then adjust R_2 so that on making or breaking, *i.e.* manipulating K down or up respectively, but preferably the former, no motion of the spot of light ensues, S being gradually cut out, as the balance gets nearer, to finally no shunt at all. Note the values of R_1, R_2.

(3) Repeat (2) for R_1 = 2000, 1220, and 1000 ohms respectively, which will be the best figures in the present circumstances.

NOTE.—With the smaller resistances it may be found advisable to increase the number of cells in B so as to obtain a more sensitive adjustment.

(4) Repeat (2) and (3) for about four different values of

standard capacity, if one of the condensers is of such a nature (as, for instance, Fig. 223) and adjustable.

(5) Compare the two capacities, and calculate the unknown from the relation—

$$C_1 : C_2 = R_2 : R_1$$

which will be observed to be just the reverse to the ordinary Wheatstone bridge law for resistances.

Tabulate your results as follows:—

Capacities.		Resistances.		Capacities.	
Number of unknown.	Standard.	R_1.	R_2.	Ratio, $\frac{C_1}{C_2}$	Unknown.

Inferences.—Prove the relation given in (5), and state any assumptions made in deducing it.

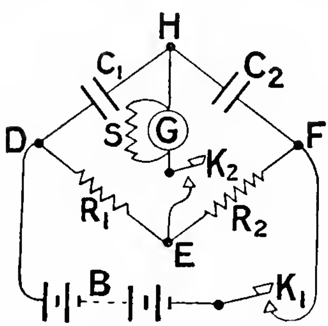
98. Comparison of Capacities (Secohmmeter Method).

Introduction.—The sensitiveness of the Wheatstone bridge method can be considerably increased by using an Ayrton and Perry secohmmeter instead of the key, K. A description of the sechommeter will be found on p. 259. It may preferably be motor driven, and should have its commutators set in the midway adjustment for this test, so that the galvanometer is reversed midway between two successive battery reversals. The rest of the apparatus is exactly the same as in the preceding bridge method, except that K is dispensed with. Connect E and H to terminals marked "bridge" on the battery side of the secohmmeter, and D and F to those marked "bridge" on the other side, the battery, B, and galvanometer, G, being directly connected to their respective marked terminals. Now rotate the instrument, and carry out the preceding experiments (1)–(5) exactly. Increasing the speed will increase the sensibility of the test without affecting the relation connecting the ratios of capacities and resistances R_1, R_2.

99. Comparison of Capacities (Wheatstone Bridge Method).

Introduction.—This is a very simple and convenient method and has the great advantage of being a null or zero one. It only differs from that of De Santy in using two ordinary spring tapping-keys instead of the "charge and discharge" key used there, and also in having the relative positions of galvanometer and battery interchanged. It is most important that both the condensers and all parts of the circuits be well insulated; and if this is the case, the battery need not be, for leakage occurring in any other part of the system but this, will cause errors, and since the capacity varies inversely as the terminal potential for a given charge, we see that a diminution of this potential through leakage during the test will cause the capacity to apparently increase.

Apparatus.—Sensitive high-resistance reflecting galvanometer, G (p. 284), and shunt, S (p. 323); two spring tapping-keys—one, K_1, ordinary, the other, K_2, highly insulated; a battery, B, of ten



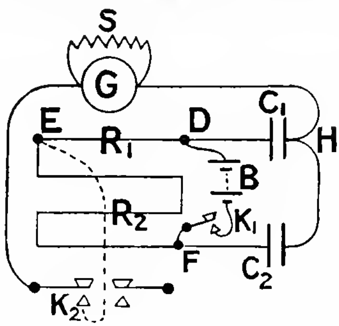


Fig. 80.

or more Leclanché cells; the two capacities, C_1, C_2, to be compared; and either two standard resistance boxes, R_1, R_2, or a P.O. bridge or a high-resistance potentiometer slide.

Observations.—(1) As a precautionary measure against damaging G, short-circuit it by S, and then connect up as in Fig. 80 (*b*), adjusting the spot of light to about zero. If the ordinary

key, K_2, of the bridge is of insufficient insulation, then dispense with it, connecting G to E through a high-insulated key.

(2) The resistances R_1, R_2 should be high for maximum accuracy, and, if a P.O. bridge is used, first set R_1 to 2220 ohms by taking all six plugs out, then adjust R_2 so that on pressing K_1 first, and 5 or 10 seconds after it K_2, no kick is produced on G, the key K_1 still being kept down, and S being gradually cut out, as balance gets nearer, to finally no shunt at all. Note the values of R_1, R_2, which give balance.

(3) Repeat (2) for $R_1 = 2000$, 1220, and 1000 ohms respectively, which will be the best figures in the present circumstances.

NOTE.—With the smaller resistances it may be found advisable to increase the number of cells in B so as to obtain a more sensitive adjustment.

(4) Repeat (2) and (3) for about four values of standard capacity, if this is adjustable (p. 354), remembering that the accuracy of the method increases as C_1 and C_2 become more nearly equal.

(5) Compare the two capacities, and calculate the unknown from the relation—

$$C_1 : C_2 = R_2 : R_1$$

which will be observed to be just the reverse to the ordinary Wheatstone bridge law for resistances.

Tabulate as follows :—

Capacities.		Resistances.		Capacities	
Number of unknown.	Value of standard.	R_1.	R_2.	$\frac{C_1}{C_2}$	Unknown.

Inferences.—Prove the relation given in (5), and state any assumptions made in obtaining it.

100. Absolute Measurement of the Capacity of Condensers (Ballistic Method).

Introduction.—The capacity of any condenser can be determined absolutely in farads F, by measuring the number of coulombs Q of electricity required to be given to either of its coatings to produce a P.D. of V volts between them. Then $Q = VF$. The accuracy of this method, of course, depends on the accuracy with which the two separate quantities Q and V are determined. The following method is simple, convenient, and accurate, and depends only on the accurate value of a resistance in ohms being known.

Apparatus.—Condenser, C, to be tested; high-insulation two-way key, S (p. 327); reflecting ballistic galvanometer, G (p. 285); any fairly constant battery, B, of a sufficient number of cells to give a convenient instantaneous deflection on G when charging C; a high resistance, r, of a known value in ohms, which may or may not be wanted, or two standard resistance boxes, r_1, r_2; a high-resistance potentiometer, XYZ, together with the necessary connecting wires.

Observations.—(1) Connect up the apparatus as indicated in Fig. 81, and adjust the galvanometer needle to zero by means of the controlling magnet.

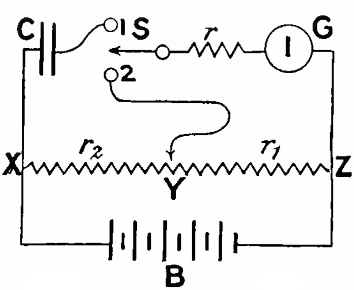


FIG. 81.

(2) Adjust the resistance of XZ to some convenient value, say, from about 2000 to 5000 ohms, depending on circumstances; and also adjust the number of cells B such that, on pressing S1 to charge the condenser, a convenient *instantaneous* deflection d_1 is obtained. Note this first throw d_1.

(3) Adjust r_1 to a suitable low value, if possible, and press S2, noting the *steady* deflection d_2 obtained on the galvanometer.

(4) Repeat (2) and (3) for about ten different values of r_1, so as to obtain deflections over the whole scale, keeping $r_1 + r_2$

constant. The condenser C must be short-circuited after every charge, in order to as nearly as possible have no residual charge left in it which would introduce errors due to residual absorption.

(5) Measure the periodic time of vibration T of the needle in seconds (*vide* p. 133), and the logarithmic decrement λ (*vide* p. 134) taken with the galvanometer terminals disconnected from the circuit.

(6) Calculate the capacity F of the condenser in farads from the formula—

$$F = \frac{T}{2\pi} \times \frac{R}{r + G} \times \frac{d_1}{d_2}(1 + \tfrac{1}{2}\lambda)$$

NOTE.—If r_1, which represents battery resistance b in taking d_2, is not small compared with $r + G$, it cannot be neglected, and we must therefore insert $b + r + G$ in the formula instead of $r + G$. If $\frac{R}{r + G}$ is expressed in megohms, then F will be given in microfarads.

Tabulate your results in the following manner :—

Periodic time of vibration of needle, T, = secs. ; galvanometer resistance, G, = ohms ; mean Napierian logarithmic decrement λ = .

Value of r.	Value of r_1.	Value of r_2.	Ratio, $\frac{r_1}{r_1 + r_2} = R$	Fling, d_1.	Deflection, d_2.	Capacity of condenser. Farads, F.	Capacity of condenser. Microfarads.	Condenser tested.

Inferences.—Show how the formula in (6) is obtained, and explain what assumptions, if any, are made in deducing it.

101. Comparison of Capacities (Method of Mixtures).

Introduction.—This method, due to Lord Kelvin, but commonly known as "Thomson's method of mixtures," has the great advantage of being a null or zero one, though it is possible to make a ready correction for a small resultant deflection, which course, however, is certainly not to be recommended. The essence of the whole method consists in charging the two capacities

with equal quantities of electricity by means of a potentiometer arrangement, with two P.D.'s bearing a known ratio to one another, and noting that their charges are equal by "mixing" the charges through a delicate galvanometer. Or, as was mentioned above, the method can be turned into a deflection one, and the resultant P.D. measured after mixing the two unequal charges. The method, which is now almost universally employed in factories, cable-stations, and in ordinary work, is, generally speaking, applicable to comparisons of both small and large capacities, or when one or both of them are long submarine or other cables having very different degrees of "inductive retardations," which, with ordinary direct-deflection methods, would entail using a ballistic galvanometer having a large periodic time T to ensure the quantity passing through the galvanometer on charge or discharge before the needle moved. For ordinary condenser work about 10 seconds should be allowed for the condensers to charge, while for long cables it should be something like 4 or 5 minutes. This method gives quite accurate results, even though the capacities may differ by as much as 1 : 6; but for wider difference residual absorption becomes very troublesome, and causes inaccuracies. The sensibility of the method will increase with the P.D.'s employed to charge, which should therefore be fairly large. It is extremely important to reduce all leakage from the condensers as much as possible in order to avoid errors. For this reason all parts of the circuit should be well insulated, apparatus clean and not dusty, and all connections kept in mid-air.

Apparatus.—Sensitive high-resistance reflecting galvanometer, G (p. 284); two adjustable high-resistance boxes, R_1, R_2, or a P.O. bridge or a high-resistance potentiometer, whichever is to hand; a highly insulated Pohl's commutator, P (p. 330); spring tapping-key, K; reversing switch, S (p. 329); battery, B, of about ten or more Leclanché cells; capacities, C_1, C_2, to be compared.

Observations.—(1) Connect up the above apparatus as indicated in Fig. 82, and adjust the spot of light from G to about zero on the scale.

(2) R_1, R_2 being adjusted to suitable *high* values, close S, and turn P over to the front (battery side) for about 10 seconds, so as to charge the condensers to different and opposite potentials, K being open.

(3) Now close K, and turn P over to the back (galvanometer side), so as to *mix* the opposite charges in C_1 and C_2. These, if equal, will just neutralize, and cause no deflection on G. This condition will rarely ever be obtained at first start.

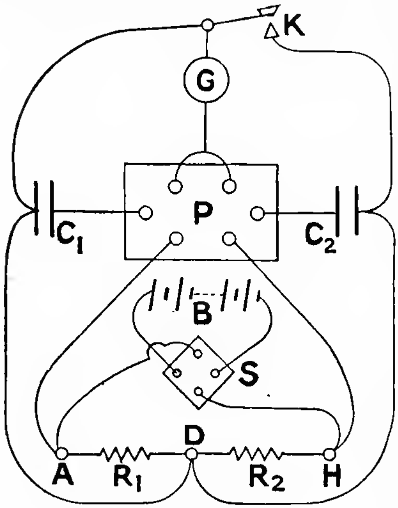


Fig. 82.

(4) Keeping R_2, say, fixed, alter R_1, so that on repeating (2) and (3) no deflection is observed on G. Then note the values of R_1 and R_2.

(5) To eliminate the effect of residual absorption in the condensers, turn S round through 90°, thus reversing B across R_1, R_2. If now there is a deflection on G, readjust R_1 to obtain none, calling its new value R_1', repeating (2)–(4) again.

(6) Repeat (2)–(5) for about four widely different values of R_2, remembering that the test will be more accurate when R_1, R_2 are large.

(7) If either of the capacities is an adjustable standard, such as that on p. 354, then repeat (2)–(6) for about three or four different values of the standard, remembering again that the test will become more accurate as the capacities become more nearly equal.

(8) Compare the two capacities, and calculate the value of the unknown from the relation—

$$C_1 : C_2 = R_2 : R_1$$

and tabulate your results as follows:—

Capacities.		Resistances.				Capacities.	
Number of unknown.	Value of standard.	Current one way, R_1.	Current reversed, R_1'.	Mean, R_1.	R_2.	$\frac{C_1}{C_2}$	Value of unknown.

Inferences.—Prove the relation given in (8), and state any assumptions made in obtaining it.

102. Comparison of Capacities (Siemens's Continued Subtraction Method).

A brief digest only of this method, devised by the late Sir W. Siemens, will be given, for the following reasons :—

(1) The method is difficult of application and manipulation.

(2) It requires great care, and in ordinary hands, except with special arrangements, would most probably give erroneous results.

(3) Leakage effects introduce errors, and the highest insulation is required in the condenser tested; for the former reason the manipulation must be as rapid as possible.

(4) For accuracy the smaller and applied condenser must attain the potential of the large one on each successive deduction, which cannot be the case unless time is given it, when leakage errors will at once creep in.

(5) Difficulty in correcting for damping.

Thus it will be seen that though the method is in one respect a pretty one, having the advantage of enabling a very large capacity to be compared with a small standard one, yet the practical difficulties and errors which creep in render it unsuitable for ordinary work.

The method is, however, as follows :—

(*a*) Charge a standard known condenser of capacity C_s to some definite potential V, and note the first throw d_s on discharging it through a ballistic galvanometer.

(*b*) Charge the large condenser of capacity C to the same potential V, and take away the charging source; then gradually but rapidly discharge it by applying C_s, n times across its terminals, completely discharging C_s after each application.

(*c*) After the nth application, discharge the standard through the galvanometer, and note the first throw d_n.

Then, neglecting damping and other corrections, we get—

$$\frac{d_n}{d_s} = \left(\frac{C}{C + C_s}\right)^n, \text{ or } C = \frac{C_s}{\left(\frac{d_s}{d_n}\right)^{\frac{1}{n}} - 1}$$

where d_n and d_s are assumed to be directly proportional to the last and first quantities of electricity flowing out of C_s through the galvanometer.

103. Comparison of Capacities (Differential Galvanometer Method).

Introduction.—This "deflection" method is applicable to any two condensers whose capacities do not differ very much from one another, and it requires that the periodic time of oscillation of the differential galvanometer used should be large compared with the larger of the two "time constants" of the two condensers compared. This being the case, it is not necessary that the periods during which the two quantities flow should be absolutely one and the same. There are obviously two modes of procedure: (*a*) that of arranging so that the charges, *i.e.* the quantities Q_1 and Q_2, of electricity which pass into or out of the two condensers, due to the same battery E.M.F., go simultaneously through the two galvanometer coils—then, if $\theta° =$ the angular throw resulting, we have $Q_1 - Q_2 \propto \sin \frac{1}{2}\theta°$; (*b*) that of shunting the galvanometer coil, which is in series, with the larger of the two capacities, whence $Q_1 - Q_2 \propto$ the quantity Q_s which passes through the shunt.

The author does not, however, purpose discussing this method by the differential galvanometer in further detail, as it is not sufficiently generally applicable for comparisons of capacity, and, moreover, is inferior to several other methods which are more easily manipulated.

Lastly, (*a*) possesses the defects inherent in all deflection methods, while with (*b*) an unknown quantity is introduced, due to shunting, a galvanometer when using transient currents.

The left-hand diagram (Fig. 79) would represent the connections for the mode of procedure (*a*) above, providing the circuit containing G and S is dispensed with, and the two coils of the differential galvanometer substituted for the resistances R_1, R_2 of that diagram.

104. Comparison of Capacities (Method of Divided Charge).

Introduction.—This method is a simple and in many respects a convenient one, and becomes most accurate, as has been shown by Mr. Kempe, when the standard capacity is about half that of the unknown. The method is very analogous to the Siemens subtraction, but here in this one only one subtraction is made.

Apparatus.—Sensitive high-resistance ballistic galvanometer, G; highly insulated charge and discharge key, K (p. 328); highly insulated ordinary spring tapping-key, K_1; Leclanché battery, B, of a suitable number of cells; standard and unknown capacity, C_s and C respectively.

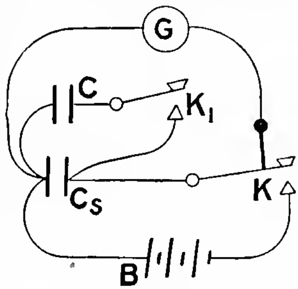


Fig. 83.

Observations.—(1) Connect up as indicated in Fig. 83, and adjust the spot of light to zero.

(2) With K_1 open, press K, thus charging the standard capacity C_s with the E.M.F. of B.

(3) Release K, thereby discharging C_s through G, and note the first throw of the galvanometer needle, and call it d_1.

(4) Again press K, then release it to an intermediate position between the upper and lower contacts, and quickly press K_1 for 2 or 3 seconds. Now release both keys, and note the first throw d_2 on the galvanometer.

(5) Repeat (2)-(4) with about four different values of standard capacity C_s, and also, say, a different E M.F. of the battery B, each time completely discharging C before applying to C_s.

(6) Compare the two capacities, and calculate the unknown from the formula—

$$\frac{C}{C_s} = \frac{d_1 - d_2}{d_2}$$

and tabulate as follows :—

Capacities.		First throws.		Capacities.	
Number of unknown, C.	Value of standard, C_s.	d_1.	d_2.	$\frac{C}{C_s}$	Unknown, C.

Inferences.—Prove the relation given in (6), and state any assumptions made in obtaining it.

105. Measurement of Specific Inductive Capacity of Dielectrics.

Introduction.—Not only is the capacity of a condenser increased by using glass, mica, gutta-percha, vulcanite, etc., instead of air as the dielectric or insulating material across which the electrostatic induction takes place, but the resistance to rupture and consequent loss of charge by sparking is considerably increased, this depending as it does on the rigidity and not on the insulating quality of the substance. The increase in capacity occurs by reason of the specific inductive capacity of the dielectric, and if C_1 = capacity of any condenser when its plates are separated by air, then its capacity C_2, when they are separated by any other dielectric, is $C_2 = C_1 \times$ specific inductive capacity of that substance. Hence the specific inductive capacity of a substance is the ratio of the capacity of a condenser when its plates are separated by this substance to the capacity of the *same* condenser when its plates are separated by air.

The following method, suggested by Professor Ayrton, will be found convenient when, as is usually the case, we want to measure the specific inductive capacity of a comparatively small specimen of insulating material or dielectric, and which is too small to use in making a condenser of sufficient capacity for ordinary testing. The material to be tested may be a sheet of glass or ebonite, etc., and should have pasted on each side of it sheets of tinfoil of equal size and about 1 inch smaller all round than the plate of dielectric, so as to reduce surface leakage to a minimum, which is exceedingly important to the working of

this method. In all cases the unpasted border should be *carefully cleaned* and *dried*, and with the same object in view, the plate should rest on a block which *only touches* the underneath sheet of foil. Thin wires should make contact with each sheet of tinfoil, and these form the terminals of this experimental condenser.

Apparatus.—Two resistance boxes, r_1, r_2; telephone, G_T, as galvanometer; charge and discharge key, K; ten-cell battery, B; air condenser, C_1; condenser, C_2, as above, with its plates separated by the dielectric to be tested.

Let A_1A_2 = total effective areas of the metal coatings, d_1d_2 = total effective distances between them for the air and other condensers, C_1 and C_2, respectively, then, for no sound in the telephone, we have $C_1r_1 = C_2r_2$.

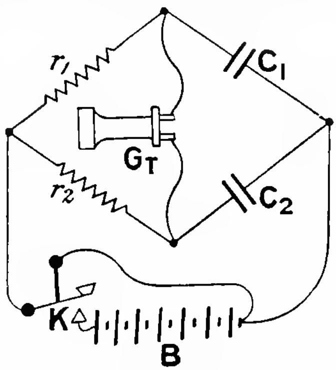


FIG. 84.

Observations. —(1) Connect up as shown in Fig. 84, and adjust r_1, say, to a convenient value, which should be fairly high.

(2) Adjust r_2 so that on rapidly actuating K up and down so as to rapidly charge and discharge C_1 and C_2, no sound is heard in the telephone. Now note the values of r_1, r_2.

(3) Repeat (1) and (2) for six widely different values of r_1, r_2, and calculate the specific inductive capacity from the formula—

$$\text{specific inductive capacity} = \frac{r_1}{r_2} \cdot \frac{A_1}{A_2} \cdot \frac{d_2}{d_1}$$

(4) Tabulate your results as follows:—

Dielectric tested.	A_1.	A_2.	d_1.	d_2.	r_1.	r_2.	Specific inductive capacity, $\frac{r_1}{r_2} \cdot \frac{A_1}{A_2} \cdot \frac{d_2}{d_1}$

Inferences.—Prove the relation given in (3), and state any assumptions made in obtaining it.

106. Capacities in Series and Parallel (Laws of Combination).

Introduction.—It is important to know the precise result of combining capacities in series, parallel, or partly in series and partly in parallel, and to be able to predict the combined or *effective capacity* of all such combinations without difficulty. This experiment is arranged with a view to proving, by means of carefully made tests, the laws of various combinations, and thus impressing the matter more firmly on one's memory. When *capacities* are connected *in series*, or "cascade," as it is sometimes called, their combined capacity is obtained from the individual capacities in exactly the same way as that of a number of known *resistances in parallel.* On the other hand, *capacities in parallel* combine in exactly the same way as *resistances in series.*

To verify these and other rather less obvious relations, the student will be expected to be already acquainted with Thomson's "method of mixtures" for comparing capacities (p. 182), and consequently the mode of performing the observations will not be repeated in what follows.

Apparatus.—All that named on p. 183 for the method of mixtures, where C_2, one of the capacities named, will now represent the respective combinations of capacities to be tested; in addition, a box of four separate and well-insulated condensers, A, B, C, and D, arranged similarly to that described on p. 347, so that they can be connected up singly or in all possible combinations C_2 of series and parallels.

Observations.—(1) Connect up as shown in Fig. 82, and carry out all the observations, (1)-(7), there mentioned in the following tests.

(2) Measure the capacities of each of the four condensers A, B, C, and D separately.

(3) Measure the capacity all in series, and also for any two and any three in series.

(4) Measure the capacity of two different pairs of any two capacities in parallel.

(5) Measure the capacity of two different pairs of any three capacities in parallel.

(6) Measure the capacity when all four condensers are in parallel.

(7) Measure the capacity of the four condensers arranged two in series and two in parallel.

(8) Calculate the *combined* or *effective* capacity obtained from the measurements from the relation—

$$C_2 = \frac{C_1 R_1}{R_2}$$

(9) Also calculate it from the following formulæ to be proved :—

$$\frac{1}{C_2} = \frac{1}{A} + \frac{1}{B} + \frac{1}{C} + \frac{1}{D} + \ldots \text{ for pure series combinations}$$

$$C_2 = A + B + C + D + \ldots \quad \text{,,} \quad \text{parallel} \quad \text{,,}$$

and tabulate your results as follows :—

Capacities.		Resistances.				Capacities.	
Combination of unknown.	Value of standard, C_1.	Current one way, R_1.	Current reversed, R_1'.	Mean, R_1.	R_2.	Measured, $C_2 = C_1 \frac{R_1}{R_2}$	Calculated, C_2.

Inferences.—Prove the relations given in (9), and state all that can be inferred from the results of your observations.

107. Constancy of the Capacity of Condensers (Proof Ballistically).

Introduction.—When a quantity Q of electricity flows into either coating of a condenser of capacity C, and raises the P.D. between the coatings to a value V, then we have the relation $Q = CV$, or $C = \frac{Q}{V}$ = a constant (approximately). Now, in an ideal condenser in which a *perfect* dielectric is used, *i.e.* a dielectric which is a perfect insulator and whose absorption and

dielectric hysteresis is *nil*, the ratio $\frac{Q}{V}$ would be constant under all conditions. A good air condenser approximates to an ideal one, but the ordinary practical condenser possesses an imperfect dielectric, which results in the ratio $\frac{Q}{V}$ having to depend to a certain extent on the initial state of the condenser and on the time it is given to charge or discharge, thus causing $\frac{Q}{V}$ to be only approximately constant.

Apparatus.—Sensitive ballistic galvanometer, G (p. 285); damping coil, with its key and cell (p. 349); highly insulated charge and discharge key, K (p. 329); a *rather leaky* condenser, C, of suitable capacity; battery, B, of ten cells or more; and a fairly high-resistance potentiometer, PS, say, of about 5000 ohms, with a moving contact or sliding key, N. Then, if, say, ten cells are permanently connected to P and S, their total E.M.F. will be very approximately constant, and the relative resistances between S and the point of contact N will at all times give the relative P.D.'s employed in charging C, which, therefore, will be a ready and convenient means of using what E.M.F. is desired from 0 to that of 10 cells in series.

Observations.—(1) Connect up as in Fig 85, and adjust the galvanometer to zero.

(2) First adjust B so that when N and P coincide, *i.e.* when the full E.M.F. is used, a full-scale deflection is obtained on G when K is pressed.

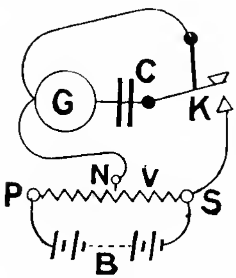


Fig. 85.

(3) Set N so that NS = $\frac{1}{10}$PS, and note the first throw d_1 on charging C by pressing K; then quickly damping G to rest, note the first throw d_2 on discharging C by releasing K.

(4) Repeat (3) for NS = $\frac{2}{10}$, $\frac{3}{10}$, $\frac{4}{10}$, . . . of PS respectively up to the full; and to make certain that the total E.M.F. of B has not altered, repeat (2) over again.

(5) Repeat (3) and (4) with, say, a 10-seconds interval of the lever of K intermediate between the two stops on discharge, and tabulate all your results as follows:—

Charging P.D., $V \propto NS$.	Throws on			Ratios, $\frac{Q}{V}$		
	Charging, d_1.	Discharging, d_2.	Discharging after 10 secs., d_2'.	$\frac{d_1}{NS}$	$\frac{d_2}{NS}$	$\frac{d_2'}{NS}$

Inferences.—What can you infer from the results of your observations? and state whether anything has been lost sight of which might affect the ratio $\frac{Q}{V}$ independently of other things.

108. Self and Mutual Induction Coefficients (Absolute and Comparative Determinations).

Introductory.—It is not the intention of the author to discuss theoretical considerations relating to the above subject, which can more appropriately be obtained from text-books dealing with that branch of the subject in detail. Some prefatorial remarks and definitions are, however, necessary.

SELF-INDUCTION.—It is well known that, when a current is started or stopped in a circuit, a certain time elapses before the current reaches a final steady value. This phenomenon is due to the lines of force, emanating from one part of the circuit, cutting another part in springing out or collapsing, and thereby causing a momentary counter E.M.F. to be set up (by Lenz's law), which retards the rise or fall, as the case may be, of the current. This effect is termed the self-induction of the circuit, and its magnitude depends on the geometrical form and permeability of the circuit and surrounding medium.

If N = number of magnetic lines of force threading the circuit due to a current A flowing through it, then $N \propto A$ if there is no magnetic material in or near the circuit; and since N varies, under such conditions, as A, we may say that $N = LA$, and therefore that $\frac{dN}{dt} = L\frac{dA}{dt}$.

Definition.—The coefficient L is called the *coefficient of self-induction*, and $L\frac{dA}{dt}$ = the counter or back E.M.F. of self-induction.

Now, since from the above $L = \frac{N}{A}$, the coefficient of self-induction may be defined as the *number of lines* threading the circuit when *unit* current is started or stopped in it.

If R = the resistance of the circuit, then $\frac{LA}{R}$ = whole *quantity of electricity* in the induced or transient current on making or breaking during the rise or fall, and if the current is reversed, this quantity is $= \frac{2LA}{R}$. Hence L can be defined as the *quantity* of electricity passing in the circuit of *unit* R when *unit current* is "made" or "broken." L is constant if the permeability of the surrounding media is constant.

If R stands for the ohmic resistance of a coil of self-induction L, then $\frac{L}{R}$ is called the *time constant* of the coil, and is the number of seconds required for the current to arrive at 0·632 of its maximum value after first closing the circuit.

Mutual Induction.—Now suppose two circuits are in proximity to each other, and that a current A be suddenly started or stopped in one of them (called the primary), while the terminals of the other (the secondary) are short-circuited. Then the quantity of electricity set up in the secondary during the rise or decay of the primary current $= \frac{MA}{R}$, where R = resistance of the whole secondary circuit, or, if the primary current is reversed, this quantity $= \frac{2AM}{R}$.

Definition.—The coefficient M is called the *coefficient of mutual induction*, and $M\frac{dA}{dt}$ = the induced E.M.F. in the secondary. The coefficient M may thus be defined as the quantity of electricity which is induced and flows in the secondary of unit resistances when unit current is made or broken in the primary, or, in other words, it is the number of lines of force common to or threading each circuit when unit current flows in one of them.

M is constant if the permeability of the surrounding media is constant. If any circuit contains magnetic material, then M or L, whichever is present, will depend for its magnitude on the

magnetic saturation of such material, and consequently on the current.

Unit.—The unit of either self or mutual induction is variously termed a secohm, Henry, or quadrant, and in the electro-magnetic system of units has an absolute measure of 10^9 cms.

Dimensions.—Its *dimension* is of the order of resistance × time, *i.e.* velocity × time, $= \frac{\text{length}}{\text{time}} \times$ time, or, in other words, *length.*

In the following measurements we shall adopt the usual notation shown in Fig. 85A, I. and II., namely—

I. for a non-inductive resistance, and
II. for an inductive resistance.

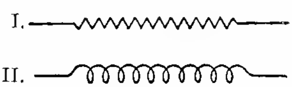


FIG. 85A.

These simple representations, if studiously adhered to, will save any confusion, and at once follow from the fact that circuits doubled back on themselves have an extremely small coefficient of induction, and for all practical purposes may be considered as non-inductive. No circuit, however, can be absolutely non-inductive.

109. Absolute Measurement of Self-induction (Maxwell-Rayleigh Method).

Introduction.—This is a ballistic or deflection method for determining a coefficient of self-induction absolutely in secohms, and is one originally proposed by Clerk Maxwell, but modified by Lord Rayleigh. The result depends on the accuracy with which five separate quantities are either observed or measured. The arrangement is substantially that of a Wheatstone bridge, and the method is capable of accurate results when sufficient care is taken and all necessary corrections applied. A null method, however, is distinctly to be preferred when available.

Apparatus.—Known standard non-inductive adjustable resistances, P, Q, R, and S; battery of fairly good Leclanché cells, B; Pohl's commutator, K_1 (*vide* p. 330); spring tapping-key, K_2; ballistic galvanometer, G (p. 283); coil of self-induction, L, secohms to be measured and of ohmic resistance, *l*.

NOTES.—In the annexed arrangement, which is the best one to use, P must = Q, *ad* being our adjustable arm, in which case they may each be single resistances, but of about the same order of magnitude, preferably as l. G should preferably be of the moving needle type, and have a resistance approximating, if possible, to the most suitable value, namely,

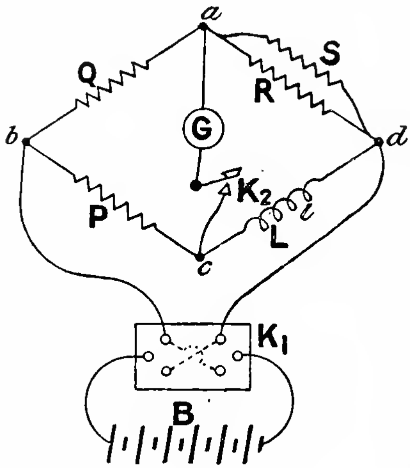


FIG. 86.

$$\frac{(Q + P)\left(\frac{RS}{R + S} + l\right)}{P + Q + 2l}$$

ohms for the above arrangement, and P = Q.

If l should be very small, it will generally be found necessary to bank it up by inserting a *non-inductive* resistance of suitable value in series with it, so that the arm *cd* now equals the sum of the two, and may still be called l.

In Obs. (5) below the inductive throws d may be taken on "making" and "breaking" the battery circuit at K_1, and the *mean* used in the calculation or preferably "half" the throw on reversing the battery connections as this throw, which is approximately double either of those at the "make" and "break," can be read more accurately, and, in addition, owing to the battery remaining on "closed circuit," its E.M.F. will be more constant than if used intermittently.

Observations.—(1) Connect up the above apparatus as indicated in Fig. 86, and adjust the galvanometer spot of light to zero on the scale by means of the controlling magnet.

(2) Assuming l to be known roughly, make P = Q = about this resistance, if possible and convenient.

(3) Remove "infinity" plug in S, and adjust R so that on closing K_1 first, and then K_2 3 or 4 seconds later, say, it is *just* too large to give no deflection on G.

(4) Reinsert "infinity" plug in S, and, starting with a much

higher resistance than is already out in R, adjust S so as to finally "balance the bridge" accurately for steady currents, and obtain no motion of the spot of light on manipulating K_1 and K_2, as in (3).

(5) Now, without altering anything, close K_2 and K_1, and then, when the spot of light is *perfectly at rest*, suddenly reverse the current by means of K_1, and note the angular throw $d°$ of the spot.

(6) Throw out the balance for steady currents obtained in (4) above by increasing S, thereby increasing the resistance of the arm *ad* by a small amount ρ, and note the steady angular deflection D° produced when both K_1 and K_2 are closed.

(7) Repeat (5) and (6) alternately a few times in order to eliminate any error due to variation of the battery E.M.F., and take the mean of each set of values for $d°$ and D° respectively.

(8) Repeat (3)-(7), first by increasing the E.M.F. of B, and thereby $d°$ and D°, which will give a more sensitive test; secondly, by employing two or three different values of ρ altogether, enabling several different determinations of the same self-induction to be made and a *mean* obtained.

(9) Determine accurately the periodic time of oscillation T of the galvanometer needle in seconds in the manner set forth on p. 133.

(10) Determine the *Napierian logarithmic decrement* λ of the galvanometer with the same arrangement of the bridge arms as in (4) above, and with K_1 open and K_2 closed, in the manner set forth on p. 134.

NOTES.—Should G be too sensitive, control its deflections by a more powerful controlling field, rather than shunt it, if possible. T may have any convenient value from about 4 seconds upwards; ρ must be kept small, and might preferably have such a value as to give $d° = D°$, nearly.

It is advisable to see the order of $d°$ prior to very accurately balancing in (4), as it may be found expedient to increase $d°$ by increasing the sensibility of G, or by altering the resistance of arm *cd* if this is "banked" up.

(11) Calculate the self-induction L measured from the formula—

$$L = \frac{T}{2\pi} \cdot \frac{\rho}{2} \cdot \frac{d}{D}(1 + \tfrac{1}{2}\lambda) \text{ secohms}$$

where d and D are in scale-divisions corresponding to $d°$ and D°.

Tabulate your results as follows:—

Periodic time, T, = secs.; log dec. λ = ; time constant of coil, $\frac{L}{l}$, = secs.

Kind of induction tested.	Ohmic resistance, l.	Throw, d.	Steady deflection, D.	Resistance, ρ.	Value of L.

Inferences.—Prove the formula given in (11), and state any assumptions made in obtaining it.

110. Absolute Measurement of Self-induction (Secohmmeter Method).

Introduction.—The following deflection method necessitates the use of a special piece of apparatus termed a secohmmeter, which has been devised by Professors Ayrton and Perry. The object of it is to increase the sensitiveness of the Maxwell-Rayleigh method by obtaining the cumulative effect of a rapid succession of throws on the galvanometer due to "make" or "break" impulses. The actual arrangement of the bridge itself may preferably be that used in the above-named method, the function of the keys K_1 and K_2 now being performed by the secohmmeter. The method is subject to the defects inherent in all deflection methods.

The secohmmeter, a further description of which will be found on p. 259, can either be rotated by hand through the medium of a handle or by a small electro-motor coupled direct to the commutator spindle. The general arrangement for the latter mode of driving is shown in Fig. 119.

Apparatus.—Known standard non-inductive adjustable resistances, P, Q, R, and S; battery of fairly good Leclanché cells, B; an ordinary sensitive Wheatstone bridge galvanometer, G, of any type, though preferably fairly dead-beat for convenience; an auxiliary key, K; the self-induction, L, secohms to be measured of ohmic resistance, l; secohmmeter, O, complete with tachometer, T, and preferably a small electro-motor, M, the electrical circuit feeding M consisting of a small secondary battery, switch,

and continuously variable rheostat for adjusting the speed and maintaining it *constant.*

NOTES.—The student is referred to those in the Maxwell-Rayleigh method, which apply here exactly with the exception of the latter part relative to the inductive throws. In addition, it may be remarked that the relative positions of the two commutators is unimportant. The greater the speed, the greater will be the deflection d, and the more accurately can it be observed and the larger the added resistance ρ will be; in other words, the sensibility of the method is increased, but the speed must not be too great to prevent the currents reaching their steady values between two consecutive reversals of the battery. That this condition is fulfilled may be ascertained by seeing whether the same value for L is obtained for a much smaller speed than the highest used. In Fig. 87 the galvanometer and battery commutators are represented symbolically by G.c. and B.c. respectively.

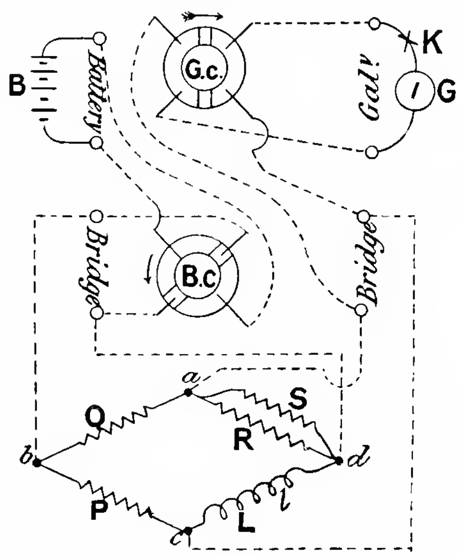


FIG. 87.

Observations.—(1) Connect up the requisite apparatus as represented in Fig. 87, the motor circuit not being shown, and adjust the galvanometer to zero.

(2) Assuming l to be roughly known, make P = Q = about this resistance, if possible.

(3) Remove "infinity" plug in S, and adjust R so that on closing the battery circuit, first by, if necessary, slightly turning the commutators into a different position for contact, and then closing the auxiliary key K a second or two after, it is just too large to give no deflection on G.

(4) Reinsert "infinity" plug in S, and making S much larger than R, adjust it so as to finally "balance the bridge" accurately for steady currents in the usual way by closing K last.

(5) Drive the secohmmeter at a low *constant* speed, N revolutions per minute by the motor, say, and note the constant deflection d scale-divisions on the galvanometer G.

(6) Stop the secohmmeter, and throw out the balance for steady currents obtained in (4) above by increasing S, thereby increasing the resistance of the arm *ad* by a small amount ρ, and note the steady deflection D scale-divisions of G.

(7) Repeat (5) and (6) alternately a few times, in order to eliminate any error due to the variation of the battery E.M.F., and take the mean of the two sets of values of d and D respectively.

(8) Repeat (3)–(7), first by increasing the E.M.F. of B for the same speed, thereby increasing d and D, and giving greater sensibility; secondly, by employing four or five different speeds N, rising by about equal increments to the maximum.

NOTES.—Strike out those values of L calculated in which the condition referred to in the introduction has not been fulfilled, and take the mean of the remainder.

The added resistance ρ may preferably have such a value as will give $d = \text{D}$ approximately.

The secohmmeter gives four reversals for one revolution of the commutator spindle.

(9) Calculate the required self-induction L from the formula—

$$\text{L} = \frac{l}{\left(\frac{\text{RS}}{\text{R}+\text{S}} + \rho\right)} \cdot \frac{\rho}{n} \cdot \frac{d}{\text{D}} \text{ secohms}$$

where d and D are in scale-divisions and n = number of reversals per second.

Tabulate your results as follows:—

P = Q = ohms.

Kind of induction tested.	Resistances.				Speed of commutators, N revolutions per minute.	Number of reversals, n, per sec., $= \frac{4\text{N}}{60}$	Deflections.		Self-induction, L secohms.
	R.	S.	l.	ρ.			d.	D.	

Inferences.—Prove the formula given in (9), and state any assumptions made in obtaining it.

111. Absolute Measurement of Self-induction (Secohmmeter Method).

Introduction.—The following "null" or "zero" method of measuring self-induction by means of the secohmmeter is a much more sensitive one than the corresponding "deflection" method. The action of the secohmmeter, the apparatus to be employed, and the arrangement of connections, is identically the same as in the preceding test by the "deflection" method. The setting of the commutators relatively to one another must be such that the galvanometer is not reversed exactly, or even very nearly midway between two consecutive reversals of the battery, since, with the former arrangement of the two, no variation in the resistance of any of the arms of the bridge can counterbalance the effect of self-induction on driving the secohmmeter; and the more nearly the commutators are placed in this midway position, the smaller will be the value of K, a constant depending on the relative positions of the two commutators, and therefore the larger the value of $\dfrac{l\rho}{\dfrac{RS}{R+S}+\rho}$ to give balance for given values of L and n.

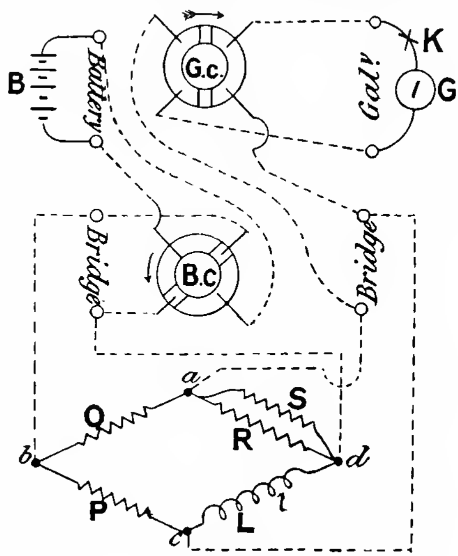


FIG. 88.

Observations.—(1) Connect up the requisite apparatus as

indicated in Fig 88, the motor circuit not being shown, and adjust the spot of light to about zero.

(2) Make P = Q = about the resistances l, if possible, and accurately balance the bridge for steady currents in the manner indicated in Obs. (3) and (4) of the preceding method.

(3) Drive the secohmmeter at some convenient constant speed so that the commutators make N revolutions per minute, and adjust the resistance of the arm *ad* so that there is *no deflection* on G, and note the values of R and S to do this.

(4) Repeat (3) for several different speeds and E.M.F.'s of B.

(5) Determine the value of K for some convenient and known speed n, experimentally, by replacing the self-induction to be measured by a *known standard* of self-induction L_s, such as Ayrton and Perry's, shown on p. 263, and then repeating (2) and (3) above, whence—

$$K = \frac{L_s\left(\frac{RS}{R+S} + \rho_s\right)n_s}{l_s\rho_s}$$

where ρ_s, R, S, and l_s are the values given by L_s and n_s.

NOTE.—The speed n should be fairly high, so as to make ρ large.

(6) Calculate the required self-induction L from the formula—

$$L = \frac{l}{\frac{RS}{R+S} + \rho} \times \frac{\rho}{n} \times K$$

where all the symbols, except K, are those pertaining to the unknown self-induction.

Tabulate your results as follows:—

L_s = secohms; n_s = revolutions per minute; constant, K, = P = Q = ohms.

Induction tested.	R.	S.	l.	ρ.	Revolutions per minute, n.	Self-induction, L.

Inferences.—Prove the formula given in (6), and state any assumptions made in obtaining it.

112. Absolute Measurement of Self-induction (Maxwell-Rimington Method).

Introduction.—The following is Mr. E. C. Rimington's modifications of the original method proposed by Clerk Maxwell for measuring a self-induction in terms of the capacity of a condenser. It has the advantage of being a "zero" method, and of obviating a succession of adjustments necessary in working the original method.

Apparatus.—Standard condenser, C, of known capacity, preferably a variable one (*vide* p. 354); self-induction, L, to be measured; sensitive ballistic galvanometer, G (*vide* p. 283); spring tapping-keys, K_1, K_2; adjustable non-inductive resistance boxes, P, R, and variable slide resistance, S; some convenient potentiometer resistance, Q, fitted with a moving plug contact capable of making contact at any point, N, of Q; battery, B.

NOTES.—Since the effect of C and L is to increase the "time constants" of the circuits *ba* and *cda* respectively, thereby retarding the rise of potential at *a* to its maximum value, the difference in the rate of rise of currents may cause G to deflect, even though C and L are accurately balanced on the bridge. To avoid this difficulty, the periodic time of vibration of G should be sufficiently large, and not less than about 4 or 5 seconds, so that the inductive actions may settle down before G has time to move.

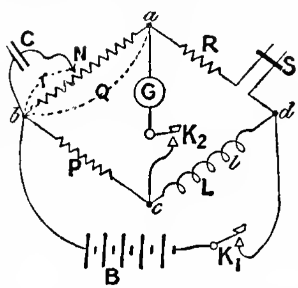


FIG. 89.

If the ohmic resistance of L is small, it may be banked up with an auxiliary non-inductive resistance in series, but in all cases $l =$ resistance of the arm *cd*.

Observations.—(1) Connect up the above apparatus as shown in Fig. 89, and adjust the galvanometer to zero roughly.

(2) Make PQ and R + S comparable with l, and finally "balance the bridge" accurately for steady currents in the usual

way by pressing K_1 first, and a second or two after K_2, S being used to obtain a delicate adjustment. It will be noticed that C does not affect this balance.

(3) Without altering anything, and keeping Q constant in magnitude throughout, alter the position of N such that no inductive throw operates on G on making K_2 first and then K_1, and note the value of r.

(4) Repeat (2) and (3) for the same value of C, but with about four different values for Q; and again for the same value of Q, but with about four different values for the capacity C.

(5) Calculate the required self-induction L from the formula—

$$L = C\frac{l}{Q}r^2 \text{ secohms}$$

and tabulate your results as follows :—

Induction tested.	Capacity, C.	r.	Q.	l.	Self-induction, L.

Inferences.—Prove the formula given in (5), and state any assumptions made in deducing it.

113. Absolute Measurement of Self-induction.

Introduction.—The following is a modification of Professor A. Anderson's method of determining a self-induction in terms of the capacity of a condenser. It has the advantage of being a "null" method, and of avoiding any succession of adjustments which would be necessary in disturbing the balance for steady currents in order to balance those due to the induction and capacity. If the ohmic resistance l of the self-induction is low, it can be banked up by means of a non-inductive resistance in series; l then will be the resistance of the combination. It may be necessary in certain cases to use a ballistic galvanometer of considerable periodic time for reasons given in the preceding method, though in most cases an ordinary Thomson astatic galvanometer is suitable.

Apparatus.—Standard condenser, C, of known capacity, preferably a variable one (*vide* p. 354); self-induction, L, to be measured; sensitive galvanometer, G (*vide* p. 284); spring tapping-keys, K_1 and K_2; battery of about ten fairly good Leclanché cells, B; four adjustable non-inductive resistances, P, Q, R, and S, in lieu of which two P.O. resistance bridges can be made to do.

Observations.—(1) Connect up the apparatus as shown in Fig. 90, and adjust the galvanometer to zero roughly.

(2) Make S = O, and with the remaining resistances "balance the bridge" for steady currents in the usual way by pressing K_1 first, and then K_2 a second or two after. Note the values of P, Q, and R.

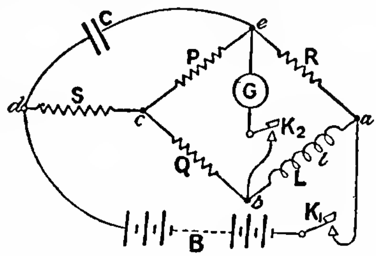


FIG. 90.

(3) Make C a suitable capacity, and adjust S so that, on pressing K_2 first and then K_1, there is no deflection on G. Note the value of S. It is to be noticed that altering S does not affect the balance obtained in (2), neither does the presence of C.

(4) Repeat (2) and (3) for about six different values of capacity C, two of the three resistances, P, Q, and R, taken as proportional arms, being preferably equal in each case.

(5) Calculate the required self-induction L from the formula—

$$L = C(Pl + RS + Sl) \text{ secohms}$$

C being in farads and all the resistances in ohms, and tabulate your results as follows:—

Induction tested.	Capacity, C farads.	P.	l.	S.	R.	Self-induction L.

Inferences.—Prove the formula given in (5), and state any assumptions made in deducing it.

114. Comparison of Two Coefficients of Self-induction (Maxwell-Niven Method).

Introduction.—The following is a modification, due to Professor C. Niven, of Maxwell's original method for comparing two unknown self-inductions for the purpose of merely getting the ratio or of finding the absolute self-induction of a coil in terms of a standard self-induction by comparing the two together. It has the advantage of being a "zero" method, and of avoiding the repeated adjustments of the resistances of the arms of the bridge in balancing the inductions after the balance for steady currents has been obtained.

The standard self-induction may conveniently be the adjustable one devised by Professors Ayrton and Perry, shown in Fig. 122, p. 263, and a description, together with calibration curves, there given.

Apparatus.—Standard of self-induction, L_1, and ohmic resistance, l_1; self-induction, L_2, to be measured of resistance, l_2; known standard non-inductive adjustable resistances, P, Q, S, R_1, and R_2; spring tapping-keys, K_1, K_2; battery, say of about ten fairly good Leclanché cells, B; ordinary galvanometer, G, of high sensibility (*vide* p. 284); plug key, K, for short-circuiting the self-induction L_2.

NOTE.—It may, in cases where L_1 is small, be found necessary to bank its resistance up by inserting a suitable non-inductive resistance in series with it in the part *bx*, when l_1 will represent the total resistance.

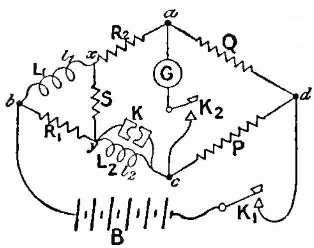


FIG. 91.

Observations. — (1) Connect up the apparatus as shown in Fig. 91, roughly setting the galvanometer G to zero.

(2) Make P = Q if convenient, and plug up K_2 and K so that these branch portions have *no resistance;* make S = infinity, then adjust R_1, and, if necessary, P and Q again, so as to balance the bridge for steady currents by

pressing K_1 first and then K_2, and note the final values of R_1, Q, and P.

(3) With S still infinity, unplug K, and adjust R_2 (only), so as still to preserve balance for steady current, as in (2).

(4) Now adjust S from infinity to such a value that, on pressing K_2 first and then K_1, there is no deflection on G due to induction.

(5) Repeat (2)–(4) for some six different values of the standard self-induction L_1, and calculate the required ratio of self-inductions and the value of the unknown from the formula—

$$\frac{L_1}{L_2} = \frac{Q(S + R_1 + l_1)}{SP}$$

Tabulate your results as follows :—

Standard self-induction, L_1, at °C. has an ohmic resistance, l_1, = ohms.							
Induction tested.	P.	Q.	S.	R_1.	L_1.	$\frac{L_1}{L_2}$	L_2.

Inferences.—Prove the relation given in (5), and state any assumptions made in obtaining it.

115. Comparison of Two Coefficients of Self-induction (Secohmmeter Method).

The sensibility of the preceding Maxwell-Niven method of comparison can be increased by using the secohmmeter described on p. 259. Its function is merely to perform the work of K_1 and K_2 in Obs. (4) of that method, and to increase the sensitiveness of the balance for induction currents. The points *b* and *d* are connected direct to terminals "bridge" on the battery side, and *a*, *c* to bridge on galvanometer side of the secohmmeter, while B and G are connected directly across terminals marked "battery" and "galvanometer" respectively. The secohmmeter can be motor-driven, as in Fig. 119, at any speed which need not be known, and the greater it is the more sensitive the test; the rate of reversal

must not, however, be too great for the currents to reach their steady values between two consecutive battery reversals. The method otherwise is exactly the same.

Fig. 92 shows diagrammatically the arrangement for comparing two coefficients, L_1, L_2, of self-induction by means of the Wheatstone-bridge-secohmmeter method, when only a P.O. Wheatstone bridge is available. The diagram practically explains itself, in which the two commutators, BC and GC, are shown symbolically by the cross-line arrangement. The standard self-induction L_2 (*vide* p. 263) is joined in series with some convenient form of slide wire and contact and the adjustable arm of the

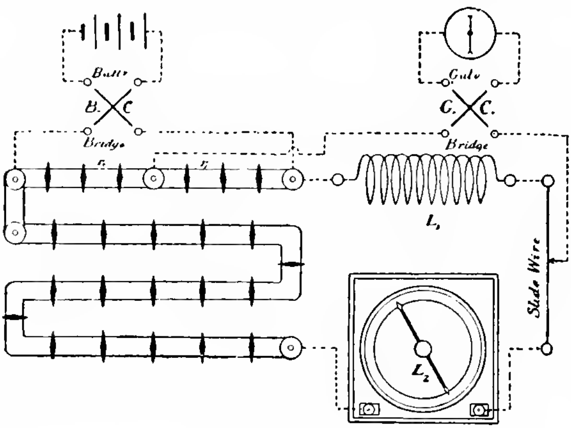


FIG. 92.

P.O. bridge, by which a perfect balance, and not one correct to 1 ohm only, can be obtained.

The ratio arms are given some convenient values, and the bridge balanced for steady currents in the manner already described. The secohmmeter is then rotated, and L_2 adjusted to again give no deflection, when we get $\frac{L_1}{L_2} = \frac{r_1}{r_2}$.

To save time, turn L_2 to its extreme positions; then, if, on rotating the secohmmeter, deflections on each side of zero respectively are obtained, a balance is possible at some intermediate position. If both positions give deflections to same side of zero, then either r_1, r_2 must be altered, or an extra standard

self-inductor added to L_2, in order to obtain balance, in which cases that for steady currents must be *reobtained*.

NOTE.—In this and all similar tests the self-inductions must be so placed that they do *not affect* one another or the galvanometer.

116. Absolute Measurement of Mutual Induction (Carey-Foster Method).

Introduction.—The following is a method, devised by Professor Carey Foster, for measuring a coefficient of mutual induction absolutely in secohms in terms of a capacity. It has the advantage of being a "zero" method, and fairly easy of manipulation.

Apparatus.—Sensitive reflecting galvanometer, G, of considerable periodic time (*vide* p. 285); standard condenser, C, of known capacity of a variable type, preferably similar to that in Fig. 223; non-inductive known variable resistances, R, S; fairly good battery of Leclanché cells, B; spring tapping-key, K; coils of mutual induction, M, to be tested, and of ohmic resistances, P and Q respectively.

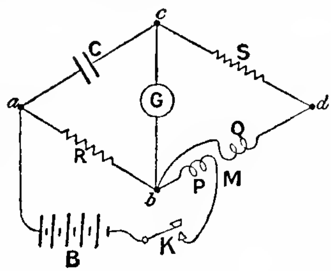


FIG. 93.

NOTES.—It should be pointed out that the figure does not represent a Wheatstone bridge arrangement.

If there is leakage between the primary coil P and secondary Q, a small permanent deflection of G will result when K is "made." This can be discarded, and only "kicks" looked for.

If the mutual induction apparatus consists of a transformer with a substantial iron core, the "time constants" for the capacity and mutual inductions will probably be very different, the condenser discharge giving a throw in one direction, and M a later throw in the opposite direction. For this reason G should have a fairly large "periodic time" of vibration, so as to enable the

inductive and capacity effects to settle down before G has a chance to move, and thereby enabling a balance to be obtained with less difficulty.

The most sensitive arrangement of the resistances will be obtained when $\frac{R}{S + Q} = \frac{G}{P + B}$, where G and B = galvanometer and battery resistances respectively.

Observations.—(1) Connect up the above apparatus as shown in Fig. 93, taking care that the secondary coil Q is so coupled up that the induction throw due to it is in the opposite direction to the capacity throw. Adjust G to zero roughly.

(2) Adjust C to some suitable value, and then vary R or S, or both, so that on "making" and breaking the primary circuit by means of K, no "kick" is observed on G.

(3) Repeat (2) for a series of values of C, from the largest downwards, with a suitable number of cells for B.

(4) Calculate the required mutual induction M from the relation—

$$M = CR(S + Q) \text{ secohms}$$

and tabulate your results as follows :—

G = ohms ; B = ohms ; primary, P, = ohms ; Q = ohms.

Induction tested.	Capacity, C.	R.	S.	(S + Q).	Mutual induction, M.

(5) Repeat (2)–(4) with the positions of the two coils interchanged.

Inferences.—Prove the relation given in (4), and state any assumptions made in deducing it.

117. Comparison of Coefficients of Mutual Induction (Maxwell's Method).

Introduction.—The method, which is practically that originally proposed by Clerk Maxwell, enables either the ratio of two coefficients of mutual induction to be found, or one measured

absolutely in secohms in terms of a standard coefficient of mutual induction. It has the advantage of being a "zero" method, and is strikingly analogous to Lumsden's or Bosscha's zero method of comparing E.M.F.'s. In fact, it is the two secondary induced E.M.F.'s of the two mutual inductions that are compared when the same current is "made" or broken in their primaries.

Apparatus.—Sensitive fairly high-resistance galvanometer, G; adjustable non-inductive resistance boxes, R_1, R_2; suitable battery, C; spring key, K; the two mutual inductions to be compared, M_1, M_2, of which P_1, P_2 are their primaries and S_1, S_2 their secondaries of ohmic resistance, r_1, r_2.

NOTE.—M_1 may be a standard of mutual induction of the form shown in Fig. 122 (p. 263), with slight modification.

Observations.—(1) Connect up as in Fig. 94, so that the induced secondary E.M.F.'s are in series and *help* each other. Adjust the galvanometer G to zero roughly.

(2) Make R_1 = some suitable value, and adjust R_2, so that on "making" and "breaking" the primary circuit with K, no deflection is produced on G.

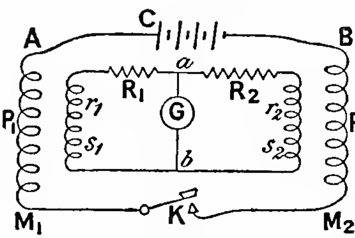


FIG. 94.

(3) Repeat (2) for different values of R_1, R_2, and also with increased E.M.F. at the battery C, and for different values of the standard.

(4) Compare the mutual inductions, and find the value of the unknown from the relation—

$$\frac{M_1}{M_2} = \frac{R_1 + r_1}{R_2 + r_2}$$

and tabulate as follows:—

r_1 = ohms at °C.; r_2 = ohms at °C.

Induction tested.	R_1.	R_2.	Standard, M_1.	$\frac{M_1}{M_2}$	M_2.

(5) Repeat (2)–(4) with the primary and secondary of both M_1 and M_2 interchanged in position.

Inferences.—Prove the relation given in (4), and state any assumptions made in obtaining it.

118. Comparison of Coefficients of Mutual Induction (Secohmmeter Method).

Introduction.—This method is merely a slight modification of the last for increasing its sensitiveness by using the secohmmeter and dispensing with K. The nature of the action of this appliance is stated on p. 259, and the general arrangement for driving it by a small electro-motor is shown in Fig. 119.

In other respects the present test is identically similar to the last, and the secohmmeter is connected up as follows: Connect the points *a* and *b* to terminals marked "bridge" on the galvanometer side of the secohmmeter, and A and B to those marked "bridge" on the opposite side. Place G and B directly across their respective pairs of terminals, and dispense with K.

The speed at which the secohmmeter is driven need not be known, but the greater it is the more sensitive the test; the rate of reversal must not, however, be too great for the currents to reach their steady values between two consecutive battery reversals.

119. Absolute Measurement of Mutual Induction by Hibbert's Standard Magneto-inductor.

Introduction. — The following ballistic or "deflection" method readily enables a coefficient of mutual induction to be determined in absolute measure or otherwise. It involves the use of a standard of magnetic induction, a convenient form of which is that devised by Mr. W. Hibbert, and described in the Appendix (p. 358).

Apparatus.—Mutual induction arrangement, M, to be tested; sensitive ballistic galvanometer, G (*vide* p. 286); standard inductor, I; standard known resistance box, *r*; suitable current measurer, A (*vide* p. 274); reversing key or switch, K (*vide* p. 329); battery, preferably of secondary cells, B (*vide* p. 337); variable rheostat, R (*vide* p. 308).

Observations.—(1) Connect up as shown in Fig. 95, preferably arranging so that I and M both deflect G individually in the same direction. Adjust the galvanometer G to zero.

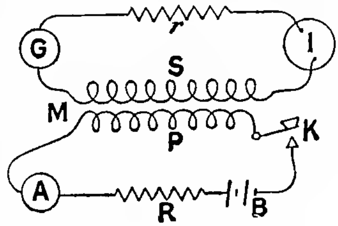


FIG. 95.

(2) Close K, and adjust the current on A to the maximum value which it may have, and then the resistance *r* to such a value as to obtain a full-scale "throw" on G on suddenly reversing the current by K. Note this throw d_1 scale-divisions and the current on A reversed.

(3) With K open and *r* as in (2), slip the inductor coil, and note the throw d_2 scale-divisions on G.

(4) Repeat (2) and (3) for about ten different values of A, decreasing regularly to the lowest convenient.

(5) Repeat (2)–(4) with the positions of P and Q interchanged.

(6) Calculate the coefficient of mutual induction M from the formula—

$$M = \frac{Kd_1}{2Ad_2}$$

where K = the constant of the inductor = N*n*, N being the total number of lines of force cut by the *n* turns of I.

NOTE.—Since the secondary circuit resistance is kept constant for each separate pair of deflections d_1 and d_2, it has not to be allowed for.

The constant K being in C.G.S. measure, A, if in amperes, must be expressed in similar measure by multiplying the numerator Kd_1 by ten, whence M will be given in centimetres or absolute measure.

If the secondary circuit resistance cannot be kept constant in each pair of observations, (2) and (3), let R_1 and R_2 be

its total resistance corresponding to d_1 and d_2 respectively. Then—

$$M = \frac{Kd_1R_1}{2Ad_2R_2}$$

Tabulate your results as follows :—

Inductor ; turns, n, = of resistance ohms at ° C. ; resistance of primary, P, = ; flux, N, = C.G.S. lines ; resistance of secondary, S, =

Induction tested.	Throws.		r.	Total resistance.		Current, A.	M.
	d_1.	d_2.		R_1.	R_2.		

Inferences.—Prove the relation for M given above, and state any assumptions made in obtaining it.

120. Determination of the Laws of Combination of Self-inductions in Parallel.

Introduction.—The present test is arranged with the object of determining the way in which the "combined" or "equivalent" self-induction of a number of individual self-inductions varies for different series and parallel combinations of the latter. The problem has an important bearing in certain branches of electrical work, especially in connection with alternating currents of electricity, and therefore attention is directed to elucidating the effect, and generally the somewhat vague notions of how self-inductions combine. The self-inductions to be experimented with may conveniently consist of four coils, I.–IV., exactly similar to each other in size, form, number of turns, and resistance, and similar to that illustrated in the Appendix (p. 265); two different coils, V. and VI., should be available to combine with these four, so as to produce unsymmetrical combinations as well. The self-inductions can be measured by one of the preceding methods, say, that on p. 205, and when measuring combinations, care must be taken in placing them in such positions as to avoid mutual induction effects between them.

Apparatus.—All that mentioned in this method and, in addition, the self-inductions as suggested above.

Observations.—(1) Measure the self-induction L and resistance R of each of the coils I.–VI. separately.

(2) Measure L and R when they are two, three, four, five, and six in series.

(3) Measure L and R when I. and II., III. and IV., I. and V., and II. and VI. are in parallel.

(4) Measure L and R when I.–IV. and I., II., V., and VI. are connected two in series and two in parallel.

(5) Measure L and R when I.–III. and I., V., and VI. are three in parallel.

(6) Measure L and R when I.–IV. and I., II., V., and VI. are four in parallel.

(7) Measure L and R when I.–VI. are all in parallel, and tabulate as follows:—

Combination of inductions.	Capacity, C.	P.	S.	R.	Resistance, l.		Self-induction, L.	
					Found.	Expected.	Found.	Expected.

Inferences—State clearly all that can be inferred from the above test.

APPENDIX

Magnetic Moments of Magnets.

Deflection Method.

Solution of Inferences.—*A. Position.*—Let the horizontal intensity H of the earth's field act parallel to the needle ns in its position of rest. Also let the force F of the bar magnet, whose length $= 2l$, on one of the poles of the needle be always at right angles to this position of rest; therefore $F = m_1 H \tan \theta$ on either pole of the needle. Now, by the law of "inverse squares," the force exerted between two magnetic poles of strengths m_1, $m_2 = \frac{m_1 m_2}{d^2}$ dynes, where

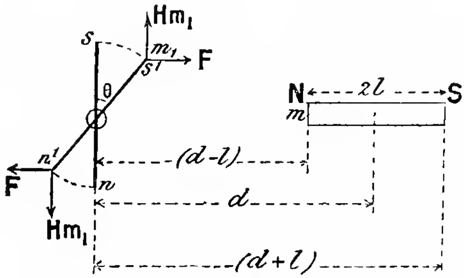


FIG. 96.

d = distance between them in centimetres, which force is a mutual attraction if the poles are of opposite kinds, and a repulsion otherwise;

Therefore force exerted by N pole on one pole of the needle $= \frac{mm_1}{(d-l)^2}$

and „ „ S „ „ „ $= \frac{mm_1}{(d+l)^2}$

where m = strength of magnet's poles, and m_1 that of the needle's.

But these forces act in opposite directions on the same pole of ns.

Hence resultant force F of the magnet on either pole of the needle $= \frac{mm_1}{(d-l)^2} - \frac{mm_1}{(d+l)^2} = \frac{mm_1 4dl}{(d^2-l^2)^2} = m_1 \mathrm{H} \tan\theta.$

But $m = \frac{\mathrm{M}}{2l}$, and equating either the forces or couples acting—

$$\text{Then } \frac{\mathrm{M}2d}{(d^2-l^2)^2} = \mathrm{H}\tan\theta$$

$$\text{and } \therefore \mathrm{M} = \frac{(d^2-l^2)^2}{2d}\mathrm{H}\tan\theta$$

Assumptions.—(1) That the poles of the magnet are at the ends, and therefore at a distance $2l$ apart.

(2) That the length $2l$ is very large compared with the length of needle, so that the mutual forces between the magnet are all parallel.

(3) That the forces are unaltered in magnitude by the deflection of the needle.

Deflection Method.

Solution of Inferences.—*B. Position.*—Let the horizontal intensity H of the earth's field act parallel to the needle ns in its position of rest in the magnetic meridian.

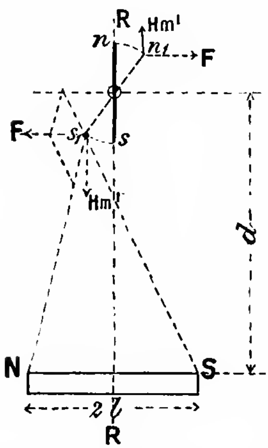


FIG. 97.

Also let $2l$ = length of the bar magnet NS,

$2l_1$ = length of the needle,

m = strength of either of its poles,

m_1 = strength of either of the needle's poles,

d = distance between centre of needle and magnet,

F = resultant force acting on ns causing a deflection θ,

Then, since the distance of either pole of the magnet NS from the needle $= \sqrt{d^2+l^2}$, by the law of inverse squares we see the force exerted between either magnet pole, and say the s pole of the needle, whether of attraction or repulsion, $= \frac{mm_1}{(\sqrt{d^2+l^2})^2} = \frac{mm_1}{(d^2+l^2)}$.

But these combine to an equivalent single F acting parallel to NS, which pulls the *s* pole of the needle counter clockwise.

$$\text{Hence } F = \frac{mm_1 2l}{(d^2 + l^2)^{\frac{3}{2}}}$$

which is the same force acting in the same direction on *n*.

Therefore couple acting on needle $= \dfrac{m2lm_1 2l_1 \cos\theta}{(d^2 + l^2)^{\frac{3}{2}}}$ due to magnet, and couple acting on needle $= m_1 H 2l_1 \sin\theta$ due to earth's magnetism; and when the deflection θ is steady, these couples just balance.

$$\text{Hence } m_1 2l_1 H \sin\theta = \frac{m2lm_1 2l_1 \cos\theta}{(d^2 + l^2)^{\frac{3}{2}}}$$

$$\text{or } H \sin\theta = \frac{M \cos\theta}{(d^2 + l^2)^{\frac{3}{2}}}$$

$$\therefore M = (d^2 + l^2)^{\frac{3}{2}} H \tan\theta$$

where $m2l = M$, the magnetic moment of the magnet.

Vibration Method.

Solution of Formula.—Let the magnet be deflected from its position of rest in the magnetic meridian, through a small angle θ; then the horizontal component H of the earth's field exerts a torque, tending to bring the magnet back to its original position of rest.

Now, let m = the strength of each of the magnet's poles, and $2l$ the distance between them, and K = moment of inertia about the axis of vibration. Then, since a magnet, whose length is large compared with its breadth, may be assumed to have two poles of equal strength, but of opposite kinds, situated at its ends—

The magnetic moment $M = 2ml$

and the force acting on each pole $= mH$

therefore the couple exerted by these two equal and opposite forces $\} = mH \times 2l \sin\theta = MH \sin\theta$

Now, for translational motion we have *force acting = mass × acceleration;* and for rotational motion we have couple acting = moment of inertia × angular acceleration;

$$\text{whence } MH \sin\theta = K\dot{\omega}$$

where $\dot{\omega}$ = angular acceleration.

But the equation of motion for a simple pendulum is—

$$g \sin\theta = l\dot{\omega}$$

where l = length of pendulum, and g the acceleration due to gravity.

Thus the periodic time of vibration $T = 2\pi\sqrt{\frac{l}{g}}$

consequently, for the magnet $T = 2\pi\sqrt{\frac{K}{MH}}$

on the assumption that θ is small, say, not more than 5° or 10°, and that the motion is one of pure rotation.

Measurement of Resistance.

Substitution Method.

Solution of Inferences.—Let g = resistance of the galvanometer, and R, r that of the known and unknown ; also let E = E.M.F. of the battery, and b its internal resistance. Then, if C is the current flowing both when R and r are separately in circuit, as is indicated by the same galvanometer deflection in each case, we have, by Ohm's law—

$$C = \frac{E}{R + b + g} = \frac{E}{r + b + g}$$

Hence $R = r$ (the unknown resistance).

This is based on the assumption that both E and b are constant throughout any pair of readings, which may not be true unless the cell has a fairly constant E.M.F. (such as that of a Daniell), and is allowed to send only a feeble current by having the external resistance as high as possible.

Wheatstone Bridge Method.

Referring to Fig. 19 (p. 37), let V_1, V_2, V_3 be the potentials of the points A, H, and D respectively ; then, when no current flows through the galvanometer, *i.e.* when the bridge is *balanced*, and there is therefore no deflection, V_2 will be the potential of the point C also. Hence, if C_1, C_2, C_3, C_4 are the currents flowing through the resistances r_1, r_2, r_3, r_4 respectively of the "arms," then, since no current passes through G, we have, by Ohm's law—

$$V_1 - V_2 = C_1r_1 = C_4r_4$$
$$\text{and } V_2 - V_3 = C_2r_2 = C_3r_3$$
$$\text{But } C_1 = C_2 \text{ and } C_3 = C_4$$

Hence, dividing up, we have—

$$\frac{r_1}{r_2} = \frac{r_4}{r_3}$$
$$\therefore r_1r_3 = r_2r_4$$

which is the law of the Wheatstone bridge.

Differential Galvanometer Method.

Referring to Fig. 21 (p. 42), let the shunts and resistances be as shown, and, moreover, let E = E.M.F. of the battery, and B its internal resistance. Then, if C_1, C_2 = currents flowing through R and x respectively, and g = resistance of each galvanometer coil, by Ohm's law we have—

$$C_1 = \frac{s_1}{s_1 + g} \cdot \frac{E}{R + B + \dfrac{s_1 g}{s_1 + g}}$$

$$\text{and } C_2 = \frac{s_2}{s_2 + g} \cdot \frac{E}{x + B + \dfrac{s_2 g}{s_2 + g}}$$

Now $C_1 = C_2$ when balance is obtained, and g does not deflect, also B is the same in both cases, and therefore cancels, then we have—

$$\frac{s_1}{s_1 + g} \cdot \frac{1}{R + \dfrac{s_1 g}{s_1 + g}} = \frac{s_2}{s_2 + g} \cdot \frac{1}{x + \dfrac{s_2 g}{s_2 + g}}$$

$$\therefore \frac{s_1}{R(s_1 + g) + s_1 g} = \frac{s_2}{x(s_2 + g) + s_2 g}$$

$$\text{or } R(s_1 + g)s_2 = x(s_2 + g)s_1$$

$$\text{Hence } x = \frac{(s_1 + g)s_2}{(s_2 + g)s_1} R = \frac{1 + \dfrac{g}{s_1}}{1 + \dfrac{g}{s_2}} R$$

If no shunts are used, $s_1 = \infty$ and $s_2 = \infty$, and we have—

$$\therefore x = R$$

High Resistance by the Substitution Method.

Referring to Fig. 22 (p. 43), let G = resistance of the galvanometer, and B that of the battery. Then, if C_1 = current flowing when R is in circuit and a deflection d_R is obtained, and C_2 = current flowing when r is in circuit and a deflection d_r is obtained, by Ohm's law, we have—

$$C_1 = Kd_R = \frac{S_R}{S_R + G} \cdot \frac{E}{R + B + \dfrac{S_R G}{S_R + G}} = \frac{S_R E}{(S_R + G)(R + B) + S_R G}$$

$$\text{and } C_2 = Kd_r = \frac{S_r}{S_r + G} \cdot \frac{E}{r + B + \dfrac{S_r G}{S_r + G}} = \frac{S_r E}{(S_r + G)(r + B) + S_r G}$$

where K = a constant which reduces the galvanometer deflections to current units.

But B will be very small compared with either R or r, and if E is constant, we have by division—

$$\frac{S_R}{(S_R + G)R + S_R G} : \frac{S_r}{(S_r + G)r + S_r G} = d_r : d_R$$

$$= \frac{1}{\left(\dfrac{S_R + G}{S_R}\right)R + G} : \frac{1}{\left(\dfrac{S_r + G}{S_r}\right)r + G}$$

$$\text{whence } d_R\left\{R\left(1 + \frac{G}{S_R}\right) + G\right\} = d_r\left\{r\left(1 + \frac{G}{S_r}\right) + G\right\}$$

If no shunts are used, then S_R = infinity, and also S_r = infinity, and we have—

$$\therefore d_R(R + G) = d_r(r + G)$$

and further, if G is small compared with R and r—

$$\text{Then } d_R R = d_r r$$

$$\text{or } r = \frac{d_R}{d_r} R$$

Low Resistance by the Potential Difference Method.

Referring to Fig. 26 (p. 51), let C be the current which flows through the two resistances R and r in series, and let V and v = potential differences across the ends of R and r respectively ;

$$\text{Then, by Ohm's law, } C = \frac{V}{R} = \frac{v}{r}$$

But the galvanometer deflections d_R and d_r are directly $\propto$ V and v respectively ;

$$\text{Hence } \frac{d_R}{R} = \frac{d_r}{r}$$

$$\therefore r = \frac{d_r}{d_R} R \text{ ohms}$$

In this we assume the current to remain constant in the time taken to observe d_r and d_R.

Laws of Combination of Resistances in Parallel.

Let there be any number of circuits of resistances A, B, C, . . . in parallel between two points, the potentials of which = V_1 and V_2 respectively. Also let R_C be their combined resistance, *i.e.* a resistance such that if it were placed across V_1, V_2 instead of A, B, C, the same total current would flow from the battery.

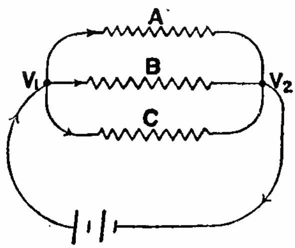


FIG. 98.

$$\text{This total current would therefore} = \frac{V_1 - V_2}{R_C}$$

$$\text{and the current through A} = \frac{V_1 - V_2}{A}$$

$$\text{That through B} = \frac{V_1 - V_2}{B}, \text{ etc.}$$

But the main current must equal the sum of the branch currents ;

$$\text{Hence } \frac{V_1 - V_2}{R_C} = \frac{V_1 - V_2}{A} + \frac{V_1 - V_2}{B} = \frac{V_1 - V_2}{C} + \ldots$$

$$\therefore \frac{1}{R} = \frac{1}{A} + \frac{1}{B} + \frac{1}{C} + \ldots$$

In other words, the reciprocal of the combined resistance is equal to the sum of the reciprocals of the resistances of the individual parallel circuits.

Magneto-inductor Method.

Referring to Fig. 27, let M = resistance of the inductor, and G = resistance of the galvanometer. Then, if the N turns on the coil

Q

of the inductor cut a number of lines of force F of the magnetic field, the whole quantity of electricity set up in the transient current is—

$$Q = \frac{NF}{M + G}, \text{ when } K_1 \text{ is closed}$$

and if Q_1, Q_2 are the quantities which flow when R_1 and R_2 are in circuit—

$$\text{Then } Q_1 = \frac{NF}{M + G + R_1}$$

$$\text{and } Q_2 = \frac{NF}{M + G + R_2}$$

and since d, d_1, and d_2 are respectively $\propto$ Q, Q_1, Q_2, by division we get—

$$\frac{Q_1}{Q} = \frac{d_1}{d} = \frac{M + G}{M + G + R_1}$$

$$\text{and } \frac{Q_2}{Q} = \frac{d_2}{d} = \frac{M + G}{M + G + R_2}$$

and by a well-known rule in proportion we have—

$$\frac{d_1}{d - d_1} = \frac{M + G}{M + G + R_1 - M - G} = \frac{M + G}{R_1}$$

$$\text{also } \frac{d_2}{d - d_2} = \frac{M + G}{R_2}$$

$$\therefore \frac{R_1}{R_2} = \frac{d - d_1}{d - d_2} \cdot \frac{d_2}{d_1}$$

Measurement of Very High or Insulation Resistance (Loss of Charge Method).

Let V_2 = P.D. first used to charge the condenser at any instant with a quantity Q of electricity, which may be taken as proportional to the deflection on the electrometer. Then, if C is the capacity of the condenser, we have—

$$Q = CV_2$$

$$\text{and } \therefore \frac{dQ}{dt} = C\frac{dV_2}{dt}$$

Now, $-\dfrac{dQ}{dt}$ = rate of loss of charge = current flowing from one coating to the other. Hence, by Ohm's law—

$$-\frac{dQ}{dt} = \frac{V_2}{R} = -C\frac{dV_2}{dt}$$

$$\therefore C\frac{dV_2}{dt} + \frac{V_2}{R} = 0$$

and integrating, we get—

$$\log V_2 + \frac{t}{RC} = A, \text{ a constant}$$

If now the P.D. falls to V_2 from V_1 in t seconds, the constant A is found by putting $t = 0$, whence $A = \log V_1$

$$\therefore \frac{t}{CR} = \log V_1 - \log V_2$$

$$\text{or } R = \frac{t}{R \log \frac{V_1}{V_2}}$$

Measurement of Galvanometer Resistance. "Equal" and "Half" Deflection Methods.

Solution of Inferences.—These will be found under the similar headings for the internal resistance of batteries, and the solutions of the inferences connected with them.

Thomson's Method.

The solution of inferences in this method is practically given in those for the Wheatstone bridge (p. 222).

Logarithmic Decrement Method.

Let λ_1, λ_2, and λ_3 be the logarithmic decrements of the galvanometer when its terminals are *free*, *short-circuited*, and *joined through a resistance* R respectively.

$$\text{Then } \lambda_1 = A + \frac{B}{R+g}, \text{ where } R = \infty$$

$$\therefore \frac{B}{R+g} \text{ also} = 0 \text{, hence } \lambda_1 = A$$

$$\lambda_2 = A + \frac{B}{g}$$

$$\therefore \frac{B}{g} = \lambda_2 - A = \lambda_2 - \lambda_1, \text{ or } B = g(\lambda_2 - \lambda_1)$$

$$\lambda_3 = A + \frac{B}{R + g}$$

$$\therefore \frac{B}{R + g} = \lambda_3 - A = \lambda_3 - \lambda_1, \text{ or } B = (R + g)(\lambda_3 - \lambda_1)$$

where A and B are constants which can be eliminated as above, and g is the required galvanometer resistance.

From the above we therefore have—

$$g(\lambda_2 - \lambda_1) = R(\lambda_3 - \lambda_1) + g(\lambda_3 - \lambda_1)$$

$$\therefore g(\lambda_2 - \lambda_1) - g(\lambda_3 - \lambda_1) = R(\lambda_3 - \lambda_1)$$

$$\therefore g(\lambda_2 - \lambda_3) = R(\lambda_3 - \lambda_1)$$

$$\text{Hence } g = \frac{\lambda_3 - \lambda_1}{\lambda_2 - \lambda_3} R \text{ ohms}$$

Internal Resistance of a Battery. Half-deflection Method.

Solution of Inference.—Let E = E.M.F. of the battery of internal resistance B, and G = galvanometer resistance. Then, if R_1, R_2 are the box resistances corresponding to deflections d_1 and d_2 respectively, we have, by Ohm's law—

$$K d_1 = \frac{E}{R_1 + B + G}$$

$$\text{and } K d_2 = \frac{E}{R_2 + B + G}$$

where K = constant which reduces the deflections to amperes.

$$\text{Hence since } d_2 = \tfrac{1}{2} d_1$$

$$\therefore \tfrac{1}{2} \cdot \frac{E}{R_1 + B + G} = \frac{E}{R_2 + B + G}$$

$$\text{or } B = R_2 - 2R_1 - G$$

If G is shunted by a shunt of resistance S, then its effective terminal resistance will $= \frac{SG}{S + G}$, which must therefore be used instead of G in the above relation.

Beetz's Method.

Referring to Fig. 38 (p. 82), let the current flowing through the cell B to be tested = A at the moment of exact balance. Then, since there is no current through G, we have, by Ohm's law—

$$A = \frac{E}{B + r_1 + r_2}$$

$$\text{and } A = \frac{e}{r_1}$$

where E and e = the E.M.F.'s of the cells having internal resistances B and b.

$$\therefore \frac{E}{e} = \frac{B + r_1 + r_2}{r_1}$$

and if r_1' and r_2' are the resistances for any other balance, we have—

$$\frac{E}{e} = \frac{B + r_1' + r_2'}{r_1'}$$

$$\therefore \frac{B + r_1 + r_2}{r_1} = \frac{B + r_1' + r_2'}{r_1'}$$

$$\text{or } B = \frac{r_1'(r_1 + r_2) - r_1(r_1' + r_2')}{r_1 - r_1'} \text{ ohms}$$

It is here assumed that $\frac{E}{e}$ is constant, which cannot quite be true.

Fall of Potential Method.

The solution of the inferences will be at once obvious by reference to Fig. 99.

Let the ordinate OA represent the E.M.F. of the cell, and let OB be the total circuit resistance. Then, if we set off OD = B, the internal resistance of the battery, draw DN perpendicular to OB and NC parallel to OB, we see that AB represent the fall of potential all round the circuit of total resistance OB = OD + DB = B + R, where R = the resistance external to the cell.

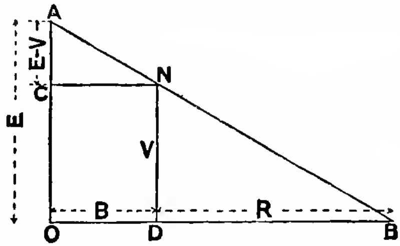


FIG. 99.

Similarly, AN is the fall down the internal resistance B of the cell. Now, evidently DN is the P.D. V at the terminals of the cell, which sends a current C through the external resistance R. Hence the P.D. taken up in the cell itself = OA − OC = E − V. Hence, by Ohm's law—

$$E = CB + CR = C(B + R)$$
$$\text{and } V = CR$$
$$\therefore C = \frac{V}{R} = \frac{E}{B + R}$$
$$\text{Hence } B = \frac{E - V}{V}R$$

or thus, by similarity of triangles—

$$ND : DB = AO : OB$$
$$V : R = E : B + R$$
$$\text{or } B = \frac{E - V}{V}R \text{ ohms}$$

Internal Resistance of Thermo-electric Generator.

Manifestly from the foregoing reasoning we have—

$$A = \frac{V}{R}$$
$$\text{Hence } B = \frac{E - V}{V}R = \frac{E - V}{A} \text{ ohms}$$

Internal Resistance of Batteries (Mance's Method).

Solution of Inferences.—Let the currents and resistances of the arms be as shown in Fig. 100. Then, by Kirchhoff's laws, we have—

$$c_1 = c + c_3 \quad \ldots \quad (1)$$
$$c_3 = C + c_4 \quad \ldots \quad (2)$$
$$c_2 = c + c_4 \quad \ldots \quad (3)$$
$$c_1 = C + c_2 \quad \ldots \quad (4)$$
$$E = c_1r_1 + c_3r_3 + CR \quad \ldots \quad (5)$$
$$0 = c_4r_4 + c_2r_2 - CR \quad \ldots \quad (6)$$
$$0 = c_3r_3 + c_4r_4 - cr \quad \ldots \quad (7)$$

Then from (6)—

$$C = \frac{c_4 r_4 + c_2 r_2}{R} = \frac{E - c_1 r_1 - c_3 r_3}{R} \text{ (from (5))}$$

$$\text{and } C = \frac{(c_3 - C)r_4 + (c_1 - C)r_2}{R} \text{ (from (2) and (4))}$$

$$\therefore RC = (c_3 - C)r_4 + (c_1 - C)r_2$$

$$\text{Hence } C = \frac{c_3 r_4 + c_1 r_2}{R + r_2 + r_4} = \frac{E - c_1 r_1 - c_3 r_3}{R} = \frac{Er_2 + c_3(r_1 r_4 - r_2 r_3)}{r_1(R + r_2 + r_4) + Rr_2}$$

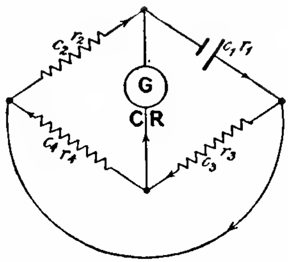


FIG. 100.

Now the condition that C is independent of c_3 is that—

$$r_1 r_4 = r_2 r_3$$

But if C be independent of c_3, it must also be independent of c, because any alteration of c affects c_3;

$$\therefore r_1 r_4 = r_2 r_3$$

when the current through the galvanometer is independent of r.

Internal Resistance of a Battery. Condenser Method.

Referring to the proof of formula in the fall of potential method, we see at once that $\frac{V}{S} = \frac{E}{B + S}$, or $V = \frac{S}{S + B} \cdot E$.

Hence, if d_1, d_2 are the first throws on the galvanometer when the condenser is charged by the E.M.F. E and P.D. V respectively, we have—

$$d_1 : d_2 = E : \frac{S}{S + B} E$$

$$\text{or } B = S\frac{(d_1 - d_2)}{d_2} \text{ ohms}$$

Electrometer Method.

The proof of the formula for this method is obtained in exactly the same manner and with the same reasoning as has just been set forth for the condenser method.

Equal Deflection Shunt Method.

Solution of Inferences.—CASE I.—If the battery of resistance B is shunted by a shunt S, then its terminal P.D. will be $\frac{S}{S+B} \cdot E$, where $E =$ its E.M.F.; and the current flowing through G will thus be $\frac{S}{S+B} \cdot \frac{E}{R+G} \propto d.$

When no shunt is used and the box resistance $= R_1$, we have the galvanometer current $\frac{E}{R_1 + G + B} \propto d$, since the deflections are equal.

$$\therefore \frac{E}{R_1 + G + B} = \frac{S}{S+B} \cdot \frac{E}{R+G}$$

$$\text{Hence } B = \frac{S(R_1 - R)}{(R + G - S)}$$

But S will be small compared with $(R + G)$, so that $(R + G - S)$ sensibly $= R + G$.

$$\therefore B = \frac{S(R_1 - R)}{R + G}$$

CASE II.—With the galvanometer shunted we have—

$$\text{The deflection } d \propto \frac{S}{S+G} \cdot \frac{E}{R + B + \frac{SG}{S+G}}$$

and unshunted—

$$d \propto \frac{E}{R_1 + B + G}$$

$$\text{Hence } \frac{1}{R_1 + B + G} = \frac{S}{S+G} \cdot \frac{1}{R + B + \frac{SG}{S+G}}$$

$$\text{Hence } B = \frac{S(R_1 - R) - GR}{G} = \frac{S(R_1 - R)}{G} - R$$

Thus, if the galvanometer is heavily shunted, R will be small, and may even be = 0.

$$\therefore \; B = \frac{(R_1 - R)}{G} \text{ ohms}$$

Comparison of Electromotive Forces. Equal Resistance Method.

Solution of Inferences.—Let A_1, A_2 = the currents produced by two cells of E.M.F.'s E_1, E_2, and internal resistances b_1, b_2 throw an external resistance R in the resistance box and G in the galvanometer. Then, by Ohm's law, we have—

$$A_1 = \frac{E_1}{R + b_1 + G}$$

$$\text{and } A_2 = \frac{E_2}{R + b_2 + G}$$

where the galvanometer is assumed to be used unshunted.

$$\text{Hence } \frac{E_1}{E_2} = \frac{A_1}{A_2} \cdot \frac{R + b_1 + G}{R + b_2 + G} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

but if (R + G), which is constant all along, is very large compared with b_1, b_2, then $R + b_1 + G$ will be very nearly $= R + b_2 + G$,

$$\text{and } \therefore \frac{E_1}{E_2} = \frac{A_1}{A_2} = \frac{d_1}{d_2} \text{ very approximately}$$

Assumptions.—(*a*) That b_1 and b_2 are so small compared with (R + G) as to be negligible.

(*b*) That the currents are proportional to the deflections, which is very approximately true in D'Arsonval and reflecting galvanometers, but in "tangent" or "sine" galvanometers $\frac{\tan \theta_1}{\tan \theta_2}$ or $\frac{\sin \theta_1}{\sin \theta_2}$, instead of $\frac{d_1}{d_2}$, as the case may be, must be used.

The disadvantages of the method are that the "law" of the galvanometer, *i.e.* the relation between current and deflection, must be known, and again, that the E.M.F.'s of the cells might vary owing to their sending a current.

Equal Deflection Method.

In the relation (1) above we have—

$$A_1 = A_2 \propto d, \text{ or } \frac{A_1}{A_2} = 1 = \frac{d}{d}$$

and if $(b_1 + G)$ and $(b_2 + G)$ are very small compared with R_1 and R_2, they can be neglected, whence we have—

$$\frac{E_1}{E_2} = \frac{R_1}{R_2}$$

This method has the advantage over the last in that, since the deflection is the same in both cases, the law of the galvanometer has not to be known. It has the disadvantage of the liability of alteration of E.M.F. through the cells sending a current.

Wiedemann's Method.

Let A_1 and A_2 be the currents flowing through R when the E.M.F.'s E_1 and E_2 are assisting and opposing one another respectively. Then, if b_1, b_2, and G are the resistances of the batteries and galvanometer respectively, we have, by Ohm's law—

$$A_1 = \frac{E_1 + E_2}{R + b_1 + b_2 + G}$$

$$\text{and } A_2 = \frac{E_1 - E_2}{R + b_1 + b_2 + G}$$

$$\text{Hence } \frac{A_1}{A_2} = \frac{E_1 + E_2}{E_1 - E_2} = \frac{d_1}{d_2}$$

Now, by adding numerator and denominator together on each side, we get $2E_1 = d_1 + d_2$, and by subtracting them we get $2E_2 = d_1 - d_2$.

$$\text{Hence } \frac{E_1}{E_2} = \frac{d_1 + d_2}{d_1 - d_2}$$

The method has the advantages that all resistance is eliminated, and therefore neither R, that of the cells or galvanometer, need be known, and that owing to R being necessarily high in order to keep the deflection d_1 on the scale, the current given by the cells is very small, and hence the polarization also.

It has the disadvantage that the current flowing through the cell of weaker E.M.F. in the *wrong* direction is liable to alter its E.M.F.

Wheatstone's Method.

Let E_1, E_2 be the E.M.F.'s of the two cells of internal resistances b_1 and b_2, and let R_0, r_0 be the respective resistances through which they send equal currents d_0, and R, r be the respective resistances through which they send equal currents d. Then, if G is the galvanometer resistance, we have, by Ohm's law—

$$d_0 = \frac{E_1}{R_0 + b_1 + G} = \frac{E_2}{r_0 + b_2 + G}$$

$$\text{and } d = \frac{E_1}{R + b_1 + G} = \frac{E_2}{r + b_2 + G}$$

Whence, if $(b_1 + G)$ and $(b_2 + G)$ are small compared with R, R_0 and r, r_0, we have—

$$\frac{E_1}{R_0} = \frac{E_2}{r_0}$$

$$\text{and } \frac{E_1}{R} = \frac{E_2}{r}$$

$$\therefore \frac{E_1}{E_2} = \frac{R - R_0}{r - r_0}$$

Lumsden's Method.

Referring to Fig. 47 (p. 95), let E_1, E_2 be the E.M.F.'s of two cells having internal resistances b_1, b_2, in series with the external resistances R and r. If A = current flowing round the circuit when G does not deflect, we have, by Ohm's law—

$$E_1 = A(R + b_1)$$

$$\text{and } E_2 = A(r + b_2)$$

$$\therefore \frac{E_1}{E_2} = \frac{R + b_1}{r + b_2}$$

And if b_1, b_2 are very small compared with R and r, they can be neglected, and we have—

$$\frac{E_1}{E_2} = \frac{R}{r}$$

Clark-Poggendorff Method.

Referring to Fig. 48 (p. 97), let E_1, E_2 be the E.M.F.'s of two cells, and r_1, r_1' the respective resistances across which they are shunted when balance is obtained, and no deflection occurs on the galvanometer. Then, if C = current flowing along ABC, we have—

$$\text{The E.M.F. across AB} = \frac{r_1}{r_1 + r_2}E = E_1 \text{ for the first cell}$$

where E = the working E.M.F. ; also—

$$\text{E.M.F. across AB} = \frac{r_1'}{r_1 + r_2}E = E_2 \text{ for the second cell}$$

$$\therefore E_1 : E_2 = \frac{r_1}{r_1 + r_2}E : \frac{r_1'}{r_1 + r_2}E = r_1 : r_1'$$

Assumptions.—(1) That the working E.M.F. E, and therefore the current through ABC, remains constant.

(2) That $r_1 + r_2$ is constant. If it is altered in any one pair of readings, then the first relation should be used.

If the galvanometer is very sensitive and of high resistance, the method can be made accurate to $\frac{1}{5000}$ of a volt.

Condenser or Ballistic Method.

Let Q_1, Q_2 be the quantities of electricity discharged through the condenser of capacity C when charged by the E.M.F.'s E_1 and E_2 respectively.

$$\text{Then } Q_1 = CE_1$$
$$\text{and } Q_2 = CE_2$$
$$\text{Hence } Q_1 : Q_2 = E_1 : E_2$$

But the respective first throws d_1 and d_2 are $\propto Q_1$ and Q_2 ;

$$\therefore E_1 : E_2 = d_1 : d_2 \text{ approximately}$$
$$\text{More accurately } E_1 : E_2 = \sin \tfrac{1}{2}\theta_1^\circ : \sin \tfrac{1}{2}\theta_2^\circ$$

where θ_1, θ_2 are the angular throws of the galvanometer in degrees.

If the galvanometer of resistance G is shunted by a shunt S, and gives a throw d_1,

$$\text{Then } E_1 : E_2 = \frac{S + G}{S} . d_1 : d_2 \text{ approximately}$$

$\frac{S + G}{S}$ being the approximate multiplying power of the shunt (*vide* p. 113).

It is assumed that there is no leakage from the condenser between the moment of cutting off the E.M.F. and discharging it through the galvanometer.

Electrometer Method.

Let V_1, V_2 be the potentials of the two pairs of quadrants above that of the earth or case of the instrument, and let V = potential of the needle above that of the earth. Then it can be shown that if d = the scale-deflection—

$$d \propto (V_1 - V_2)\{V - \tfrac{1}{2}(V_1 + V_2)\}$$

or if V is large compared with V_1 and V_2, we have—

$$d \propto (V_1 - V_2)V$$

If, then, deflections d_1, d_2 are obtained when the quadrants are charged by E.M.F.'s E_1 and E_2, we have—

$$d_1 \propto (V_1 - V_2)V \propto E_1V$$
$$\text{and } d_2 \propto (V_1 - V_2)V \propto E_2V$$
$$\text{Hence } \frac{E_1}{E_2} = \frac{d_1}{d_2}$$

since the P.D. $(V_1 - V_2)$ between the quadrants must be the same as E_1 and E_2.

It is assumed that the potential V of the needle is constant throughout.

Action of Shunts on Galvanometer Deflections.

Solution of Inferences.—Let A = current in the main circuit,

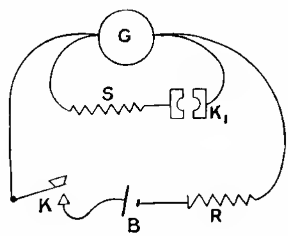


FIG. 101.

A_s and A_G = current through shunt and galvanometer respectively,

and s and g = resistances of shunt and galvanometer respectively. Then, if V = P.D. at the terminals of the galvanometer and shunt, we have, by Ohm's law—

$$A_S = \frac{V}{s}$$

$$\text{also } A_G = \frac{V}{g}$$

$$\text{Hence } \frac{A_G}{A_S} = \frac{s}{g}$$

or, by the theory of proportion—

$$\frac{A_G}{A_G + A_S} = \frac{s}{s+g}$$

$$\text{and } A = A_S + A_G$$

$$\therefore \frac{A_G}{A} = \frac{s}{s+g}$$

Now the deflections d_1 and d are respectively proportional to A_G and A.

$$\text{Hence } \frac{d_1}{d} = \frac{s}{s+g}$$

From these relations we have—

$$A = \frac{s+g}{s} A_G$$

$$\text{or } d = \frac{s+g}{s} d_1$$

where $\frac{s+g}{s}$ is called the *multiplying power* of the shunt.

Again, let R_{sg} = resistance of the shunted galvanometer; then, since $A = A_s + A_g$, we have—

$$\frac{V}{R_{sg}} = \frac{V}{s} + \frac{V}{g}$$

$$\therefore \frac{1}{R_{sg}} = \frac{1}{s} + \frac{1}{g}$$

$$\text{or } R_{sg} = \frac{sg}{s+g}$$

Determination of the Apparent Increase of Resistance of a Ballistic Galvanometer when Shunted.

Solution of Inferences.—Let Q_1, Q_2 be the quantities of electricity which pass into the condenser when it is charged with P.D.'s V_1 and V_2. Then, if C = capacity of the condenser—

$$Q_1 = CV_1$$
$$Q_2 = CV_2$$
$$\therefore \frac{Q_1}{Q_2} = \frac{V_1}{V_2}$$

But $Q_1 \propto$ deflection d_1, and $Q_2 \propto$ deflection $d_2 \times \frac{S + G + K}{S}$, where S = shunt used and K = constant (p. 113), and G = galvanometer resistance ;

$$\text{Hence } \frac{Q_1}{Q_2} = \frac{V_1}{V_2} = \frac{d_1}{d_2 \frac{S + G + K}{S}}$$

Also, since the P.D.'s V_1 and V_2 are directly proportional to the resistances r_1, r_2, we have finally—

$$\frac{r_1}{r_2} = \frac{d_1}{\frac{S + G + K}{S} d_2}$$

$$\text{or } K = S\left(\frac{d_1 r_2}{d_2 r_1} - 1\right) - G \text{ ohms}$$

Calibration of a Ballistic Galvanometer. Earth Inductor Method.

Solution of Inferences.—Let the earth-coil or inductor *ab* be set to cut the horizontal component of the earth's field whose lines of force run from N to S, as shown. Then the number of lines of force enclosed by *ab* in its zero position (*i.e.* with its plane perpendicular to NS) will be AF, where A = mean area of coil in square centimetres and F the horizontal component in C.G.S. units or lines per square centimetre. Now, on turning *ab* about its vertical axis *xy* through 180°, the front half *a* will cut FA lines in one direction, and the back half *b* the same number of lines FA in the opposite direction. The

E.M.F.'s induced while *ab* is in motion will be $\propto$ NAF units in the front half and the same in the back, and since the directions in which these act coincide round the ring, the total E.M.F. is $\propto 2\text{NAF}$, whence the total quantity of electricity Q in the transient current thus generated $\propto \frac{2\text{NAF}}{\text{R}} \propto \sin \frac{1}{2}\theta^\circ$, where θ° = the first throw of the galvanometer needle. Hence the quantity of electricity per division, which we call the "galvanometer constant"—

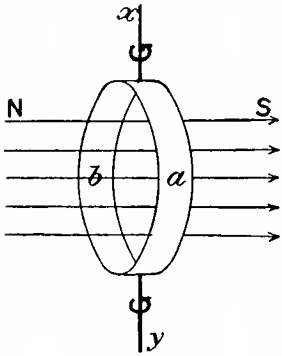


FIG. 102.

$$\text{K} = \frac{2\text{NAF}}{\text{R} \sin \frac{1}{2}\theta^\circ} = \frac{2\text{NAF}}{10^9\text{R} \sin \frac{1}{2}}$$

where R = the total circuit resistance in ohms, and N = the total number of turns on *ab*.

This value of K neglects damping in the galvanometer; hence, if λ = the logarithmic decrement of the galvanometer, we finally have—

$$\text{K} = \frac{2\text{NAF}}{10^9\text{R} \sin \frac{1}{2}\theta(1 + \frac{1}{2}\lambda)} \text{ C.G.S. units of quantity per unit deflection.}$$

Standard Solenoid Method.

Solution of Inferences.—It has already been sufficiently shown in the introduction to the test that the throw θ measures a quantity of electricity $\text{Q} = \frac{4\pi\text{CTAN}}{10^{10}l\text{R}}$, whence accurately the quantity per unit deflection $\text{K} = \frac{4\pi\text{CTAN}}{10^{10}l\text{R} \sin \frac{1}{2}\theta(1 + \frac{1}{2}\lambda)}$ C.G.S. units.

Standardization of Standard Magneto-inductor (Capacity Method).

Solution of Inferences.—Let N = the number of turns on the standard inductor tested, and F = the total number of lines of force crossing the gap. Then the total linkage of lines of force = NF, and R = the total resistance of the inductor circuit in ohms, then the

whole quantity of electricity Q_1 set up in the transient current producing a throw d_1 is $\frac{NF}{R}$;

$$\therefore d_1 \propto Q_1 \propto \frac{NF}{R}$$

Again, if C = the capacity in microfarads, and V = the P.D. in volts to which it is charged, then the whole quantity of electricity Q_2 in the discharge giving a first throw d_c is CV micro-coulombs;

$$\therefore d_c \propto Q_2 \propto CV$$

$$\text{Hence } \frac{d_1}{d_c} = \frac{NF}{R} \div CV = \frac{NF}{RCV}$$

But since a micro-coulomb $= \frac{10^{-1}}{10^6}$ or 10^{-7} C.G.S. units of quantity, and an ohm $= 10^9$ C.G.S. units of resistance;

$$\therefore F = \frac{10^{-7} \times 10^9 RCV}{N} \cdot \frac{d_1}{d_c} = \frac{100RCV}{N} \cdot \frac{d_1}{d_c} \text{ C.G.S. lines}$$

For maximum accuracy d_1 should $= d_c$.

Measurement of Magnetic Permeability (Magnetometer Method).

Solution of Inferences.—A POSITION OF SPECIMEN.—It has already been proved on p. 219 that if M = the magnetic moment of the specimen—

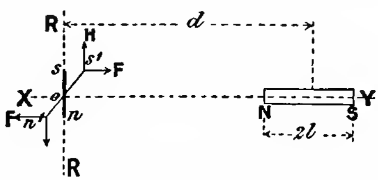


FIG. 103.

$$\text{Then } M = \frac{(d^2 - l^2)^2}{2d} H_E \tan\theta = K_1 \tan\theta^\circ$$

where K_1 is $= \frac{(d^2 - l^2)^2}{2d} H_E$.

Now, the volume of the specimen $V = 2ls$, where s = the sectional area of the specimen; therefore the intensity of magnetization $I = \frac{M}{V} = \frac{K_1}{V} \tan\theta^\circ$.

Also the magnetic induction at the centre of the specimen, namely, $B = 4\pi I + H$;

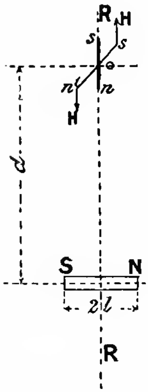


FIG. 104.

Hence $B = 4\pi I + H$

$$= \frac{4\pi(d^2 - l^2)^2 H_E \tan\theta}{V2d} + H \text{ C.G.S. lines}$$

and we therefore get—

the magnetic permeability μ $\Big\} = \frac{B}{H} = \frac{4\pi(d^2 - l^2)^2 H_E}{V2d} \cdot \frac{\tan\theta}{H} + 1$

B POSITION OF SPECIMEN.—It has also been proved on p. 220 that for the "B Position of Gauss," the magnetic moment of the specimen $M = (d^2 + l^2)^{\frac{3}{2}} H_E \tan\theta$. Hence it at once follows from the above reasoning that—

$$B = 4\pi I + H$$

$$= \frac{4\pi(d^2 + l^2)^{\frac{3}{2}} H_E \tan\theta}{V} + H \text{ G.C.S. lines}$$

$$\text{whence } \mu = \frac{B}{H} = \frac{4\pi(d^2 + l^2)^{\frac{3}{2}} H_E}{V} \cdot \frac{\tan\theta}{H} + 1$$

C POSITION OF SPECIMEN.—Let d and D be the distances of the two poles of the specimen from the centre of the magnetometer needle as shown, and let the upper pole of the specimen and the needle be in the same horizontal plane. Also let the force F of the specimen on the needle always act perpendicular to the original zero position of the needle, and therefore to the earth's field. Then, if m' is the strength of either of the poles of this needle, and m that of either pole of the specimen, we have, from the "law of inverse squares," that the stress exerted between N and s, which is one of attraction, $= \frac{mm'}{d^2}$, and stress exerted between S and s, which is one of repulsion, $= \frac{mm'}{D^2} \cdot \frac{d}{D}$, and these act in opposite directions.

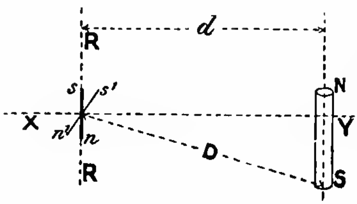


FIG. 105.

$$\text{Hence } F = \frac{mm'}{d^2} - \frac{mm'}{D^2} \cdot \frac{d}{D} = H_E \tan\theta.$$

where θ = the angle of deflection = the angle sos'.

But $m = \frac{BS}{4\pi}$, and assuming m' to be of *unit* strength, we have—

$$\frac{BS}{4\pi}\left(\frac{1}{d^2} - \frac{1}{D^2}\cdot\frac{d}{D}\right) = H_E \tan\theta$$

$$\text{or } B = \frac{4\pi H_E \tan\theta}{S\left(\frac{1}{d^2} - \frac{d}{D^3}\right)} \text{ C.G.S. lines}$$

where s = sectional area of specimen,

$$\text{whence } \mu = \frac{B}{H} = \frac{4\pi H_E}{S\left(\frac{1}{d^2} - \frac{d}{D^3}\right)} \cdot \frac{\tan\theta}{H}$$

H being the *actual effective* magnetizing force.

Comparison of Capacities Ballistically (Direct Deflection Method).

Solution of Inferences.—

Let Q_1, Q_2 = charges or quantities of electricity stored in the two condensers,

C_1, C_2 = the capacities of the two condensers,

V_1, V_2 = the potential differences to which they are charged.

$$\text{Then } Q_1 = V_1C_1$$
$$\text{and } Q_2 = V_2C_2$$

If now $\theta_1°$ and $\theta_2°$ are the angular first throws in degrees on the ballistic galvanometer produced by the passage of Q_1, Q_2 through its coils either on charge or discharge, and if λ = logarithmic decrement of the galvanometer with its terminals disconnected from the circuit, and Q_0 = the quantity of electricity which gives unit angular deflection, we have—

$$Q_1 = Q_0 \sin \tfrac{1}{2}\theta_1°(1 + \tfrac{1}{2}\lambda)$$
$$\text{and } Q_2 = Q_0 \sin \tfrac{1}{2}\theta_2°(1 + \tfrac{1}{2}\lambda)$$
$$\text{Hence } \frac{Q_1}{Q_2} = \frac{\sin \frac{1}{2}\theta_1°}{\sin \frac{1}{2}\theta_2°} = \frac{V_1C_1}{V_2C_2}$$

If $\theta_1°$, $\theta_2°$ are small, which they will be even for full-scale throws, d_1, d_2 divisions, on the usual galvanometer and scale (being about 6° or 7° only)—

$$\text{Then } \frac{V_1C_1}{V_2C_2} = \frac{d_1}{d_2}$$
$$\text{or } \frac{C_1}{C_2} = \frac{V_2}{V_1}\cdot\frac{d_1}{d_2}$$

when, if the same P.D. is used to charge each condenser, V_1 will = V_2, and we have finally—

$$C_1 : C_2 = d_1 : d_2 \text{ approximately}$$

Assumptions.—(1) That the whole discharge or charge through the galvanometer takes place before the needle begins to move, or, in other words, that the whole of the kinetic energy of the needle after the flow has occurred is employed in overcoming the magnetic forces due to the earth's magnetism and induced current in the galvanometer coils from the motion of the needle.

(2) That there is no leakage between the coatings of either condenser due to defective insulation in the condenser or circuit which would cause one or other or both of the throws to be less than it should be.

(3) That there is no "residual absorption" in the condensers due to a soaking in of a certain fraction of the charge which would also make the deflections smaller than they ought to be.

(4) That the P.D. is quite constant throughout the pair of readings with C_1 and C_2.

(5) That the angular throws are small enough to make $\dfrac{\sin \frac{1}{2}\theta_1{}^\circ}{\sin \frac{1}{2}\theta_2{}^\circ} = \dfrac{d_1}{d_2}$, the ratio of the two scale-deflections simply.

The above therefore constitute the sources of error to which this method is liable, and a more accurate relation would be—

$$C_1 : C_2 = \sin \tfrac{1}{2}{}^\circ{}_1\theta : \sin \tfrac{1}{2}{}^\circ{}_2\theta$$

From the preceding remarks we see that λ, the damping correction, does not enter into the final relation, as it affects d_1 and d_2 in the same ratio; also that for maximum accuracy we should have $d_1 = d_2$. High insulation of the battery is not essential, but that of the rest of the circuit is.

If, instead of using the method of connecting shown on p. 175, the condensers are charged or discharged by a simple key, then special care must be taken to get rid of any residual charge by short-circuiting the condenser after each discharge through the galvanometer. The diagram of the connections suggested by the author will be found to be the best in many ways, principally in getting rid of or nullifying the effects of leakage owing to the discharge being effected instantaneously after breaking the battery circuit.

Comparison of Capacities (Wheatstone Bridge Method).

Solution of Inferences.—Let V_1 be the potential of the point E, and V_3 be the potential of the point H. Then, when balance is obtained, no deflection of the galvanometer G results, or the potential of the points D and F must be equal. Let its value at either be V_2.

Then if Q_1, Q_2 = the quantities of electricity flowing into the condensers of capacities C_1, C_2, through the resistances R_1, R_2 respectively, and since C_1 and C_2 are charged to the same potential $V_2 - V_3$,

$$\therefore\ Q_1 = C_1(V_2 - V_3)$$
$$\text{and } Q_2 = C_2(V_2 - V_3)$$
$$\text{or } \frac{Q_1}{Q_2} = \frac{C_1}{C_2}$$

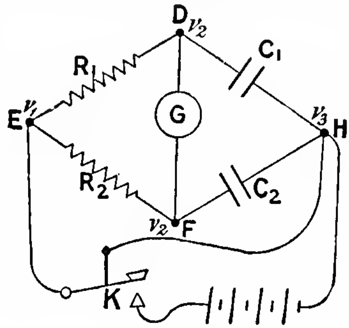


FIG. 106.

If now A_1, A_2 are the currents flowing respectively through R_1 and R_2 during a very short interval of time t, we shall have—

$$Q_1 = A_1 t$$
$$\text{and } Q_2 = A_2 t$$
$$\text{or } \frac{Q_1}{Q_2} = \frac{A_1}{A_2}$$

but, by Ohm's law, the currents will be inversely as the resistances, or $\frac{A_1}{A_2} = \frac{R_2}{R_1}$. Hence, finally, we have—

$$\frac{C_1}{C_2} = \frac{R_2}{R_1}$$

The galvanometer G should have a resistance which approximates to the sum of R_1 and R_2 for maximum sensitiveness.

Absolute Measurement of the Capacity of Condensers (Ballistic Method).

Solution of Inferences.—Let Q = quantity of electricity required to be given to the condenser of capacity F in order to raise the potential difference between its coatings to a value V. Then, as before, we have—

$$Q = VF$$

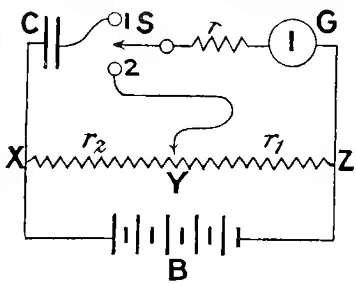


FIG. 107.

Again, if a first throw θ_1° is obtained when the quantity Q is sent through the ballistic galvanometer, whose needle has a periodic time of oscillation T seconds, and a logarithmic decrement λ, and, moreover, if a steady current A produces a steady deflection θ_2° on the galvanometer, then, for any

ballistic galvanometer, when θ_1° and θ_2° are small, which they nearly always are in ordinary reflecting instruments (6° to 7° merely), we have—

$$Q = \frac{T}{\pi} . A . \frac{\sin \frac{1}{2}\theta_1^\circ}{\tan \theta_2^\circ}(1 + \tfrac{1}{2}\lambda) = \frac{T}{\pi} . A . \tfrac{1}{2}\frac{d_1}{d_2}(1 + \tfrac{1}{2}\lambda)$$

where d_1, d_2 = the deflections in scale-divisions corresponding to θ_1° and θ_2°.

$$\text{Hence } Q = VF = \frac{T}{\pi} \cdot \frac{A}{2} \cdot \frac{d_1}{d_2}(1 + \tfrac{1}{2}\lambda)$$

$$\text{or } F = \frac{T}{2\pi} \cdot \frac{A}{V} \cdot \frac{d_1}{d_2}(1 + \tfrac{1}{2}\lambda)$$

But V = the whole P.D. between X and Z used in charging the condenser, and A = the steady current through the galvanometer due to a fraction $= \frac{r_1}{r_1 + r_2}$ V of this total potential.

$$\text{Hence } A = \frac{\frac{r_1}{r_1 + r_2}V}{b + r + G} \text{ by Ohm's law}$$

$$\therefore \frac{A}{V} = \frac{RV}{(b + r + G)V} = \frac{R}{b + r + G}$$

where R stands for $\frac{r_1}{r_1 + r_2}$, and b = the equivalent battery resistance.

$$\text{Hence } F = \frac{T}{2\pi} \cdot \frac{R}{b + r + G} \cdot \frac{d_1}{d_2}(1 + \tfrac{1}{2}\lambda)$$

Assumptions.—(1) That the total E.M.F. of the battery has remained constant in the time taken in obtaining any pair of readings d_1 and d_2.

(2) That the needle of G does not begin to move until the whole discharge is completed, and hence the value of T must be large enough to ensure this.

(3) That θ_1° and θ_2° are really sufficiently small enough to make $\frac{\theta_1^\circ}{\theta_2^\circ} = \frac{d_1}{d_2}$ *very* nearly true.

Comparison of Capacities.

Method of Mixtures.

Solution of Inferences.—Let Q_1, Q_2 be the quantities of electricity with which the two condensers, of capacities C_1, C_2 respectively, are

charged, and let V_1, V_2, V_3 be the potentials of the points A, D, and H respectively when P is turned to the front, so as to send a steady current through the combined resistance AH.

Then the P.D. between A and D $= V_1 - V_2$
and " " D " H $= V_2 - V_3$

whence, by Ohm's law, $\dfrac{R_1}{R_2} = \dfrac{V_1 - V_2}{V_2 - V_3}$

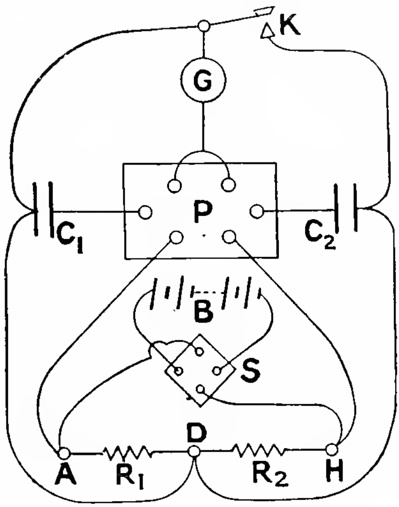


FIG. 108.

But if C_1, C_2 are the capacities of the two condensers, we have—

$$Q_1 = C_1(V_1 - V_2)$$

and $Q_2 = C_2(V_2 - V_3)$

But $Q_1 = Q_2$ when no deflection occurs ;

hence $C_1(V_1 - V_2) = C_2(V_2 - V_3)$

or $\dfrac{V_1 - V_2}{V_2 - V_3} = \dfrac{C_2}{C_1}$

Thus $\dfrac{C_2}{C_1} = \dfrac{R_1}{R_2}$

or $C_1 : C_2 = R_2 : R_1$

Method of Divided Charge.

Solution of Inferences.—

Let C = capacity of the condenser to be tested,

C_s = ,, ,, standard known condenser,

V = the P.D. used to charge the standard known condenser.

Then, when V is applied to C_s, a quantity Q_1 of electricity flows into the condenser, and when discharged through G produces a first throw d_1.

Now, when C_s is again charged to the same potential V, and the unknown capacity C put in parallel, the charge Q_1 in C_s divides in proportion to the capacities C and C_s in such a way that a part Q_2 remains in the standard, while the remainder Q_3 flows into the unknown, Q_2, and now gives a first throw d_2 when discharged through G. We therefore have—

$$Q_1 = C_sV$$
$$Q_3 = CV_1$$
$$Q_2 = C_sV_1$$

where V_1 = common potential after the charges divide.

$$\text{Hence } \frac{Q_2}{C_s} = \frac{Q_3}{C}$$

$$\text{and by proportion each } = \frac{Q_2 + Q_3}{C_s + C}$$

$$\text{But } \frac{Q_2 + Q_3}{C_s + C} = \frac{Q_1}{C_s + C}$$

$$\therefore \frac{Q_2}{C_s} = \frac{Q_1}{C_s + C}$$

But neglecting air-damping, we have—

$$Q_1 : Q_2 = d_1 : d_2,$$

where d_1 and d_2 are the first throws due to discharges Q_1 and Q_2;

$$\text{hence } \frac{d_2}{C_s} = \frac{d_1}{C_s + C}$$

$$\text{or finally } \frac{C}{C_s} = \frac{d_1 - d_2}{d_2}$$

Capacities in Series and Parallel (Laws of Combinations).

Solution of Inferences.—Suppose there are any number of capacities C_1, C_2, C_3 . . . all *in parallel*, which arrangement is shown

in Fig. 109 (only three capacities being depicted symbolically). Then, since the capacity of a condenser is proportional to the area of either of its coatings, it follows that all the left-hand coatings are +, and equivalent to one large coating = sum of the respective ones. Similarly for the right-hand coatings. Hence, if C is the combined or effective capacity, then—

$$C = C_1 + C_2 + C_3 + \ldots \text{ etc.}$$

Suppose there are any number (say three again) of capacities $C_1, C_2, C_3 \ldots$ all *in series*, as shown in Fig. 110, and let V_1, V_2, V_3 be the P.D.'s between the respective coatings of each condenser.

Then P.D. between A and B $= V_1 + V_2 + V_3 + \ldots = V$, say, and the quantity of electricity on each coating of every condenser must be

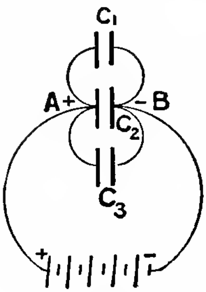


FIG. 109.

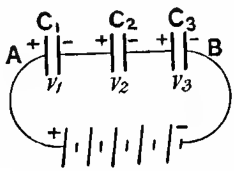


FIG. 110.

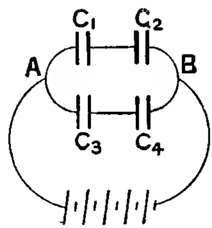


FIG. 111.

the same. Then $Q_1 = Q_2 = Q_3 =$ etc. $= Q$, say, since adjacent coatings are connected metallically, and an equal and opposite charge resides on the coating of every condenser.

$$\text{But } Q_1 = C_1V_1, \text{ and } Q_2 = C_2V_2, \text{ and } Q_3 = C_3V_3$$

$$\therefore V_1 = \frac{Q_1}{C_1}, \text{ and } V_2 = \frac{Q_2}{C_2}, \text{ and } V_3 = \frac{Q_3}{C_3}$$

$$\text{Hence } V = \frac{Q}{C} = V_1 + V_2 + V_3 + \ldots = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} + \frac{Q_3}{C_3} + \ldots$$

$$\therefore \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots \text{ etc.}$$

If four capacities were arranged, two in series and two in parallel, as shown in Fig. 111, then, from the above, we clearly get the combined capacity $C = \frac{C_1C_2}{C_1 + C_2} + \frac{C_3C_4}{C_3 + C_4}$.

Also from any two capacities, C_1 and C_2, four different ones are obtainable, viz C_1, C_2, $C_1 + C_2$, and $\frac{C_1C_2}{C_1 + C_2}$.

Absolute Measurement of Self-induction. Maxwell-Rayleigh Method.

Proof of Formula.—In the first part of the experiment, when the bridge is balanced for steady currents in the ordinary way, there will be no deflection when $Ql = P \cdot \frac{SR}{S+R}$, where Q, l, P, S, and R represent the ohmic resistances of the various parts of the bridge as shown.

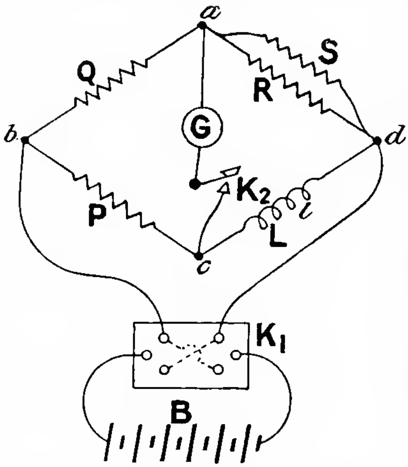


FIG. 112.

In the second part, where nothing has been altered except the order of manipulation of the keys K_1 and K_2, the impulse or throw of the galvanometer, namely, $d°$, will be proportional to the self-induction L, and will represent a portion of the total quantity of electricity developed. In other words, the discharge through G will be that due to the self-induced E.M.F. LC_1 acting independently of the battery, and as if this latter was non-existent, where C_1 = the steady current in *cd* after balance is disturbed by adding ρ to the arm *ad*. Similarly, in the third part of the experiment, the steady current through G, giving the steady deflection D°, is that due to an added E.M.F ρC_2 acting as if B were non-existent, where C_2 = steady current in arm *ad* after adding ρ. We therefore have—

$$\frac{\frac{1}{2}K}{A} = \frac{LC_1}{\rho C_2}$$

$$\text{whence } L = \frac{C_2}{C_1} \cdot \frac{\rho}{2} \cdot \frac{K}{A} \text{ secohms}$$

where K = discharge through the ballistic galvanometer in coulombs, causing throw $d°$, and A = steady current through the ballistic galvanometer in amperes, causing the steady deflection D°, and only half the throw is taken for reversal of battery current; but—

$$K = \frac{T}{\pi} \cdot A \cdot \frac{\sin \frac{d°}{2}(1 + \frac{1}{2}\lambda)}{\tan D°} \text{ coulombs}$$

where T = periodic time of vibration of the needle in seconds, and λ = the Napierian logarithmic decrement of the galvanometer.

Hence, substituting, we have finally—

$$L = \frac{C_2}{C_1} \cdot \frac{\rho}{2} \cdot \frac{\frac{T}{\pi} \cdot A \cdot \frac{\sin \frac{d^\circ}{2}(1 + \frac{1}{2}\lambda)}{\tan D^\circ}}{A} = \frac{C_2}{C_1} \cdot \frac{\rho}{2} \cdot \frac{T}{2\pi} \cdot \frac{d}{D}(1 + \tfrac{1}{2}\lambda) \text{ secohms}$$

Since in a reflecting galvanometer d° and D° are necessarily small, whence—

$$\frac{\sin \frac{d^\circ}{2}}{\tan D^\circ} = \frac{1}{2}\frac{d}{D} \text{ very approximately}$$

d and D now being in scale-divisions instead of degrees.

If ρ is small, then—

$$\frac{C_2}{C_1} = 1 \text{ very nearly}$$

$$\text{and } \therefore L = \frac{T}{2\pi} \cdot \frac{\rho}{2} \cdot \frac{d}{D}(1 + \tfrac{1}{2}\lambda) \text{ secohms}$$

If ρ is not sufficiently small, then the value of $\frac{C_2}{C_1}$ must be allowed for, and can be taken as $\dfrac{P + \frac{SR}{S + R}}{P + \rho + \frac{SR}{S + R}}$.

Secohmmeter Method.

Proof of Formula.—In the first part of the experiment the bridge will be "balanced" for steady currents accurately when $Ql = P \cdot \frac{SR}{S + R}$, and there will be no deflection on G; P, Q, R, S, and l being the ohmic resistances of the various parts of the bridge as shown. In the second part, when the secohmmeter is running at constant speed, the steady deflection d, or the continuous discharge through G, as it may be termed, is the result of a rapid succession of impulses due to and proportional to the self-induction L. In other words, d is due to the self-induced E.M.F., LC_1n acting independently of the battery, and as if this latter was non-existent where n = number of reversals per second, and C_1 = steady current in arm cd.

The steady deflection D produced by adding ρ to the arm *ad* is due to the added E.M.F. ρC_2 acting as if B were non existent, where

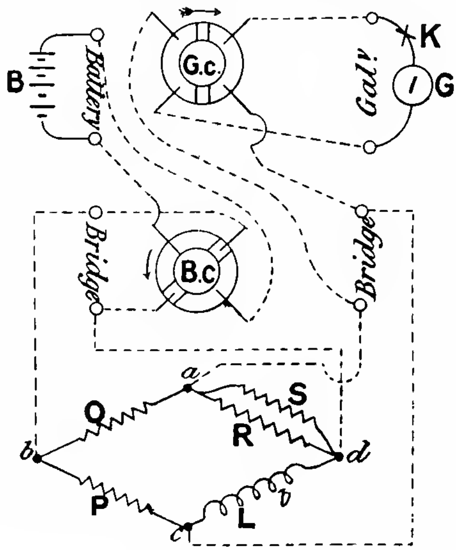


FIG. 113.

C_2 = steady current in arm *ad* after adding ρ to it; we therefore have—

$$\frac{LC_1 n}{\rho C_2} = \frac{d}{D}$$

$$\text{or } L = \frac{\rho}{n} \cdot \frac{C_2}{C_1} \cdot \frac{d}{D}$$

$$\text{But } \frac{C_2}{C_1} = \frac{l}{\frac{RS}{R+S} + \rho}$$

$$\text{Hence } L = \frac{\rho}{n} \cdot \frac{l}{\frac{RS}{R+S} + \rho} \cdot \frac{d}{D} \text{ secohms}$$

Maxwell-Rimington Method.

Proof of Formula.—

Let V_1 = potential of the point *b*,
V_2 = ,, ,, points *a* and *c*,
V_3 = ,, ,, point *d*,

C_1 = current flowing through P, and therefore through l,
C_4 = „ „ Na, „ „ R,
C_2 = „ „ r,
C_3 = „ „ the condenser.

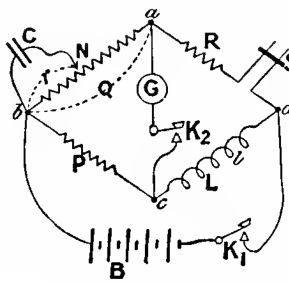


FIG. 114.

Then from the balance for steady current, we have—

$$lQ = RP$$

$$\text{or } l = \frac{RP}{Q}$$

$$\text{Also } V_1 - V_2 = C_1P = C_4(Q - r) + C_2r$$
$$= C_4Q - C_4r + C_4r - C_3r \ldots$$
$$\text{since } C_2 = C_4 - C_3$$
$$\text{Hence } C_1P = C_4Q - C_3r$$
$$\text{and therefore } C_4Q = C_1P + C_3r$$

$$\text{But } C_3 = C\frac{d(C_2r)}{dt} = Cr\frac{dC_2}{dt}$$

$$\therefore\ C_3 = Cr\frac{P}{Q}\cdot\frac{dC_1}{dt}$$

$$\text{and } C_2r = \frac{r}{Q}(V_1 - V_2)$$

$$\therefore\ C_2 = \frac{V_1 - V_2}{Q} = \frac{C_1P}{Q}$$

$$\text{Hence } C_4Q = C_1P + Cr^2\frac{P}{Q}\cdot\frac{dC_1}{dt}$$

and multiplying all through by $\frac{R}{Q}$, we get—

$$C_4R = C_1\frac{RP}{Q} + Cr^2\frac{RP}{Q \times Q}\cdot\frac{dC_1}{dt}$$
$$= C_1l + Cr^2\frac{l}{Q}\cdot\frac{dC_1}{dt}$$

$$\text{since } l = \frac{RP}{Q}$$

$$\text{But } V_2 - V_3 = C_4R = C_1 l + L\frac{dC_1}{dt}$$

$$\therefore C_1 l + L\frac{dC_1}{dt} = C_1 l + Cr^2\frac{l}{Q}\cdot\frac{dC_1}{dt}$$

and by equating the coefficients of $\frac{dC_2}{dt}$, we have—

$$L = Cr^2\frac{l}{Q} \text{ secohms}$$

C being the capacity in farads.

Absolute Measurement of Self-induction.

Proof of Formula.—

Let the current flowing in the arm $ae = C_1$,

" " " $bc = C_2$,

" " " $ce = C_3$.

Then the current flowing in the arm cd will $= C_2 + C_3$

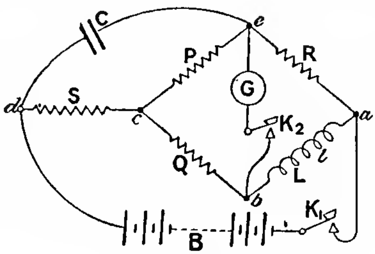


FIG. 115.

Now, when balance for steady currents is obtained, as in Obs. (2), we have $Pl = RQ$; and also $PC_3 = QC_2$, since the total fall of potential down P must equal that down Q for the balance. For similar reasons—

$$C_1R = C_2 l + L\frac{dC_2}{dt}$$

Now, the P.D. across the condenser terminals, *i.e.* between d and e, is $= C_3P + (C_2 + C_3)S = C_2S + (P + S)C_3$, and the quantity in the condenser $= C\{C_2S + (P + S)C_3\}$; also the current through the condenser $= C\left\{(P + S)\frac{dC_3}{dt} + S\frac{dC_2}{dt}\right\} = C_1 - C_3$;

$$\therefore\ C_1 = C_3 + C\left(P + \frac{SQ}{P}\cdot\frac{dC_2}{dt} + S\frac{dC_2}{dt}\right)$$

$$\text{Hence } C_1R = C_2l + L\frac{dC_2}{dt} = \frac{RQ}{P}C_2 + RC\left(Q + \frac{SQ}{P} + S\right)\frac{dC_2}{dt}$$

But $l = \frac{RQ}{P}$, and equating the coefficients of $\frac{dC_2}{dt}$, we have—

$$L = RC\left(Q + \frac{SQ}{P} + S\right) = C\left(RQ + S\frac{RQ}{P} + RS\right)$$

$$\therefore\ L = C(RQ + Sl + RS) \text{ secohms}$$

where C = capacity in farads, and R, Q, S, and l = resistances in ohms.

Absolute Measurement of Mutual Induction (Carey-Foster Method).

Proof of Formula.—Suppose a spring tapping-key, K_c, to be inserted in the condenser arm, *ac*, of the figure, and one, K_s, in the secondary circuit, say at *d*.

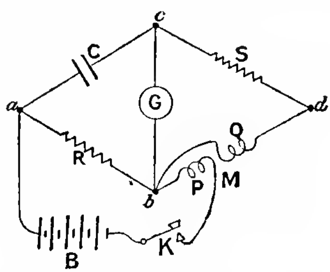


FIG. 116.

If C = the capacity of the condenser in farads,
G = „ galvanometer resistance in ohms,
P = „ primary „ „
Q = „ secondary „ „
M = mutual induction of primary and secondary required,
R and S = two resistances,

then, with K_c open and K_s closed, on "making" or "breaking" the battery key K, the steady current C_1 in the primary will produce a flow of current in the secondary, the total quantity of which $= \frac{MC_1}{Q + G + S}$, corresponding to a throw of d_1 scale-divisions on G.

If now K_c be closed and K_s opened, and R as before, then, on making and breaking the battery circuit with K, the quantity of electricity in the transient current passing through G = CRC_1, corresponding to a throw d_2 scale-divisions on G.

$$\text{Hence } \frac{MC_1}{Q + G + S} : CRC_1 = d_1 : d_2$$

$$\therefore M = CR(Q + G + S) . \frac{d_1}{d_2}$$

But if, as in the test, these quantities of electricity neutralize one another, there will be no current through G, and therefore no deflection, whence $d_1 = d_2 = 0$ and G disappears, and we get—

$$M = CR(Q + S) \text{ secohms}$$

Absolute Measurement of Mutual Induction (by Standard Magneto-inductor).

Proof of Formula.—If M = coefficient of mutual induction required, A = current reversed in the primary, $R_1 R_2$ = total secondary circuit resistance for the throws d_1, d_2 respectively, then, when the primary current is reversed, the whole quantity of electricity set up in the transient current = $\frac{2AM}{R_1}$, which is $\propto d_1$, the throw resulting.

Similarly, on slipping the inductor, the whole quantity = $\frac{Nn}{R_2}$, which is $\propto d_2$, where d_2 = throw resulting, N = number of lines of force, n = number of turns on the inductor.

Hence we have—

$$\frac{2AM}{R_1} : \frac{Nn}{R_2} = d_1 : d_2$$

$$\text{or } M = \frac{Nn}{2A} . \frac{R_1}{R_2} . \frac{d_1}{d_2}$$

But if $R_1 = R_2$, and we call K the constant of the inductor,

$$\therefore K = Nn$$

$$\text{and } \therefore M = \frac{Kd_1}{2Ad_2}$$

APPARATUS

The Secohmmeter.

THE instrument or appliance known as the secohmmeter was devised by Profs. Ayrton and Perry, and, in conjunction with an ordinary Wheatstone bridge arrangement, is a contrivance for interchanging alternately the connections from the battery and galvanometer respectively to their usual points of junction on the bridge. The throws due to self-induction are thus commutated into a steady unidirectional deflection. The secohmmeter, as constructed up to quite recently, is shown in Fig. 117. It consists of two rotatory two-part commutators

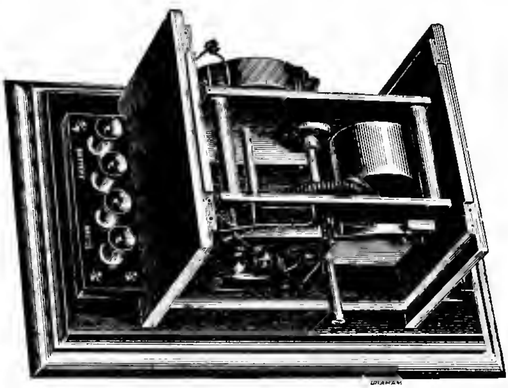


FIG. 117.

fixed to and rotated by a horizontal spindle, which is prolonged at one end beyond one of the commutators for the purpose of coupling it to a speed indicator if necessary. This spindle also carries two small toothed pinions, which gear into and are driven respectively by two toothed wheels carried by a second parallel and horizontal spindle capable of being rotated by a handle. A flywheel carried by a third spindle is driven by means of another pinion gearing into the larger of the toothed wheels. The speed ratios between handle and commutator spindles are 1 : 2 and 1 : 6, and to alter from one to the other,

press down the end of the locking lever (not seen in figure) on the right, and give the handle spindle an axial motion to the front or back, turning it slightly to assist the toothed wheels engaging properly; then release the locking lever. Four stationary brushes press against each commutator, the two pairs of opposite brushes on one commutator going to the two pairs of terminals one end of the secohmmeter, and similarly for the remaining part. Only one set of terminals (on the left) and the front commutator can properly be seen in Fig. 117.

One commutator (GC) is for periodically interchanging the points to which the galvanometer terminals are attached, the other (BC) for doing the same with the battery. By means of a set screw, the relative positions of GC and BC can be varied so that both sets of interchanges can be made to occur simultaneously, or one a little before or after the other, or one midway between two successive interchanges of the other. From the preceding speed ratios, it is seen that one revolution of the handle can be made to give either eight or twenty-four interchanges of galvanometer and battery.

Dehydrated paraffin oil may be used to lubricate the spindles and commutators where pressed by the brushes, but in the latter case a mere drop will suffice.

The latest method of construction of the secohmmeter is shown in Fig. 118. As seen, the commutators and brush-holders are outside a

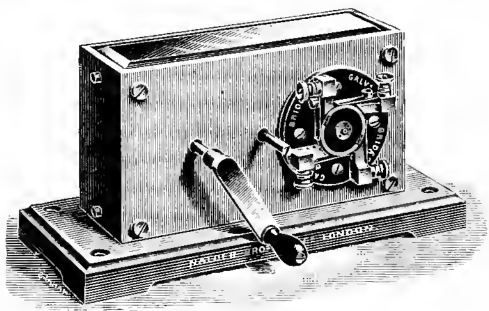


FIG. 118.

brass case at opposite ends of the same spindle, which is prolonged for driving a speed indicator. The plate carrying one set of brushes is arranged so that it can be rotated through an angle up to 90°, and the "lead" thus far more easily varied than in the previous manner. Two separate spindles are provided for altering the speed ratio, and the handle can drive either of them at pleasure. Only one commutator and set of brushes are shown, the other being at the back.

The secohmmeter may preferably be driven by an electro-motor

direct-coupled to the commutator spindle. Fig. 119 shows a convenient arrangement. On the left is the secohmmeter with cover and one side removed, with the handle for hand driving when required. In the centre is an electro-motor of the Cuttriss type, and on the right is a Young's speed-indicator. The spindles of all three are in alignment, and are coupled by spiral springs. In the secohmmeter shown the author has devised a different arrangement of brushes to those shown in Fig. 119, consisting of metal plungers or pistons so fitted that their rounded ends are pressed against the rotating commutators by light spiral springs. The connections to the terminals are soldered

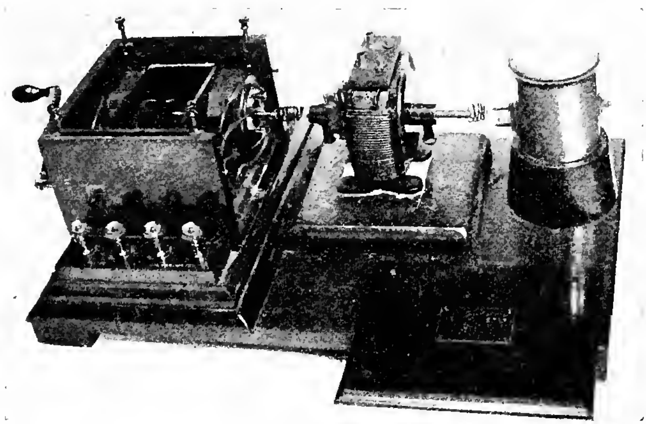


FIG. 119.

to the ends of the four plungers of each commutator. This arrangement has been found much more satisfactory than the old one in the matter of obtaining a reliable rubbing contact with minimum wear of commutator and brushes. If the time of one revolution of the commutators is small compared with the period of the galvanometer needle, the steady deflection thus produced is that which would be given by a current Qn, where n = number of reversals per second of the battery, and Q the quantity of electricity set up at each. From this we see that the sensibility increases with the speed. The latter must not, however, be too great for the currents to reach their final steady values between two consecutive battery reversals. The action of the secohmmeter will perhaps be more fully understood by reference to Fig. 120. The commutators BC and GC of course rotate isochronously, and are shown as set with a "lead" given by the angle O, and represented by the distance between GC and BC on the bottom figure. If the angular advance of GC over BC = 15°, we have a "lead" of $\frac{15}{360} = \frac{1}{24}$, which means that BC makes $\frac{1}{24}$ of a revolution before the battery circuit is

closed, after the galvanometer is closed. Letting BC and BB stand for battery closed and broken respectively, and GC, GB for galvanometer closed and broken, then in the following diagram, starting at

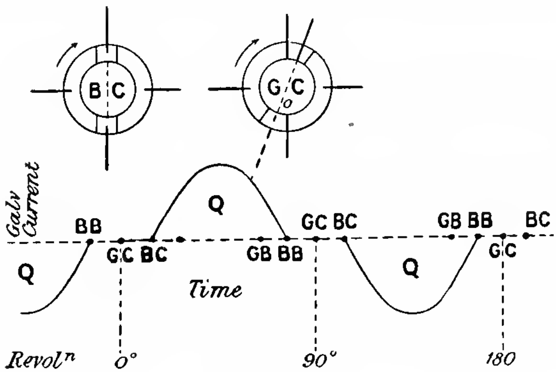


FIG. 120.

GC, we see the current induced through galvanometer rises from o at BC to a maximum, and falls to o again at BB, GB taking place $\frac{1}{24}$ of a revolution earlier. GC is next made, and the current starts again in the opposite direction at BC, and so on. Thus here only the "make" impulses go through G, and since GB happens always just before BB, and GC after BB and just before BC, these impulses are all unidirectional through G, though B is being periodically reversed. Hence the areas of all the curves are equal to one another, and represent the quantity Q of electricity set up at each reversal.

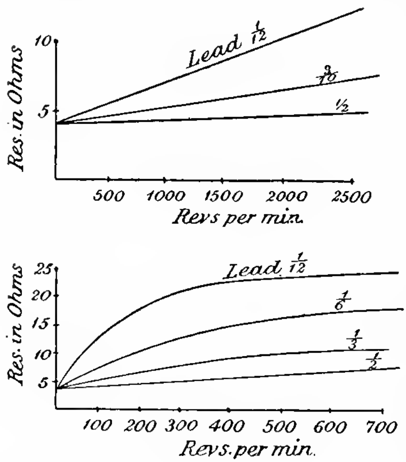


FIG. 121.

In the "zero" method of measuring a coefficient of self-induction (p. 201), the effect of an alteration of "lead" is well illustrated in Fig. 121. The top figure refers to a test on a solenoid alone, the lower one to that of the same solenoid with its iron core. The ordinates of the curves represent the added resistance necessary to preserve balance on driving the secohmmeter at the speeds given by

the abscissæ. It is noticeable that with a given coil the smaller the "lead" and greater the speed the larger (ρ) the added resistance becomes and the more sensitive the test up to a certain limit of speed.

Standards of Self and Mutual Induction.

These are constructed in two forms, viz. *fixed standards*, having only one value of induction which is invariable, and *variable standards*, the coefficient of induction of which is alterable from 0 to some definite maximum by almost any desired amount. The latter kind are undoubtedly of greater utility for experimental work. Profs. Ayrton and Perry devised the variable standard of self-induction seen in Fig. 122. It consists of a fairly large flat coil of insulated copper

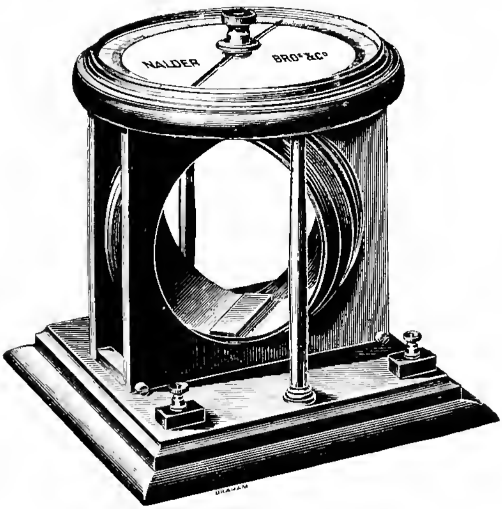


FIG. 122.

wire wound in two rather narrow channels and fixed with its plane vertical in the framework shown, which carries a fixed scale at the top. A second similar coil is capable of rotating about a vertical axis inside the fixed coil, and close to it. The plane of this moving coil is also vertical, and its upper spindle carries a pointer which moves over the scale graduated, say, in degrees one side and secohms on the other for a semicircle; the fixed and movable coils are in series, and joined to

the two terminals shown. Thus it can be seen that all values of self-induction can be obtained from 0, when the coils are in the same plane and the current flows round them in opposite directions, to the maximum, 180° further round. It is more convenient to attach each coil to its own pair of terminals, and connect those in series or parallel by a temporary wire as required. The instrument can then be used as a *variable standard of mutual induction* as well as of self-induction. The above instrument is also improved by the addition of a circular plane mirror to the scale interior, to avoid errors due to parallax in reading the indications of the pointer.

The calibration curves, for self and mutual induction, of an Ayrton and Perry standard arranged in the manner just mentioned are shown in Fig. 123, the maximum values of self and mutual inductions attainable being 0·770 and 0·155 secohm respectively. The reason why the

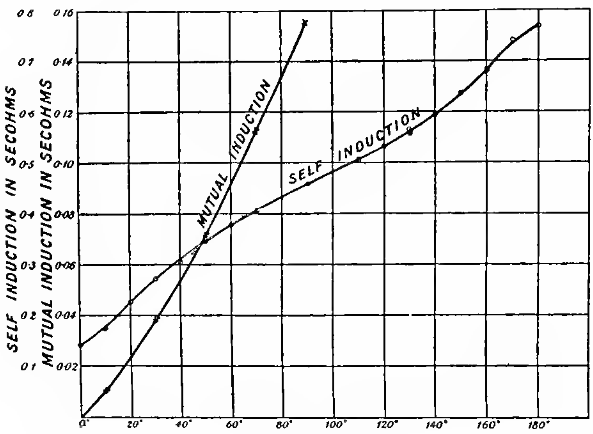


Fig. 123.

self-induction curve does not pass through the origin is because the number of turns of wire on the outer fixed coil is considerably greater than on the inner movable one; thus, after partial neutralization there is a balance of self-induction due to the fixed coil. Another form of variable standard of self-induction is made by Messrs. Nalder Bros. and Co., a general view of which is seen in Fig. 124 and a diagrammatic one in Fig. 125, which shows only the bottom disc or base of the instrument. The base or bottom disc of ebonite contains the terminal, T, a scale, S, and two flat coils, *c*, of fine insulated wire. The upper disc of ebonite also carries a terminal and two similar coils to the lower ones, and is capable of turning round the central pin. The four coils are in series and connected to the two terminals, and

when the top disc is turned, the top pair of coils rotate in a horizontal azimuth over and very close to the lower pair. When the pointer or index on the top disc points to its lowest scale reading (about 4 milli-henrys), the top coils are opposing the bottom ones.

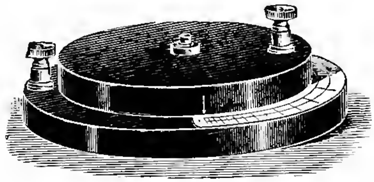


Fig. 124.

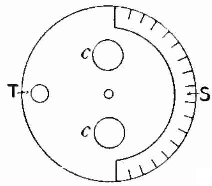


Fig. 125.

On turning through 180° the two pairs assist one another, and the self-induction is a maximum. Intermediate positions of the coils give intermediate values of L, the self-induction.

Fig. 126 shows a rather convenient form of experimental test coil or practising the measurement of self-induction on. It consists of

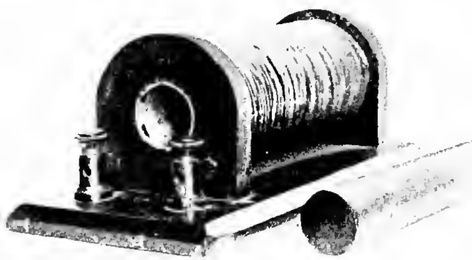


Fig. 126.

one thousand turns of insulated copper wire wound on a thin tube of insulating material with mahogany ends. The coil is connected to the two terminals, and is provided with a soft-iron core, shown by its side, which can be inserted in the coil to increase its self-induction. Four exactly similar coils to this form a very useful arrangement to experiment upon.

Galvanometers.

Of these there are two main classes, viz. those with moving coils and those with moving magnetic needles. In each class there are

numerous instances in which both light aluminium pointers and mirrors attached to the moving part are used for the purpose of indicating deflections. In the latter case, small concave mirrors are most generally used, having a focal length of about 36 to 40 inches. If, then, a parallel beam of light from a double convex lens (having a 4-inch focal length, say), and due to a luminous source in the focus, strikes the mirror, an image of a slit or wire just in front of the lens will be formed by the mirror on a scale placed some 36 inches away.

By this mirror contrivance very small deflections can easily be observed without the aid of a very long pointer, which, however light, would increase the moment of inertia of the moving system and render it sluggish. It will be obvious that a spot of light brought to a focus on a scale say 36 inches from the moving mirror, is equivalent to using a pointer 36 inches long fixed to the moving system and moving over a scale of that radius. Hence the enormous gain in sensitiveness of mirror instruments over those with pointers. It should also be remembered that in reducing scale deflections to degrees, or *vice versâ*, in reflecting instruments, the angular motion of the reflected ray is twice the angular motion of the mirror. This latter, in ordinary reflecting instruments, does not exceed about 6° or 8°.

To make this clearer, take the following numerical example. Suppose a deflection d of, say, 1 millimetre (which can very easily be detected) is obtained, with the spot of light on a scale placed at a distance L of 1 metre away from the mirror of the galvanometer, which is a very usual distance in practice. Then, since 1 mm. = 0·1 cm. and 1 metre = 100 cms., the angular motion θ° of the mirror with its attached needle $= \frac{1}{2} \cdot \frac{d}{L} \cdot \frac{180^\circ}{\pi} = \frac{1}{2} \cdot \frac{0{\cdot}1}{100} \cdot \frac{180}{3{\cdot}1416} = \mathbf{0{\cdot}0287^\circ}$, where $\tan 2\theta = \frac{d}{L}$; and, since reflecting galvanometer scales are usually not more than about 60 cms. long from end to end, the angular motion, θ°, of the above mirror for a deflection of the spot of light from the centre to one end (30 cms.) of the scale $= 0{\cdot}0287 \times 300 = \mathbf{8{\cdot}61^\circ}$. For small values of θ°, we have $\tan \theta = \frac{d}{2L}$ very nearly, which is what we have used above.

By suitably curving the scale, we can make $\tan \theta$ almost exactly proportional to d, the error introduced by assuming this being a very small fraction of 1 per cent.

With regard to the method of supporting moving coils and needles, the usual fine fibre of cocoon silk, or, better still, the quartz fibre, introduces far less friction or torsional resistance than a pivot and the best jewel, and is much cheaper than the latter. One disadvantage does, however, apply to fibre suspensions, namely, that levelling screws must always be provided to enable the moving system to be centred accurately, whereas with pivots this is not required.

The sensitiveness of galvanometers varies vastly according to the

particular type of instrument. Where the only resistance is in the wire of the coils, which is almost always copper, we may infer that a high-resistance galvanometer is also one of high sensitiveness, or "figure of merit," as it is often called. Thus, probably the most sensitive galvanometer yet made is that of the Thomson type wound to 360,000 ohms, owing to there being such an enormous number of turns of such fine insulated wire, arranged in such a way as to get a maximum of turns as close to the needles as practicable. The D'Arsonval, or moving coil galvanometer, as made at the present day, has a high figure of merit, and can be wound to a considerable resistance. For all but very high resistance work, it is one of the best and most convenient galvanometers to use.

Tangent Galvanometer.

A most convenient and useful form of galvanometer that can be employed for a number of different tests is that known as the post-office tangent galvanometer, of which Fig. 127 shows a very slightly modified type to that in general use in the postal system. A diagrammatic sketch of the instrument is shown in Fig. 128.

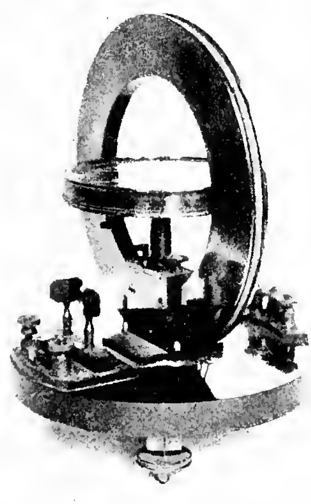


FIG. 127.

It consists of a circular flat brass bobbin, in the narrow channel of which are wound three distinct coils of insulated copper wire, each represented in Fig. 128 by only one, or nearly one, complete turn for clearness. The coil between terminals *a* and *b* consists of 3 turns only of thick wire; that between *b* and *c* of 12 of the same wire wound in the opposite sense. Hence only 12 − 3 or 9 effective turns would act on the needle if a current is sent between *a* and *c*. Consequently this arrangement by itself constitutes a low-resistance galvanometer having three distinct sensibilities (3, 9, and 12 turns), according to whether we use terminals *a* and *b*, or *a* and *c*, or *b* and *c* respectively. The third coil between brass block 2 and terminal T_2 consists of a great many turns of fine silk-covered copper wire having a resistance of exactly 320 ohms.

The resistance r consists of a coil of wire of exactly 750 ohms placed under the base of the galvanometer; a $\frac{1}{9}$ shunt having a resistance of $\frac{320}{9}$, or 35·55 ohms, is connected between. Block 3 and T_2, and the two terminals T_1, T_2 can be short-circuited by releasing a springing lever, K. Six different sensibilities are now possible. Galvanometer coil only between T_1 and T_2 (*i.e.* left plug in, right out). Galvanometer coil shunted (*i.e.* both plugs in); each of these arrangements *with* and *without* the resistance r. We are thus able now to get a high-resistance galvanometer, the highest resistance between T_1 and T_2 being $750 + 320 = 1070$ ohms, with the left-hand plug out and the right in. The action of K is to bring the deflected needle to rest quickly by short-circuiting T_1 and T_2. The galvanometer needle n, to which is attached a long light pointer, is pivoted in a scale-box at the centre of the coil. One half of the circular scale is degree-divided, and the other is usually graduated in numbers *proportional*, but not equal, to the tangents of those degrees, and is commonly called the "tangent scale."

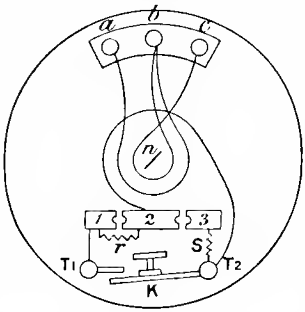


FIG. 128.

When possible, it is desirable to obtain deflections on a tangent galvanometer in the vicinity of 45°, as the sensitiveness of this type of galvanometer is then at its maximum, owing to a given variation in the current producing greater effect in that region of the scale than in any other. The clamp (seen underneath the scale-box in front) is for raising the needle and pointer off the pivot when the galvanometer is moved about. The $\frac{1}{9}$ shunt, when across the galvanometer coil, reduces the resistance between block 2 and T_2 from 320 to $\frac{320 \times 35}{320 + 35}$, *i.e.* to **31·3** ohms approximately, and allows one-tenth of the main current to flow through the galvanometer coil, and nine-tenths through the shunt. In other words, the total current will be ten times that indicated by the deflection of the needle. It may perhaps be superfluous to remark here that the tangents of the angles of deflection are directly proportional, though not equal, to the currents producing them respectively. To obtain a relation of equality, the former must be multiplied by a "constant" K for that particular arrangement of coils in use, and we thus get *current* = K tan $\theta°$, where $\theta°$ = the angular deflection of the needle or pointer.

It is often the case that students, at least many of those commencing the subject, jump to the conclusion that any galvanometer they may see must be a tangent one. If, however, he will only study the two following conditions, which must hold in any galvanometer for it to be a tangent one, he will not go wrong afterwards in his surmises;

(1) the controlling force must be constant in magnitude and direction; (2) the deflecting force must always act in the same direction and at right angles to the controlling force. Whenever these two conditions hold, the current will be proportional to tan $\theta°$.

Helmholtz Tangent Galvanometer.

Fig. 129 represents a good form of standard tangent galvanometer, known as the Helmholtz pattern. It consists of two exactly similar ring-shaped coils, one on either side of the magnetic needle. The coils, which are of large diameter, are placed symmetrically with respect to the needle, in such a position that the distance of the centre of each from that of the needle, along their common axis, passing through the centre of the needle, is half the radius of either coil. The turns of wire composing each coil are so wound that they each form part of a coil which, if continued, would have its apex at the centre of the needle. Thus the solid angle subtended by each coil at the centre of the needle is the same, which practically eliminates the error due to the needle not being extremely short compared with the coil diameter; and also, owing to the coils not being very broad, the mean distance of each turn from the needle's centre is practically the same. The elementary theory of the tangent galvanometer will be found on p. 115.

FIG. 129.

The short magnetic needle has a light aluminium pointer fixed to it, and is suspended by a fine silk fibre from the top of a tube carried by the scale box. In some cases, however, a small agate centre is let into the centre of the needle, which is then pivoted inside the scale box on a fine-pointed pivot. In almost all cases a clamp, actuated by a milled-headed nut under the scale box, can be raised so as to lift the needle and pointer when the galvanometer is carried about, thus avoiding damage to the pivot. As seen, the base rests on three levelling screws, and carries two pairs of terminals, one pair to each coil, which must be so connected that the current flows round both coils in the same direction. The two large terminals seen are joined to the two thick copper turns, wound one on each ring and joined in series. These are intended for heavy currents when necessary.

An alterable form of tangent galvanometer is shown in Fig. 130, and is suitable for determining the effect of a current flowing in a circular coil of a variable number of turns and different diameters on a magnetic needle placed in different positions in the plane of the coil and along its axis. The arrangement consists of a base-board, to one end of which is fixed a standard carrying scale box and magnetometer needle. Two ring-shaped bobbins are wound with coils of insulated copper wire. The larger one, fixed to a base capable of sliding either parallel or perpendicular to its own plane, is wound with two distinct coils of 10 or 30 turns, respectively connected to two pairs of terminals on its base. Thus 10, 20, 30, or 40 effective turns can be obtained by using the coils singly and in helping or opposing series. The smaller bobbin is wound with only one coil of 10 turns, connected to the two terminals on its base, which, when placed on the base of the larger coil, brings its centre to coincide with that of the needle. The main base-board is provided with scales and grooves at right angles, by means of which the coils can be guided to any distance from the needle in either of two directions at right angles, when the pins are inserted in the base of the large coil. When in use, the small coil, whose diameter is half that of the large one, is used as shown in the figure, on the base of the larger, the two being kept concentric with one another by making the index marks on their bases coincide.

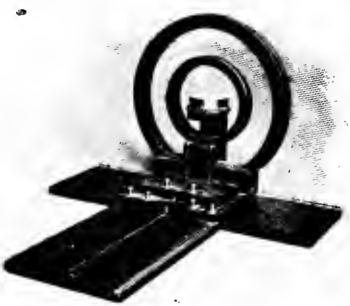


FIG. 130.

Portable Galvanometer.

A portable form of galvanometer, which can be used with a Wheatstone bridge when a very great degree of sensitiveness is not required, is shown in Fig. 131. It is convenient for use when a reflecting instrument cannot be employed or obtained in just the position desired. It consists of two flat coils, wound with fine silk-covered copper wire, of the form shown in Fig. 132. There is a large narrow opening in the centres of each of the coils, and they are fixed so that their magnetic axes

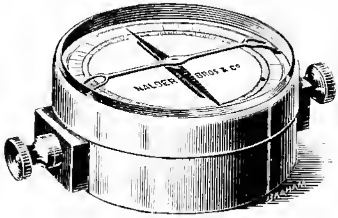


FIG. 131.

coincide, the ends, such as E, being a short distance apart to allow of the free rotation of a delicate vertical spindle passing down between them and running in jewelled centres. This spindle carries a long light pointer at the top and a light magnetic needle lower down, which can just rotate freely inside the rectangular openings in the coils. The coils are so connected together in series between the two terminals that their unlike poles are adjacent; thus, when the axis of the little needle is at right angles to their magnetic axis and a current is sent through the coils, the poles of the needle are attracted by the dissimilar ones at the outside ends of the coils, the pointer at the same time indicating the deflection of the needle inside on the degree-divided scale above. The type of galvanometer shown is wound usually to about 1000 ohms resistance, and may have a figure of merit of something like $\frac{1}{1000000}$ ampere per 1° deflection on the scale. It should be noted that before using the instrument it should be turned round so that the pointer points to zero on the scale, for then the magnetic axes of needle and coils will be perpendicular.

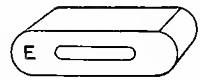


FIG. 132.

Fig. 133 shows another convenient form of galvanometer, devised by Dr. R. M. Walmsley and Mr. T. Mather, and having a considerable degree of sensitiveness. The construction and action will be more

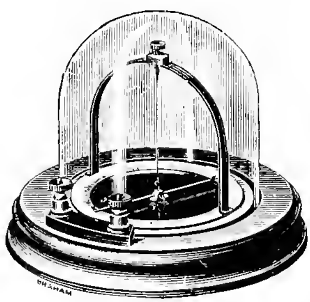


FIG. 133.

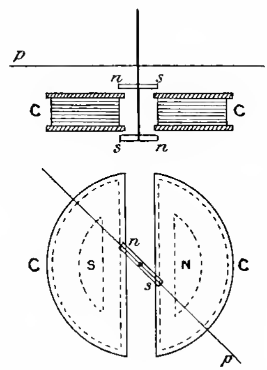


FIG. 134.

readily understood by reference to Fig. 134 as well. It consists of two semicircular coils, C, fixed inside a well turned out of a base, which is supported on three levelling screws (Fig. 133). The coils are connected in series, so that the upper ends are a north and south pole respectively, and are joined to the two terminals shown. Suspended symmetrically with regard to the coils, by a fine silk fibre, is a light magnetic needle, *ns*, which can just rotate freely above them, its length being just

a little greater than the distance between the parallel edges of C, C. A pointer, *p*, is rigidly attached to *ns* by the thin aluminium wire fixed to both. For no current passing *ns* lies parallel to the straight edges of C, C, but on the passage of a current, unlike poles attract, causing a deflection, which in this type of galvanometer is directly proportional to the current up to about 60°, owing to the needle, when deflected, moving into a more powerful part of the deflecting field. The suspending arrangements are shown in Fig. 133, which, together with the scale and pointer, etc., are under a glass shade to protect the needle from draughts. As seen, a circular plane mirror is used inside the degree-divided scale for the purpose of avoiding errors due to parallax.

Astatic Combination.

When a still more sensitive instrument of this form is wanted, an "astatic combination" of needles can be used instead of one needle only. This consists of two nearly similar highly magnetized steel needles rigidly fixed to a thin aluminium wire, with their like poles pointing in opposite directions. In the above galvanometer it will be seen that the sensitiveness is more than doubled by such an addition, because while the earth's controlling force on the system is much diminished the movement of the deflecting force on the needles is doubled, since they are at opposite sides of C, C. In practice, perfect astaticism is undesirable, owing to the earth's magnetism being unable to exert any controlling force on the system, causing the needles to wander or rest in any position. Hence a small directive force may preferably be given to the system by making one of the needles slightly stronger than the other. It may also be done by placing a controlling magnet a trifle nearer one needle than the other, so as to act more strongly on that one.

Sine Galvanometer.

The preceding Walmsley-Mather galvanometer can be used as, what is commonly called, a sine galvanometer, by placing the instrument as it stands on another baseboard capable of turning in a horizontal azimuth about a centre which is in a vertical line with that of the needle. In any galvanometer the current flowing through it will be directly proportional to the sine of the angle of deflection, providing—

(1) The controlling force is constant in magnitude and direction.

(2) The deflecting force, although variable in its direction in space, is fixed in direction relatively to the deflected needle.

In other words, any galvanometer which is controlled by a distant magnet or the earth's field, and can be rotated round an axis passing through the centre of the needle, can be used as a sine galvanometer.

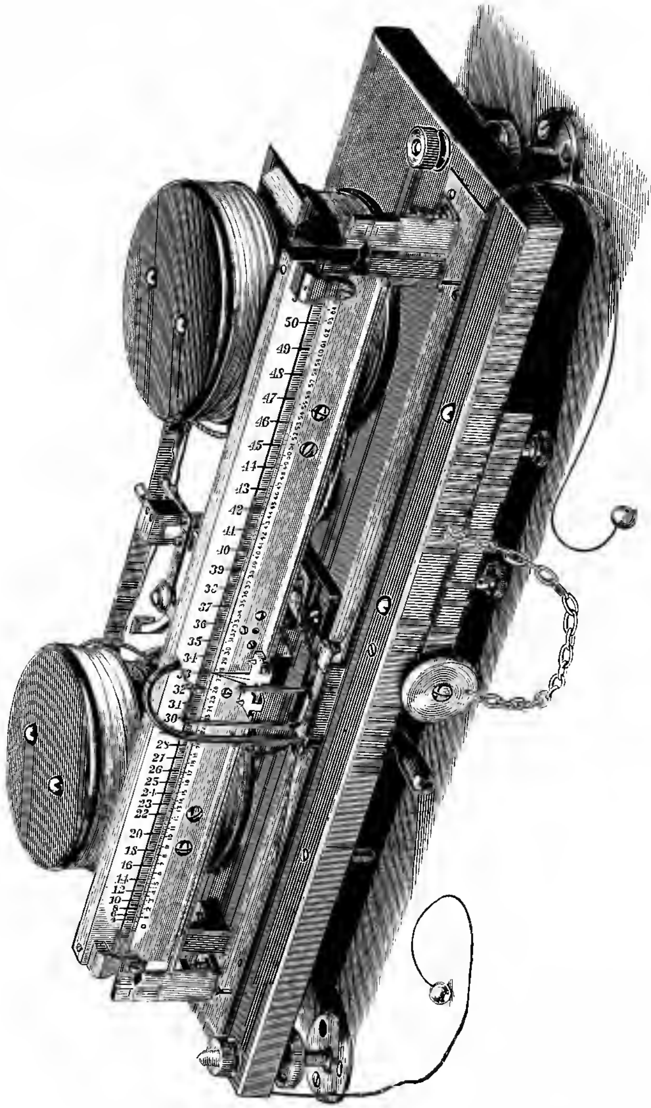


Fig. 135.

Standard Direct-reading Electric Balance.

Fig. 135 shows the general appearance (with glass cover removed) and Fig. 136 a part symbolical elevation of Lord Kelvin's centi-ampere balance.

1. The instrument is founded on the principle of action of the mutual forces, discovered by Ampere, between movable and fixed

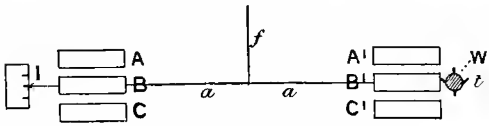


FIG. 136.

portions of an electric circuit. The shape chosen for the mutually influencing portions is circular, and each such part will be called for brevity an ampere ring, whether it consists of only one turn or of any number of turns of the conductor.

2. In this balance, each movable ring, B and B′, is actuated by two fixed rings, AC and A′C′—all three approximately horizontal. There are two such groups of three rings—two movable rings attached to the two ends of a horizontal balance arm pulled, one of them up and the other down, by a pair of fixed rings in its neighbourhood. The current is in opposite directions through the two movable rings to practically annul disturbance due to horizontal components of terrestrial or local magnetic forces.

3. The balance arm is supported by two trunnions, each hung by an elastic ligament of fine wire, *f*, through which the current passes into and out of the circuit of the movable rings.

4. The mid-range position of each movable ring is in the horizontal plane nearly midway between the two fixed rings which act on it.

5. The current goes in opposite directions through the two fixed rings, so that the movable ring is attracted by one of the fixed rings and repelled by the other. The position of the movable ring, equidistant from the two fixed rings, is a position of minimum force, and the sighted position, for the sake of stability, is above it at one end of the beam and below it at the other, in each case being nearer to the repelling than to the attracting ring by such an amount as to give about $\frac{2}{10}$ per cent. more than the minimum force.

6. The balancing is performed by means of a weight which slides on an approximately horizontal graduated arm attached to the balance; and there is a trough, *t*, fixed on the right-hand end of the balance into which a proper counterpoise weight, W, is placed, according to the particular one of the sliding weights in use at any time (sect 9 below).

For the fine adjustment of the zero a small metal flag is provided, as in an ordinary chemical balance. This flag is actuated by a fork having a handle below the case outside, as shown in the drawing, Fig. 135. *To set the zero* the left-hand weight is placed with its pointer at the zero of the scale, and the flag is turned to one side or the other until it is found that, with no current going through the rings, the balance rests in its sighted position.

7. *To measure a current* the weight is slipped along the scale until the balance rests in its sighted position. The strength of the current is then read off approximately on the fixed scale (called the inspectional scale), with aid of the finely divided scale for more minute accuracy, according to the explanations given in sect. 11 below. Each number on the inspectional scale is twice the square root of the corresponding number on the fine scale of equal divisions.

8. The slipping of the weight into its proper position is performed by means of a self-releasing pendant, hanging from a hook carried by a sliding platform, which is pulled in the two directions by two silk threads passing through holes to the outside of the glass case.

9. Four pairs of weights (sliding and counterpoise), of which *the sledge or carriage and its counterpoise constitute the first pair*, are supplied with the instrument. These weights are adjusted in the ratios of 1 : 4 : 16 : 64, so that each pair gives a round number of amperes, or half-amperes, or quarter-amperes, or of decimal subdivisions or multiples of these magnitudes of current, on the inspectional scale.

10. The useful range of each instrument is from 1 to 100 of the smallest current for which its sensibility suffices. The range of this instrument is from 1 to 100 centi-amperes. The following table shows the value per division of the inspectional scale corresponding to each of the four pairs of weights :—

	Centi-amperes per division.
First Pair of Weights	0·25
Second ,, ...	0·50
Third ,, ...	1·0
Fourth ,, ...	2·0

11. The fixed inspectional scale shows, approximately enough for most purposes, the strength of the current ; the notches in the top of the aluminium scale show the precise position of the weight corresponding to each of the numbered divisions on the fixed scale, which practically annuls error of parallax due to the position of the eye. When the pointer is not exactly below one of the notches corresponding to integral divisions of the inspectional scale, the proportion of the space on each side to the space between two divisions may be estimated inspectionally with accuracy enough for almost all practical purposes. Thus we may readily read off 34·2 or 34·7 by estimation

with little chance of being wrong by 1 in the decimal place. But when the utmost accuracy is required, the reading on the fine scale of equal divisions must be taken, and the strength of current calculated by aid of the table of doubled square roots given at the end of this book. Thus, for example, if the reading is 292, we find 34·18, or say 34·2, as the true scale reading for strength of current; or, again, if the balancing position of the pointer be 301 on the fine scale, we find 34·70 as the true reading of the inspectional scale.

12. The centi-ampere balance, with a thermometer to test the temperature of its ampere rings, and with platinoid resistances up to 1600 ohms, serves to measure potentials of from 10 volts to 400 volts.

CONSTANT OF THE CENTI-AMPERE BALANCE WHEN USED AS A VOLTMETER.

Weight used.	Resistance in circuit.[1]	Volts per division of fixed scale.
First Pair of Weights ...	400	1.0
,, ,, ...	800	2·0
,, ,, ...	1200	3·0
,, ,, ...	1600	4·0

If the second pair of weights is used, the constants will be double of those noted above.

13. **Instructions for the Adjustment of the Standard Balances.**—The instrument should be levelled in accordance with the indications of the attached spirit-level, by means of the levelling screws on which the sole-plate of the instrument stands.

14. In this centi-ampere balance, the beam can be lifted off its supporting ligaments by turning a handle attached to a shaft passing under the sole-plate of the instrument. This shaft carries an eccentric, on the edge of which rests the lower end of a vertical rod, which is fixed at its upper end to a tripod lifter. When the instrument is to be packed for carriage, or when it is to be removed by hand from place to place, the lifter should be raised; but when it is fixed up for regular use, it is advisable to keep the beam always hanging on the ligaments.

15. The carriage is fitted with an index to point to the movable scale, and is intended to remain always on the rail. One or other of the weights is to be placed on the carriage in it in such a way that the small hole and slot in the weight pass over the conical pins. The weights are moved by means of a slider, which slides on a rail fixed to the sole-plate of the instrument, and carries a pendant with a vertical arm intended to pass up through the rectangular recess in the front of the weight and carriage. The slider and weight are shown in position in the figures. The slider is moved by silk cords, which pass out at the ends of the glass case. When the cords are not being

[1] Including resistance of the instrument, which is about 50 ohms.

pulled for shifting the weight, their ends should be left free so that the pendant may hang clear of the weight. When a weight is to be placed on or removed from the carriage, the slider should be drawn forward at the top until it is clear of the weight, and then pushed to one side until the weight is adjusted, when it may be replaced in position in a similar manner.

16. Cylindrical counterpoise weights with a cross-bar passed through them are supplied for the purpose of balancing the sliding weights when they are placed at the zero of the scale. The sliding weight should be placed so that the index of the carriage points to the zero of the scale, and the proper counterpoise weight should be placed in the trough, fixed to the right-hand end of the beam, with its cross-bar passing through the hole in the bottom of the trough. The flag which is attached to the cross-trunnion of the beam should then be turned by means of the handle projecting from under the sole-plate, until the index on the end of the movable scale points to the middle one of the five black lines on the fixed scale opposite to it. *Care must be taken when making this adjustment that the fork which moves the flag is not left in contact with it, as this would impede the free swing of the beam.* The fork should be turned back a little after each adjustment of the flag, and, when the flag is being adjusted, it is better to watch the flag itself, and make successive small adjustments until the beam stands at zero, than to make successive trials by pushing round the handle while watching the position of the index.

If the ligament has stretched since the instrument was standardized, the index at one end of the movable scale will be found to be below the middle line on its vertical scale, when the index at the other end is correctly pointing to the zero position. The error so introduced would be a small one, but it may be easily put right by slightly loosening the screws fixing the pillared frame, which supports the movable beam, to the base plate, and raising it by slipping one or two thicknesses of paper below it until the indices simultaneously point to their zero position.

17. A lens is supplied with each instrument for facilitating accurate observation, either when reading the position of the weight or when adjusting the zero.

18. The vibrations of the beam may be checked so as to facilitate reading by bringing the pendant, which moves the weight, lightly into contact with it, in such a way as to give a little friction without moving the weights.

19. In using the centi-ampere balance as a voltmeter when great accuracy is required, care must be taken that the effect of change of temperature in changing the resistance of the coils of the instrument, and of the external resistance coils, is allowed for; and in this use of the instrument it is advisable to employ currents such as can be measured by the lightest weight on the beam. When the instrument is to be used as a voltmeter, four resistances are provided, three of which are each 400 ohms, and the fourth is less than 400 ohms by the

resistance of the coils of the instrument at a certain specified temperature. The smallest resistance is intended to be included by itself in the circuit when the lowest potentials are being measured, and in series with one or more of the others when the potential is so high as to give a stronger current than can be measured with the lightest weight on the beam. The correction for temperature is, for the copper coils of the balance, about 0·38 per cent. per degree Centigrade, and for the platinoid resistances, about 0·024 per cent. per degree Centigrade.

Adjustable Magneto-static Current Meter.

1. The magneto-static current meter (Fig. 137) consists essentially of a small steel magnet or system of magnets suspended in the centre of a uniform field of force due to two coils, each having one or more turns of copper ribbon or wire, and also under the directive influence of two systems of powerful steel magnets.

FIG. 137.

2. The suspended system of magnets is attached to one end of a vertical shaft passing down centrally through an opening in the sole-plate of the instrument from an indicating needle, which is supported by a jewelled cap resting upon an iridium point.

3. The two systems of directive magnets are circular in form, and each ring is composed of two semicircular magnets placed in a brass cylindrical frame with their similar poles together. Each system is securely fixed to a circular brass frame, which fits on to the cylindrical case of the instrument in such a manner that the systems are capable of being turned round, together or separately, as explained below.

4. The instrument has a "tangent scale," which is adjusted in its position before the instrument is sent out, so that the needle indicates equal differences of readings for equal differences of current. The scale consists of a hundred divisions, and for most purposes it is convenient to set the field magnets in such a position that the needle points to 0, and to use the scale from that point upwards towards 100. Sometimes, however, it may be found convenient to measure currents, whose direction is being occasionally reversed, without being at the trouble of reversing the electrodes in the contact clip ; in that case the

zero should be set to the division 50 at the middle of the scale, and readings taken on each side of it. It must be remembered that when the point taken as zero is changed, the *constant*, by which the indications of the instrument have to be multiplied to give the current in amperes, is changed in proportion to the cosine of the angle between the zero point and the middle of the scale ; and as this angle is 60°, the *constant* with the zero at 50 on the scale is exactly double the *constant* with the zero at 0 on the scale.

5. The instrument is provided with a "lifter," which serves to raise the needle off the iridium point when it is being moved about from place to place. This lifter is in the form of a ring placed below the needle, and may be raised or lowered by turning the handle attached to an eccentric passing through the side of the instrument on a level with the scale. It also serves as a checker, by bringing it lightly into contact with the pointer, so as to stop its vibrations.

6. The instrument has an advantage, important for some practical purposes, of being available as an accurate direct-reading current meter, through a continuous range of from 1 to 100 times its smallest current, which may be anything from half a milliampere to 4 amps., according to the number of turns in the coils supplied with the instrument. It is not, however, available as an alternate current instrument, and it must be remembered that the magnetism of the steel directing magnet does not remain *absolutely* constant. With good quality of steel, a proper preliminary *ageing* of the magnet (by heating it several times in boiling water and cooling it again, and subjecting it to somewhat varied rough usage) brings it to a condition in which its magnetism is found to remain exceedingly nearly constant month after month and year after year. Still, it should never be relied upon as *absolutely* constant, and for accurate laboratory work it is therefore necessary to occasionally standardize it.

7. Another advantage which the instrument has is that, when a standard instrument is available, its constant is capable of being varied to any desired value down to one-tenth of that which it has with its directive magnets in their strongest position. Thus if the constant should be 3 amps. per division of the scale, with the similar poles of the magnets coinciding, it may be adjusted to any value down to 0·3 amp. per division.

8. **Instructions for Use of the Magneto-static Current Meters.**—The instrument should be levelled, in accordance with the attached spirit-level, by means of the levelling screws.

9. *To adjust the Pointer to Zero.*—(*a*) Loosen the two lower milled-headed screws clamping the magnet frame, and turn the frame round till the pointer stands at zero. (*b*) Reclamp the frame by tightening the two screws.

10. *Adjustment of the Scale.*—The scale, as stated above (sect. 4), is firmly clamped in its place before sending the instrument out, and this position is marked by two lines on the outside of the case, one horizontal and the other vertical, just below the 0 of the

scale. The horizontal line is engraved below the movable top of the instrument, and the vertical one on the side of the case. Should the top of the instrument have been inadvertently moved, and the scale thus put out of adjustment, it may be set right by slightly loosening the two slotted screws and turning the top round till the extremities of the two lines coincide.

11. If the needle should by accident be slightly bent,[1] and so render a new adjustment of the scale necessary, this may readily be made in the following manner: Set the zero, by the field magnets, to the division 50 at the middle of the scale, then join the instrument in series with another current instrument of convenient form, and pass a current through both sufficient to give a deflection of about 40 divisions on the magneto-static instrument; reverse the current on the magneto-static instrument only, and set the scale so that equal deflections, read in divisions, are given on each side of the zero for equal currents, as indicated on the auxiliary instrument. The zero must, of course, be reset by the magnets every time the scale is moved. When the scale has been adjusted to this position, firmly clamp the top of the instrument by the two slotted screws, and again mark the position of the horizontal line on the outside of the case.

12. *Adjustment of Constant.*—The constant may be quickly varied as follows: Join the instrument in series with any reliable current instrument of known accuracy, such as the deci-ampere balance, and pass a convenient current through both instruments, observing the readings. Break the current, loosen the two upper pairs of milled-headed screws, and turn the top system of magnets relatively to the lower, so that the similar poles of the two systems are brought closer together or moved further apart, according as it is desired to make the instrument respectively less or more sensitive. Reclamp the screws and adjust the zero as described in sect. 10. Again make the current, and note the reading on the two instruments. The desired reading on the magneto-static may be obtained quickly after one or two approximations, care being always taken to readjust the zero after each movement of the top magnets.

13. When convenient it is always best to standardize the instrument in the place where it is to be used; but when it is intended to move it from place to place, it should be standardized in such a position that when the needle is pointing to zero it is in a direction approximately east and west.

Reflecting Galvanometers.

A simple form of reflecting galvanometer is shown in Fig. 138, and consists of two coils of fine silk-covered copper wire wound on a wooden

[1] If it is bent so largely as to be perceptible to the eye, it ought to be straightened by hand as nearly as may be.

bobbin, having a small hole through its centre and one end hollowed out trumpet-shaped. The bobbin is fixed to two brass standards on a base supported on three levelling screws. The ends of each coil are connected to the right and left hand pairs of terminals respectively on the base. A wooden plug terminating in a knob at one end is inserted in the hole through the coil, and extends just as far as the centre of the coil. A pin is inserted in the edge of the other end of this plug, and the mirror, with its one or more needles fixed to its back, is suspended from it by a short fine silk fibre. Two small pieces of fine watch-spring, magnetized and fixed to the back of the mirror with like poles together, form a very good needle. The inner turns of the coil are brought as close to the needles as possible, to increase the sensitiveness of the galvanometer, and the end of the bobbin is trumpet-shaped to allow of a wider angle of reflection of the mirror. A controlling magnet, resting on a pin attached to a clamp, can be clamped in any position up or down the slot in the front standard, by the milled nut shown. Thus the magnitude of the control can be altered at will, and the magnet turned on the pin to bring the spot of light reflected from the mirror to zero on the scale. The "figure of merit" of this form of galvanometer is very high considering its simple construction, and its resistance can be made anything desired.

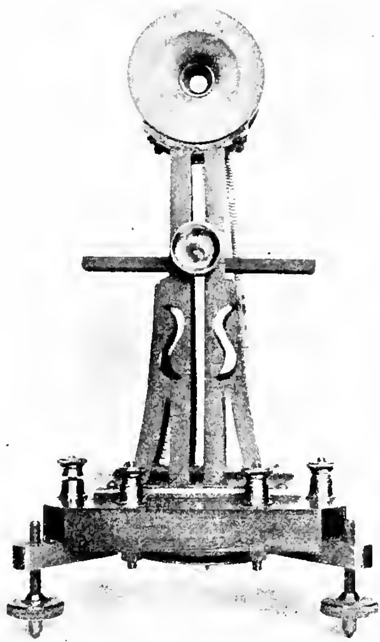


FIG. 138.

With it possessing two coils, these can be used either singly or in series helping or opposing one another, or in parallel, thereby giving an extended range both in resistance and sensitiveness.

Differential Galvanometer.

The instrument just described is well adapted for forming a differential galvanometer, the only difference being the final adjustments of the number of turns and resistance of its two coils. Suppose the coils were coupled up to a battery as in Fig. 139. Then, if the *resistance* of coil ABC equals that of ADB, equal currents will flow through them, but in opposite directions; hence, if in addition the *magnetic effects* of the two coils are equal, no deflection of the needle will take place whatever E.M.F. is placed across the points AB. It will now be at once obvious that other things remaining the same, any added unknown resistance inserted in the circuit of coil ACB will cause a deflection, and that this can be nullified by inserting an exactly equal resistance to circuit ADB.

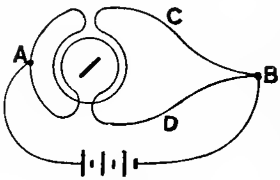


FIG. 139.

Thus an instrument possessing two coils acting on the same magnetic needle with *equal magnetic effect*, and having *equal resistances*, can be used for comparing two resistances, or of measuring that of an unknown coil in terms of a standard, and is termed a "differential galvanometer." The last-named galvanometer can be made differential as follows: Wind both coils on to the bobbin as close together as possible, and off two reels of exactly similar wire; then test and cut, so that each coil has the same resistance, though not necessarily the same length. Send now a current through the two coils in "opposing series," and unwind just so many turns or fraction of a turn as will result in no deflection being obtained; then, without cutting this off, coil it up in the base of the instrument. In this way equal magnetic effect and equal resistance is obtained, and any slight adjustment can afterwards be made by means of the levelling screws, so as to suitably tilt the needle in the right direction to make the galvanometer absolutely differential.

Astatic Ballistic Galvanometer.

The reflecting galvanometer shown in Fig. 140 is one devised by Mr. T. Mather, for the purpose, primarily, of comparing quantities of electricity, though it can of course be used for all ordinary experimental work in which a galvanometer with a very considerable degree of sensitiveness is required. It can be made either "astatic," "ballistic," or both, and consists of two flat bobbins with very thin flanges on one side of each, wound with fine silk-covered copper wire, and connected to their respective pairs of terminals on the base supported

on three levelling screws. The centres of each coil at the thin flange end are hollowed out trumpet-shaped as much as possible, and these two ends of the coils brought close together and fixed by ebonite angle brackets on the outer ends of the coils. On an aluminium wire are fixed three sets of little magnetic needles, together with the small concave mirror at the top, as seen. The middle set can just move freely in the hollowed coil centres, and forms an astatic combination with the remaining two sets close to the top and bottom of the coils.

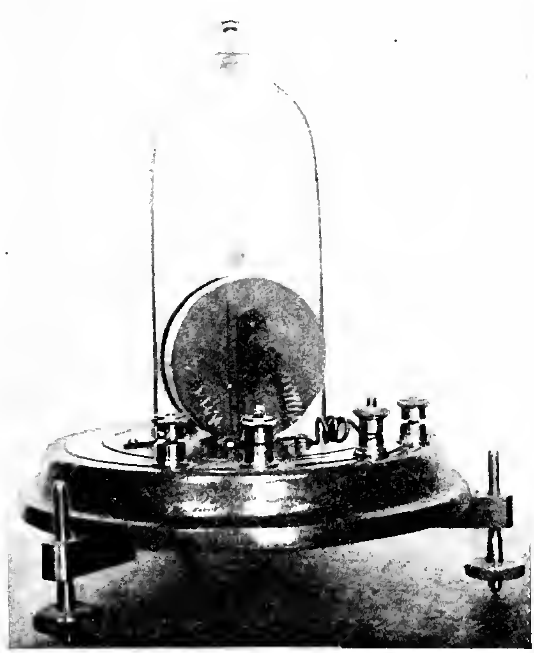


FIG. 140.

This magnet system is suspended by a fine silk fibre from a milled-headed pin, which can move up or down in the top of a standard of the shape shown, and a glass shade protects these parts from air draughts. It will now be seen that, when the coils are joined up so that opposite polarities are adjacent, the two outside sets of magnets are affected by the outside convolutions of the two coils, and will both be deflected in the same direction as the middle set. Thus the deflecting couple will be that due to the sum of the couples acting

on the three sets of needles individually. To obtain greater damping due to air resistance, a light mica strip may be used instead of the aluminium. To make the galvanometer ballistic use the wire, and in addition weight the needle system so as to increase its periodic time of oscillation.

Thomson High-resistance Astatic Galvanometer.

Fig. 141 represents a delicate and highly sensitive galvanometer, commonly known as the Thomson high-resistance astatic galvanometer, which is capable of detecting exceedingly small currents, such as might flow through the insulation of a cable. It consists of four coils of very fine silk-covered copper wire, mounted in four ebonite coil boxes, and arranged in two pairs with the magnetic axes of each pair in the same straight line. Each coil is slightly hollowed out at the centre, and when these hollows of a pair of coils are close together and just opposite, sufficient room is obtained for the motion of the small suspended magnetic needles inside.

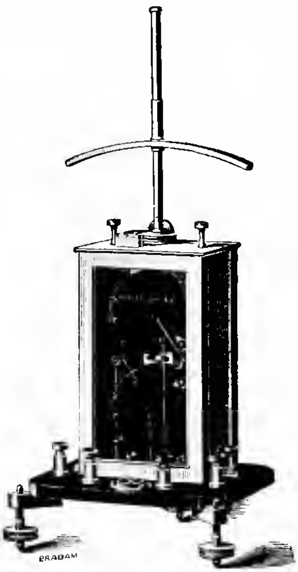


Fig. 141.

The two pairs of coil boxes are supported one above the other by two corrugated ebonite pillars at the sides, and the two front boxes are hinged and secured by means of spring buttons to the fixed boxes at the back. Suspended by a fine silk fibre, attached to a point over the top of the upper coils, are two sets of magnetic needles, rigidly fixed to a light aluminium wire, and arranged astatically. The needles of each set have their like poles facing one way and close together, so that each set can move freely in the hollow opening between the centres of each pair of coils. The mirror, together with a light mica vane, is fixed to the aluminium wire midway between the coils, as seen. This is a much better plan than that of attaching the mirror to one set of magnetic needles, which necessitates having a large opening in the front coil of that pair, and prevents the turns of wire being brought so close to the needles, which is of the utmost importance. A rectangular shade or cover protects the interior from draughts, and at the same time supports on its top a vertical rod, which can be rotated by a worm and wheel, and carries the curved controlling magnet. The ebonite base, which carries two pairs of terminals connected to the

two pairs of coils, and also a spirit-level, is supported on three levelling screws, to enable it to be carefully levelled. These galvanometers are wound to from 5000 to 100,000 ohms or more.

High-resistance Astatic Ballistic Galvanometer.

Fig. 142 shows a very sensitive form of high-resistance ballistic galvanometer, which will be found very useful for measuring quantities of electricity, for capacity, and other accurate work. The principle and action is identically the same as that of the Mather galvanometer

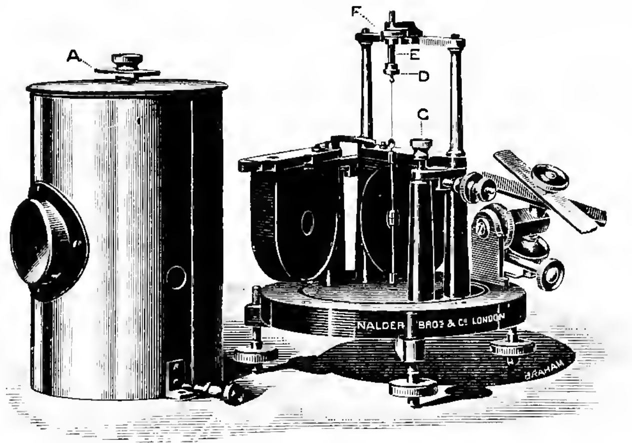


FIG. 142.

described on p. 283. The figure practically shows all the details of construction, but there are nevertheless some characteristic points about it that may be noted. The magnetic needles are bell-shaped, and those at the top and bottom of the coils together form an astatic combination with those at the centre of the two coils. To obtain complete accessibility to the magnet system, the front coil is hinged, as shown. By making all except the brass parts of ebonite very high insulation resistance is obtained, which is of the utmost value. The instrument, which can be wound to any resistance, usually totals about 10,000 ohms. Two controlling magnets are provided, one on the top of the brass case, and one capable of sliding along a rod at the back of the instrument ; both can rotate on their spindles.

The two terminals, instead of being on the base, which would lower the insulation resistance, are attached to two brass rods, which pass through much larger holes on each side of the case, and screw into

brass blocks carried by ebonite pillars, and to which the coil ends are brought. Ebonite plugs slide along the terminal rods, and fit the case closely to prevent the entrance of dust. When, however, the galvanometer is in use, these must be pulled out to increase the insulation resistance of the instrument.

Reflecting D'Arsonval Galvanometer.

Fig. 143 shows a very convenient form of reflecting D'Arsonval galvanometer suitable for ballistic purposes. It consists of a powerful permanent steel magnet of the form shown, let into and supported on the ebonite base, which rests on three levelling screws. A standard at the back, which is fixed to the base, holds the magnet ends firmly in position, and carries a light brass pillar, to the head of which the suspension is soldered. A rectangular coil of fine wire, wound on a light

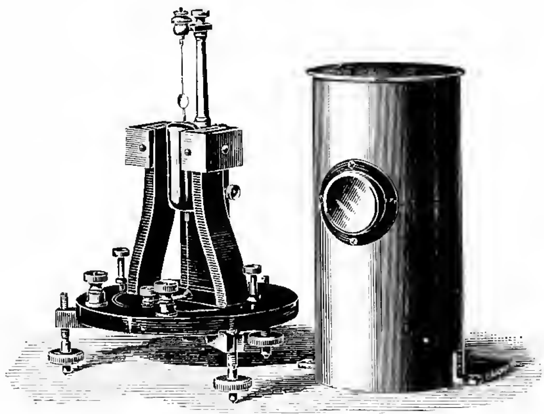


FIG. 143.

non-metallic frame, carries a small concave mirror, and one end is attached to the phosphor-bronze strip suspension, while the other is attached to a similar strip underneath, the lower end of which is held down by a spring strip on the base. The coil encircles a cylindrical piece of good soft iron, often termed the armature, fixed to the standard, and which concentrates the flow of lines of force through the coil from pole to pole, thus making the galvanometer more sensitive. One terminal on the base is connected with the top suspension, and the other with the bottom. Thus a source of E.M.F. placed across the terminals will send a current round the coil, which will be deflected in such a direction that its own lines of force tend to coincide in direction

with those of the fixed field. The periodic time of oscillation can be made almost anything within reason, and of course depends on the moment of inertia of the coil. This type of galvanometer can, on the contrary, be made very aperiodic by winding the coil on a light aluminium or thin copper frame. Then, when it begins to move, eddy or Foucault currents are induced in the frame, because of it cutting the lines of force due to the fixed field, which damp its oscillations. These galvanometers can be made very sensitive, and the great advantage of them is that not only are the scale deflections directly proportional to the current, but such deflections can be immediately damped down to zero by short circuiting the coil with a suitable key.

Non-reflecting D'Arsonval Galvanometer.

A non-reflecting form of D'Arsonval is that devised by Carpentier, and shown in Fig. 144. It is a very sensitive and extremely useful type in that it is a zero instrument, and the currents are observed without the aid of a mirror, etc. It consists of a rectangular coil of very fine wire wound on a light aluminium frame to make the instrument aperiodic. This encircles a soft iron armature, and is suspended between the poles of a powerful laminated steel magnet by means of a fine spiral phosphor-bronze spring. The upper end of this spring is attached to a torsion head, carrying a pointer, which moves over a uniform scale supported by the same standards as the torsion head and magnets. A second spring is fitted underneath the coil, but in the reverse sense to the top one, in the same way as the strip in the previous D'Arsonval.

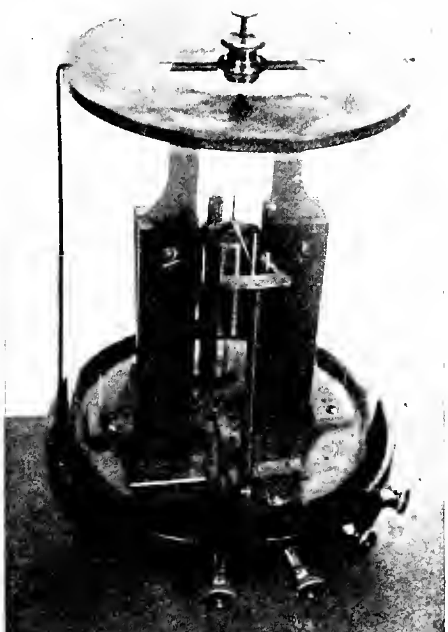


Fig. 144.

The two large terminals shown are connected to the coil through the two springs. The smaller pair of terminals are an addition to the galvanometer for a special purpose, and need not be further mentioned. A light aluminium index arm is rigidly fixed to the top of the coil, and moves over a short index scale carried on the top of the pillar shown in front, and constitutes the zero arrangement. The topmost milled head when turned raises or lowers the coil, and turns with the torsion head just under it. The galvanometer is protected by a glass shade from draughts, and rests on three legs. The moving coil can be wound to have either a high or low resistance as required.

Weston Voltmeter.

Fig. 145 shows a Weston voltmeter giving a full-scale deflection for 1 volt, and having a resistance of 100 ohms. The seven terminals on the right are connected to anti-inductive platinoid resistances contained in the cubical box on the right. These being

FIG. 145.

multiples of 100 ohms, the arrangement at once enables higher voltages than 1 volt to be measured. This question, however, will be deferred to our volume on Advanced Testing in Electrical Engineering, as also the construction of the Weston voltmeter; suffice it merely to say that it is a pivoted D'Arsonval galvanometer with a spring control, and when used as a shunt to any of the standard low resistances described on p. 311, it forms a most convenient means of measuring current by the principle of "Ohm's law." It will be found most useful for measuring current in the iron tests (*vide* p. 146, *et seq.*).

Quadrant Electrometer.

A simple and convenient form of quadrant electrometer, and one that is susceptible of a high degree of sensitiveness, is that made by M. J. Carpentier, a slight modification of which is shown in Fig. 146. It consists of a permanent laminated steel **U**-shaped magnet, let into and fixed by brass angles to a polished ebonite base on three levelling screws. Between the two magnet poles is fixed the moving needle and fixed quadrants seen in Fig. 147. This arrangement

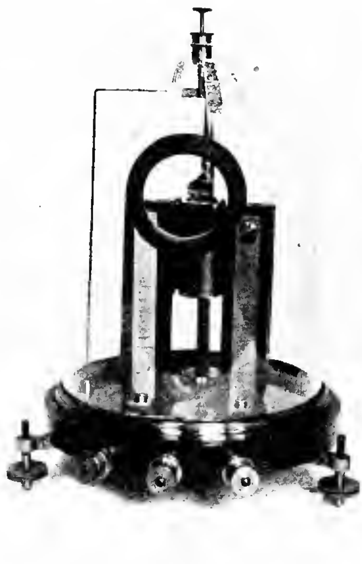


FIG. 146.

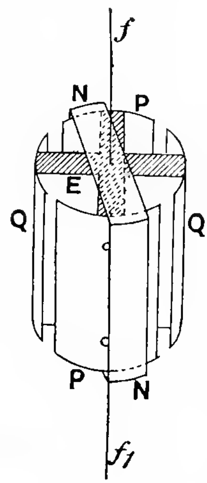


FIG. 147.

consists of four quadrant-shaped iron plates, PP, QQ, fixed, in a cylindrical form, to a piece of ebonite, E, of the same length as the plates, and having a cross-section as shown in the shaded cross at the top. A rather broad rectangular needle or frame, N, made of thin aluminium strip, is suspended by a fine metallic strip, f, from a pin carried at the end of a squared ebonite rod, which can be raised or lowered by the top screw on the brass pillar at the back. The lower fibre, f_1, is attached to the end of a spring strip of brass, and keeps the needle N central with PP and QQ. Outside these latter, but not shown

in Fig. 147, are four exactly similar quadrant plates of brass screwed to them with distance pieces between, so that N can just rotate freely between the brass and iron plates, of which opposite pairs are metallically connected together, and to the outside terminals of the three shown. The quadrants are carried by the standard at the back. The middle terminal is connected to N through the upright angle wire seen on the left and the suspension *f*. The whole of the interior is protected by a glass shade from draughts of air. *f* supplies the requisite torsion, when the torsion head is turned, for turning N to any desired position inside the quadrants. This quadrant electrometer is rendered aperiodic, or dead-beat, as it is usually called, by the powerful magnetic field which passes across the needle, inducing eddy currents in this latter immediately it moves, which, by Lenz's law, tends to stop the motion. In this and all electrometers the difficulty of sufficient insulation makes itself felt. If this form be used idiostatically, *i.e.* with the needle joined to one pair of quadrants, and therefore no initial charge, it can be made to give about 25 cms. deflection per 75 volts on a scale placed at the usual distance from the mirror fixed to N ; or if it is used heterostatically, and the needle charge separately to 100 volts, a deflection of 1 cm. per volt is obtained.

Electrometers.

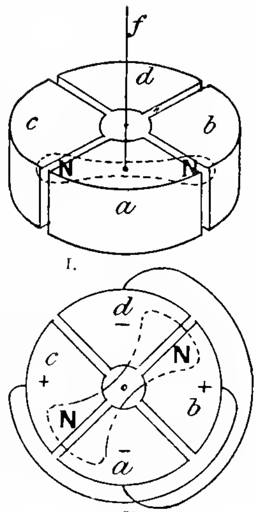
Introductory.—As distinguished from the class of instrument termed a galvanometer, which in any form whatsoever measures differences of electric potential by means of the *electro-magnetic* effects set up by the actual passage of currents of electricity due to such P.D.'s, the electrometer is an instrument for measuring P.D.'s by the effects of *electro-static* attraction or repulsion between two bodies at different potentials. Thus it will be seen that one great distinctive difference exists between the galvanometer and the electrometer, namely, that the former passes a current, while the latter passes *no current* when used to measure P.D.'s. This feature in many classes of measurement renders one or other of the instruments totally unsuitable for the work.

There are a large number of different forms of electrometers, but these may be classified into four distinct types, depending, in general, on the configuration of the particular instrument, and are as follows :—

(i.) Repulsion electrometers ; (ii.) attracted disc electrometers ; (iii.) capillary electrometers ; (iv.) symmetrical electrometers.

In the following pages we shall restrict ourselves to the type (iv.) of symmetrical electrometer, and of the particular design commonly known as the "*quadrant form*," which was originally devised by Lord Kelvin, then Sir William Thomson, and hence, even up to the present time, is commonly called by the latter name. Though there are many

patterns of electrometer constructed after this particular symmetrical form, as, for instance, those of Kelvin, Clifton, Carpentier, etc., the principle underlying the action of each is the same, and will be understood by reference to Fig. 148, I. and II., which shows the arrangement used in the Kelvin electrometer. It consists of a light metallic needle, NN, made of the shape shown, out of the thinnest aluminium sheet that will maintain its shape from the point of view of rigidity.



II.
Fig. 148.

This needle is suspended by a fibre of cocoon silk (two being used in the actual Kelvin electrometer), so that it can turn in a horizontal plane, like a compass needle, inside four brass quadrant boxes, *a*, *b*, *c*, *d*, which form together a flat hollow cylindrical box as seen, and which is placed horizontally. Each quadrant box is highly insulated from the next and rest of the instrument by being supported by a glass stem or rod, not seen in the figures. Opposite boxes, *a*, *d* and *b*, *c*, are connected electrically by wires, and are therefore always at the same potential. The principle of action is as follows: Suppose the needle, NN, is positively charged in some convenient manner, to be described later; then, if quadrants *a* and *d* are connected to earth or to the negative pole of source of E.M.F., while *c* and *b* are joined to the other, and therefore have a positive charge, NN will be repelled out of boxes *b* and *c*, and attracted into *a* and *d*, thus causing a deflection, which will be greater the higher the potential of NN, or the greater the E.M.F. applied to the two pairs of quarter cylinders. When the needle and quadrants are at the same potential, *i.e.* when it is not in use, NN is made to take up the position of *symmetry*, with respect to the quadrants, shown in Fig. 148, II. For the shape of the needle shown, and the fact that the quadrants are very close together, the field of force produced inside the quadrants and acting on the needle will be very approximately uniform in and near this zero position, so that the deflection will be very approximately proportional to the P.D. between the two pairs of quadrants. We will now consider in some detail the somewhat elaborate form of quadrant electrometer designed by Lord Kelvin and made by Messrs. White of Glasgow.

The requirements which this instrument satisfies are as follows:—

(1) A means of maintaining, varying, and indicating the electrification of the needle.

(2) A means of controlling the needle, and of ensuring that this is constant with time and use.

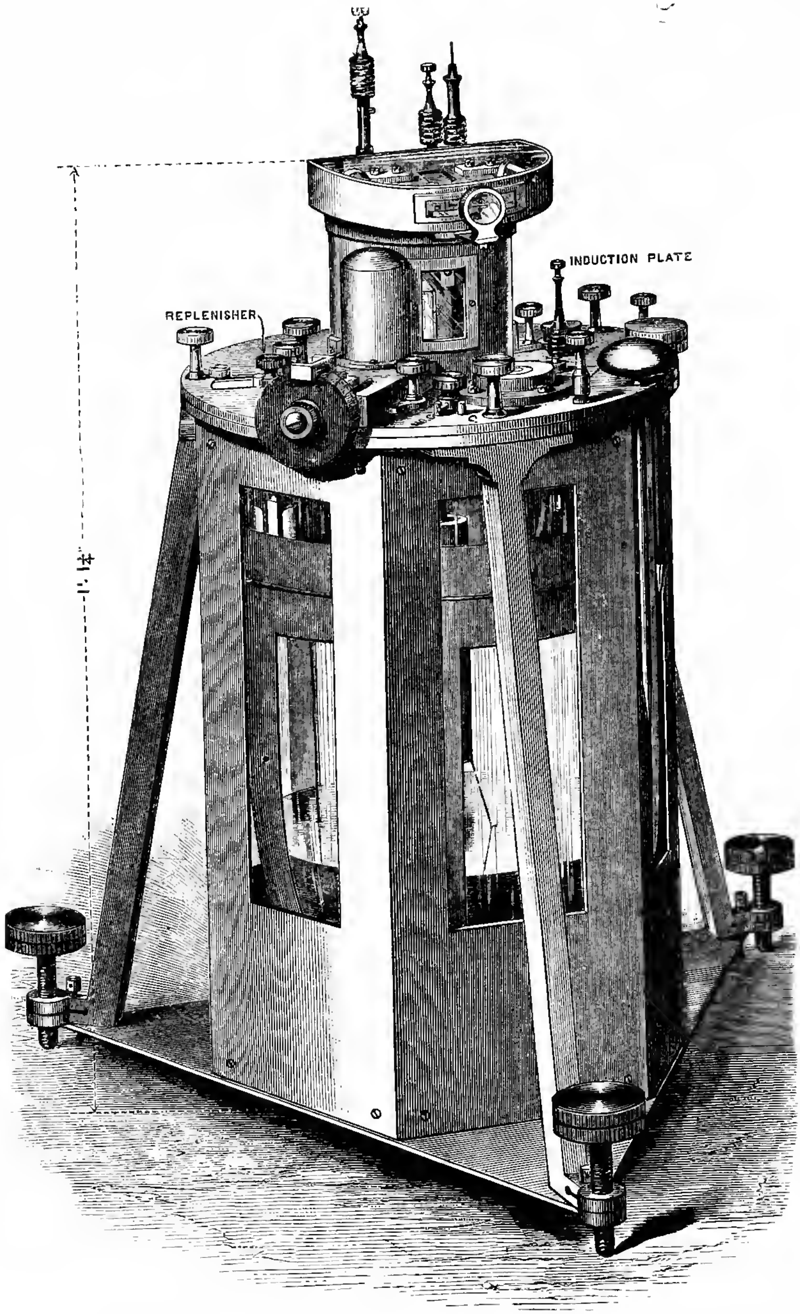


FIG. 149, I.

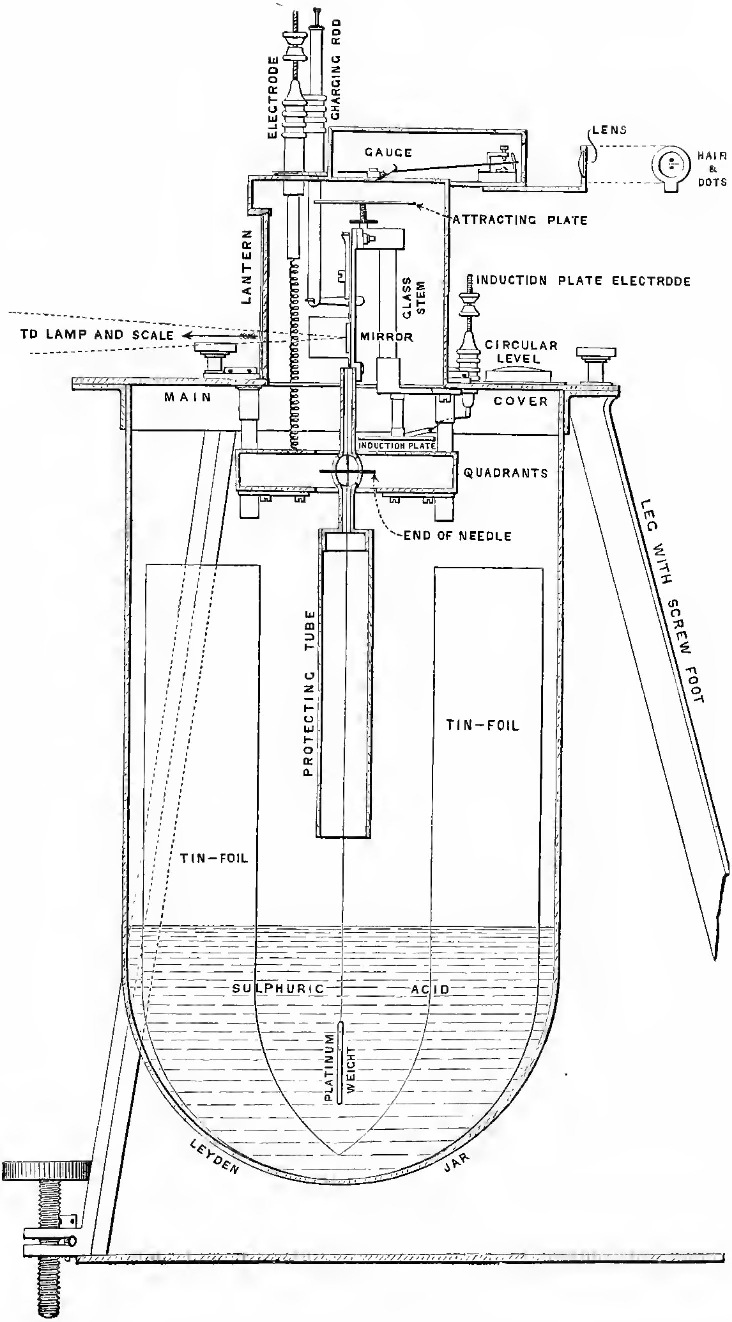

ELECTRODE
CHARGING ROD
GAUGE
LENS
HAIR & DOTS
ATTRACTING PLATE
LANTERN
INDUCTION PLATE ELECTRODE
GLASS STEM
TO LAMP AND SCALE
MIRROR
CIRCULAR LEVEL
MAIN
COVER
INDUCTION PLATE
QUADRANTS
END OF NEEDLE
LEG WITH SCREW FOOT
PROTECTING TUBE
TIN-FOIL
TIN-FOIL
SULPHURIC ACID
PLATINUM WEIGHT
LEYDEN JAR

FIG. 149, II.

(3) A means of varying slightly the directive force of the control, and still ensuring that it remains constant afterwards.

(4) A means of indicating the motions of the needle accurately.

The electrometer, which is largely used for insulation resistance and other testing in connection with cable work, etc., is shown in perspective in Figs. 150 and 149, I., and in sectional elevation in Fig. 149, II. The body of the instrument consists of an inverted glass shade, carried in an outer brass framework provided with a number of rectangular apertures, and supported on three ebonite levelling screws. This inverted glass shade, or bell jar, is coated externally with strips of tin-foil in such a way as to leave openings or windows at intervals through which the interior can be seen.

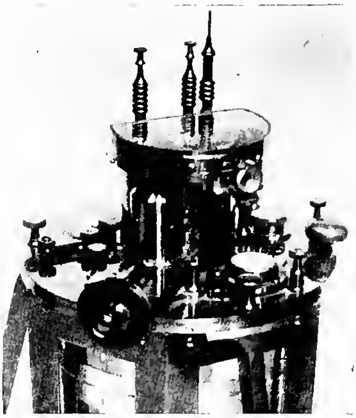


FIG. 150.

The foil is in electric connection with the outer brass framework. With strong sulphuric acid contained in the jar, the arrangement constitutes and is termed the "*Leyden jar*" of the electrometer. The acid not only keeps the interior of the jar dry, owing to its avidity for moisture, but, from its being a conductor, it forms the other coating of the Leyden jar.

The top of the jar is closed by a flat metal lid, which we shall term the "main cover" (*vide* Fig. 149, II.), and which carries the whole mechanism of the instrument. Visible on the top of this main cover (*vide* Figs. 149 and 150) is an ebonite *micrometer head*, which when turned causes one of the quadrant boxes to close up to or recede from the others. This head, which is finely graduated on its periphery, rotates past a fixed index mark, and is seen slightly to the left of the front. On the right of Fig. 149, I. and II., can be seen the electrode or terminal of the *induction plate*, which will be referred to again later on.

The head of the *replenisher* is seen on the left (Figs. 150 and 149, I.). A circular spirit-level, seen most clearly in Fig. 150 to the front, is for the purpose of carefully levelling the instrument, so that the needle hangs freely, and always in the same position relatively to the quadrants.

The two similar-looking terminals (Fig. 150), seen above the segment-shaped glass top at the back of it, are the two electrodes of the two pairs of quadrants, and that by the side of them, to the right, is the charging electrode for charging the needle. The former can by

raising them be disconnected from the quadrants. The erection in the centre of the main cover is called the *lantern*, and has as seen a segment-shaped glass top and a flat glass side (at the back) not seen. Inside this is an *attracting plate* supported by a glass stem (Fig. 149, II.), which carries the *mirror* and *needle* by a *bifilar suspension*.

In the uppermost portion of the lantern is the *gauge*, by means of which the potential of the needle is regulated, and which is really an *attracted disc electrometer*.

The four quadrants and the induction plate above one of them are supported from the under side of the main cover by glass stems. The needle previously mentioned is of the shape there given, and is about 4 square cms. in area, and weighs about $\frac{1}{12}$ grm.

The Gauge is shown more in detail in Fig. 151, I. Above the metal attracting plate just mentioned, in the upper part of the lantern, which plate is connected with the needle electrically, and hence with the inside of the Leyden jar, but insulated from all other parts of the instrument, is an *arm*, *h*, turning on a horizontal axis, *f*, consisting of a stretched platinum wire, *f*, supported by two pins or bridges, *bb*. The lighter end consists of a square plate, P, of thin sheet aluminium, which lies parallel to the attracting plate when the arm *h* is horizontal. This last-named plate charges P inductively, but with the opposite kind of electricity, and therefore attracts it. The heavier end of the arm *f* terminates in a horizontal fork, K, across which a fine hair is stretched. A white enamel index fixed to the stand, and having two dots on it, projects up through the fork, and both this and the hair, which plays in front of them, are observed through a lens, L, just in front, and seen in the figures. The arm and dots at the onset are so adjusted that when the needle is properly charged, the hair appears exactly midway between the dots. If the needle is overcharged, P is attracted further down, and closer to the plate under it, and the hair rises; if, however, the needle is insufficiently charged, the hair falls.

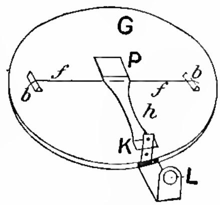


FIG. 151, I.

Fig. 151, II., shows the mirror and its bifilar suspension, together with the circular attracting disc at the top.

The Suspension, which is *bifilar*, consists of two silk fibres, the upper ends of which are fixed to and wound round two pins, *c* and *d*. These may be turned slightly in their sockets by a square pointed key seen hanging at the edge of the main cover (Fig. 149, I.); and in this way they equalize the tensions on the two fibres, and at the same time adjust the height of the needle so that it is midway between the upper and lower surfaces of the quadrants. *a* and *b* are two screw pins pivoted in blocks carried by the spring strips *e* and *f*. When, therefore, *a* and *b* are turned by a key clockwise, they butt up against a fixed plate at the back of *e* and *f*, and throw forward the suspending pin *c* and *d*, and the reverse is the case when they are screwed out.

In this way the needle can be given a slight turn until it takes the position of symmetry (Fig. 148, II.), the quadrants at the time being short-circuited and earthed. h is a conical pin which when pressed in forces the plates to which c and d are separately fixed apart, and so diminishes the sensibility of the needle to a deflecting force. Any adjustment of the distance between c and d can by this means be effected from $\frac{1}{20}$ inch to $\frac{1}{8}$ inch. The lower ends of the fibres are

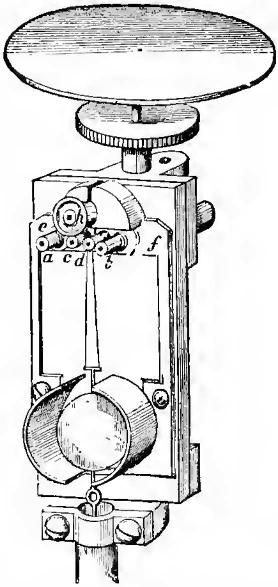


FIG. 151, II

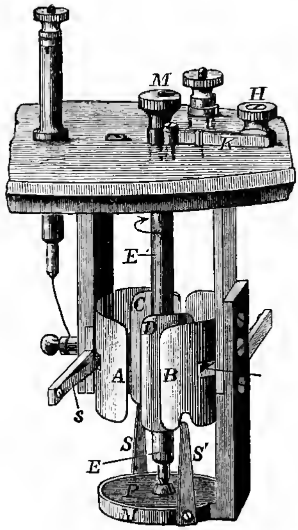


FIG. 151, III.

attached to a cross or T-piece formed on the upper end of a light but stiff platinum wire, to which the mirror and needle are rigidly fixed.

The "Replenisher," which was devised by Lord Kelvin, is for the purpose of maintaining the charge of the needle; in other words, to supply what portions of the charge are lost by leakage, and thus maintain the hair of the *gauge* midway between the dots.

It is an accumulating influence machine which builds up potential on what is usually termed "the *compound interest law*." Fig. 151, III., shows this Thomson replenisher in perspective, and Fig. 152 symbolically in plan. Two brass carriers, C and D, are fixed eccentrically to an ebonite arm, carried by the vertical ebonite spindle E (Fig. 151, III.), which can be rotated by the milled ebonite head M. A and B are two *inductors*, and S, S′ two springs connected by a wire, M (Fig. 152), and which touch C and D as they rotate. In

Fig. 151, III., S, S′ are connected by the brass strip M, which passes round the edge of the ebonite centre plate P, which carries the lower bearing of E. Fig. 151, III., shows, without need of further explanation, how the various parts are disposed and supported.

The action is as follows: When the carriers C, D simultaneously touch S, S′, any charges they contained neutralize, and the next instant they are acted on inductively by the charges in the inductors A and B; and on touching, *s*, *s′* part with their charges to A and B, and in this way the charges on A and B rapidly accumulate.

In practice there is always a sufficiently large P.D. between A and B to start the action on turning the spindle E. To avoid, however, A and B losing the greater part of their charge, when the replenisher

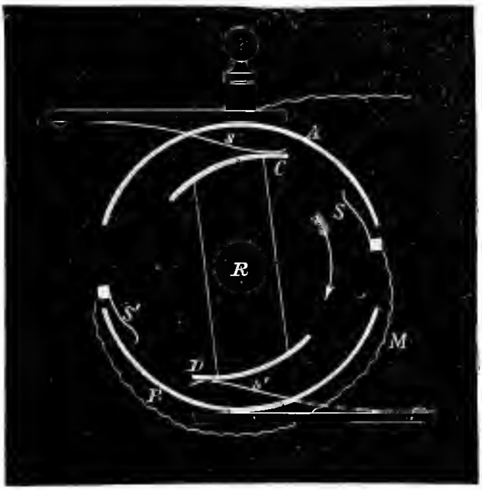


FIG. 152.

is not in use, by touching C and D accidentally, the milled head M is prevented from turning by a pin fixed to the head H, catching a hole in M when H is turned to a certain position. When H is again turned slightly, the pin disengages from M, which is therefore free to revolve. H is lightly locked in one or other of two positions by a spring, K, pressing on a flat formed on H. In one position H locks M in turn, and in the other it does not.

With the bell jar constructed of good glass, carefully washed with distilled water and dried thoroughly before the pure sulphuric acid is put in, the Leyden jar can be made to insulate so well as not to lose so much as 1 per cent. of its charge in the course of 24 hours, and quite a few turns of the replenisher will supply a much greater loss than this usually in a few moments.

The Induction Plate is an addition to the electrometer proper, for the purpose of enabling P.D.'s up to say 100 volts to be measured. As with the ordinary arrangement, where the P.D. is placed directly across the quadrants, the instrument is so sensitive that E.M.F.'s not greater than 5 volts only can be dealt with. The use of the induction plate diminishes the sensibility considerably. Referring to Fig. 153, I is the induction plate insulated and carried by a glass stem or rod, *g*, fixed to the under side of the main cover M, as shown. It is connected with the brass terminal or electrode E by a wire passing up through the centre of the ebonite pillar that carries E. The plate I is placed directly over one of the quadrant boxes Q. If now it is required to measure a P.D. too high for the ordinary arrangement, E is used for one electrode instead of the electrode making contact with the particular quadrant under I, the latter electrode being raised out of contact with that pair of boxes. Thus the potential of these latter is only that induced by I, and is therefore considerably less than that of the body under test depending on the distance of I from Q.

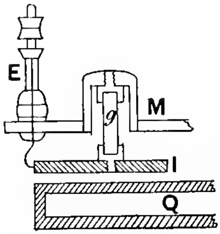


FIG. 153.

The needle is connected with the inner coating of the jar by an attached platinum wire, which passes down a wide protecting tube and dips into the acid. A small platinum weight is fixed to the end, and is completely immersed, and acts as a damper to the motions of the needle. A small brass arm, attached to a vertical axis passing through the main cover, can by its milled head on the top be turned so as to touch the quadrant under the induction plate, and so discharge it.

Electrometer Reversing and Short-circuiting Key.

Some special form of "reversing and short-circuiting" key, having a high-insulation resistance, must be employed when using an electrometer. Fig. 154 show the general form, and Fig. 155 the connections of the one devised by Lord Kelvin. *a*, *b*, *c*, and *d* are brass studs, each connected with a terminal. Only two of these terminals, T_1 and T_2, which are connected to *a* and *b*, are shown; those joined to *c* and *d* are at the back of the key. To *a* and *b* are attached two brass strip springs, s_2 and s_4, capable of pressing at the same time against a brass stud, *p*, joined to *d*. Two other strip springs of brass, s_1 and s_3, connected together and to *c*, just clear s_2 and s_4 when these latter both press against *p*. An eccentric or tail-shaped piece of ebonite, E, is capable of turning on a pin when the ebonite arm A (to which it is fixed), terminating in the ebonite handle *h*, is raised or lowered. In Fig. 155, if *h* is raised to mark 2, E presses s_4 outwards and into contact with s_3; while if lowered to mark 1, s_2 is forced out into contact with s_1, and s_4 returns

to contact with p. It will be noticed that, owing to the cams being in exactly opposite positions in Figs. 154 and 155, as seen, the contacts between the two pairs of springs will be made in just the reverse order

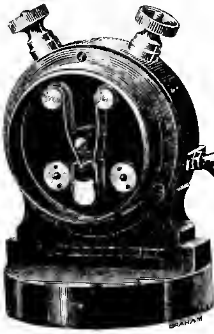


FIG. 154.

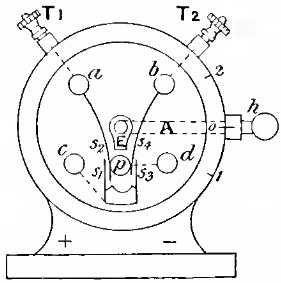


FIG. 155.

to that mentioned in the key (Fig. 155), when h is moved up or down. The key is used as follows in practice :—

Connections to Key.—a and b, *i.e.* T_1 and T_2, are joined to the two terminals of the two pairs of quadrants of the electrometer, of which we will suppose that the pair of quadrants joined to b is also to case, and therefore earthed. c and d are joined to the positive and negative poles respectively of the battery.

Action of Key.—(α) *With h at* O, *a and b are short-circuited* through p, and are both earthed, thereby completely *discharging the quadrants* while the battery is on open circuit.

(β) *With h pressed to* 1, s_2 breaks contact with p and makes contact with s_1 ; *therefore the positive battery is joined to* T_1, *and the negative battery to* T_2 *and to earth.*

(γ) *With h raised to* 2, s_2 makes and s_1 breaks contact with p, and s_3 and s_4 are connected ; *therefore the positive battery is joined to* T_2, *and the negative battery to* T_1.

Thus in moving h from 1 to 2, the quadrants are charged, then shorted, and then charged in the reverse sense.

It may here be convenient and advisable to note the relation existing between the deflection D of the spot of light on the scale and the potentials of the various parts of the electrometer. Let V_1, V_2 be the respective potentials of the two pairs of quadrants above that of the framework (earth) of the instrument, and V that of the needle above earth. Then we have—

$$D \propto (V_1 - V_2)\{V - \tfrac{1}{2}(V_1 + V_2)\} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

From which we see that D becomes greater, *i.e.* the electrometer more sensitive, as V, the potential of the needle, gets greater.

There are two principal methods or ways of using an electrometer.

(*a*) "*Idiostatically*," *i.e.* with the needle *not charged* to any separate or independent potential, but connected metallically to one pair of quadrants. In this case $V = V_1$ (suppose);

$$\therefore D \propto (V_1 - V_2)(V_1 - \tfrac{1}{2}V_1 - \tfrac{1}{2}V_2)$$
$$\propto (V_1 - V_2)(V_1 - V_2)\tfrac{1}{2}$$
$$\therefore D \propto \tfrac{1}{2}(V_1 - V_2)^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Hence the deflection is proportional to the square of the P.D. between the quadrants.

(*b*) "*Heterostatically*," *i.e.* with the needle charged to a separate and independent high potential when equation (1) gives the relation which exists. In this case it can be seen that if V is very large compared with V_1 or V_2, that $\{V - \tfrac{1}{2}(V_1 + V_2)\}$ will be very nearly the same as V.

$$\therefore D \propto (V_1 - V_2)V$$

Hence the deflection in this case is proportional to the P.D. (simply) between the quadrants, assuming V to remain *constant*.

The Quadrant Electrometer.

The following are the directions for the adjustment and use of the quadrant electrometer, drawn up by W. Leitch:—

I. **Short Directions for placing and using an Electrometer already adjusted** (sect. 1–10).—1. Set down the instrument where it is to be used, and put in the prepared sulphuric acid according to the directions in sect. 14.

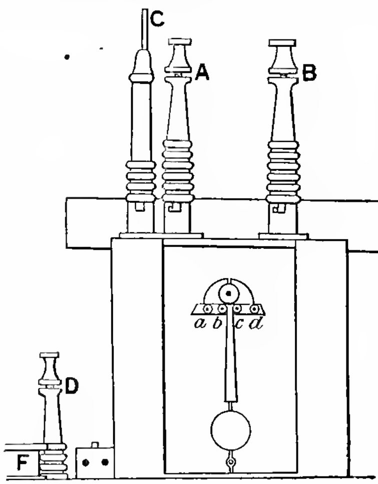


FIG. 155*a*.

2. Twist a fine copper wire round the charging rod C, fasten it by the clamping screws to the two chief electrodes A and B, to the electrode D of the induction plate, and to one of the binding screws on the cover of the jar. Put the charging rod in connection by raising it slightly and turning it in the direction of the hands of a watch.

3. Level the instrument by means of the circular spirit-level fixed on the cover, and see that the needle lies symmetrically, as represented in Fig. 148, II.

4. Place the scale at the proper distance, and so that the reflected image of the wire, which is stretched vertically before the light, may stand at the middle of the scale. The two ends of the scale should be equally distant from the centre of the mirror. [The proper distance between the mirror and the scale depends on the curvature of the mirror, and is found experimentally for each instrument by observing what distance gives the best-defined image of the stretched wire upon the scale. A note of this distance is sent along with the instrument.] The instrument and scale should always stand in this relative position, that is, with the reflected image in the middle of the scale when all electrical disturbance has been guarded against.

5. Now charge the jar through C, previously disconnected from the wire, which must still connect the electrodes with the cover. [The charge may be put in by an electrophorus, an electrical machine, a Leyden jar charged from a machine, or any source from which a small positive charge may be obtained. It is found that a positive charge is less liable to be dissipated from the sharp edges of the needle than a negative.]

6. When the proper potential has been reached, it is indicated by the hair of the aluminium balance rising. Use the replenisher to adjust the charge exactly, so that the hair may stand between the black spots when observed through the lens. When the lever carrying the hair is at either extremity of its range, it is apt to adhere to the stop. In using the replenisher to bring it from either limit, it is necessary to free it from the stop by tapping the cover of the jar with the fingers.

7. If the charge has caused the reflected image to be deflected from the middle of the scale, it may be brought back to that position by turning the micrometer screw which moves the fourth quadrant, and, if necessary, sliding out or in one or more of the other quadrants, and the instrument is ready for use.

8. The small percentage of the charge lost from day to day may be recovered by using the replenisher.

9. In using the instrument to measure a small difference of potentials, such as that of the opposite poles of a single Daniell's cell, the poles are connected with the studs of a key, from the springs of which wires are led to the two chief electrodes A and B. The wire led to A is also connected with one of the binding screws on the cover of the jar, and that led to B is led also to the electrode D of the induction plate. When the key is put down, the image will be deflected over a number of scale divisions proportional to the difference of potentials to be measured; on reversing the key, an equal deflection should take place in the opposite direction.

10. To measure a much greater difference of potentials, such as that of the opposite poles of a battery of thirty cells, the connections are the same, except that the electrode B is raised from the quadrant beneath it. If the act of disconnecting the electrode from the quadrant should induce a charge in the latter indicated by a deflection of the

image, the quadrant is to be earthed by making contact with the disinsulator situated behind the lantern and beside the circular level; the milled vulcanite head of the disinsulator is to be turned so that the small pin projecting horizontally from it may point to the letter "C" (connect) engraved on the cover. This being done, the image will be brought back to its proper position of rest, and the quadrant must be insulated again by turning back the disinsulator to the position "D" (disconnect). With this arrangement differences of potential, corresponding to about a hundred cells and under, will give deflections within the limits of the scale.

II. **Method of Adjustment.**—At present all the quadrant electrometers made are adjusted and tested in the laboratory of the works before being sent to their destinations. The following is the process of adjustment, which it may possibly be necessary to repeat at the place where the instrument is to be used, in the event of the adjustment being disturbed during transit.

11. In the small pasteboard box which accompanies the instrument will be found two square pointed keys, a brass guard tube with a narrow neck, and a fine platinum wire with a small loop at one end and a small platinum weight at the other. The wire and tube must be placed in their proper position in the instrument. Take out the screws numbered 5, 6, and 7, which fasten the cover of the jar. The cover being lifted off, and held in the hand of an assistant, or properly supported about 18 inches above the table, it will be observed that the stiff platinum wire to which the needle is attached just appears below the narrow guard tube enclosing it, in the centre of the quadrants, and terminates in a small hook. The loop at the end of the fine platinum wire is to be slipped over this hook, so that the fine wire and weight may hang from it. The wide guard tube found in the box, when in its proper position, forms a continuation of the upper guard tube, so as to enclose the fine platinum wire just suspended. It must therefore be passed upwards over the suspended wire, and neck foremost, until the neck embraces the lower part of the upper guard tube, where it must be fixed by the pin which is observed screwed into the latter; this pin is to be screwed out by means of one of the square-pointed keys fitting the square hole in its head, and screwed into its place again after passing through the small hole made for the purpose in the neck of the lower guard tube. This being done, replace and fasten the cover.

12. Lift off the lantern, after taking out the two screw pins numbered 3 and 4, which fasten it to the cover, and ascertain whether the four quadrants are hanging properly in their places, with their upper surfaces in one horizontal plane when the instrument has been levelled according to the indication of the circular spirit-level on the cover of the jar. It will be observed that the needle and mirror have been secured during transit by a pin passing through the ring in the platinum wire just above the guard tube, and screwed into the brass plate behind. Screw out this pin with the long steel square-pointed

key, remove it carefully upon the point of the key, and screw it into the hole made for it in the cover, just behind the main glass stem. The needle will now hang by the fibres.

13. The two quadrants in front of the mirror should now be drawn outwards from the centre as far as the slots allow, by sliding outwards the screws from which they hang, and which project above the cover of the jar with their nuts resting upon flat oblong washers. A better view will thus be obtained of the needle. Its surfaces ought to be parallel to the upper and under surfaces of the quadrants, and midway between them. This will be best observed by looking through the glass of the jar just below the rim. If the needle requires to be raised or lowered, it is done by winding up or letting down the suspending fibres, that is, by turning the proper way the small pins *c*, *d*. The suspending wire which passes through the centre of the needle should also be in the centre of the quadrants. This is best observed when the quadrants have been moved to their closest position, as in Fig. 148, II. The fourth quadrant is moved out or in by the micrometer screw with the graduated disc overhanging the edge of the cover. A deviation of the suspending wire from its proper central position, if in the direction perpendicular to the mirror, may be corrected by means of the small screws *a*, *b*; by screwing these inwards, that is, as the hands of a watch move, the points of suspension are brought forward towards the operator, and *vice versâ*. A deviation in the transverse direction it may not be possible to correct otherwise than by sliding back the quadrants to which it is nearest. It will be observed while turning the screws *a*, *b*, that the mirror and needle turn through an angle while only one of the screws is being turned, and that it is by this means that the needle is made to lie in the proper direction, that is, with the black line on the top parallel to the transverse slit made by the edges of the quadrants, when these are symmetrically arranged, as in Fig. 148, II.

14. The sulphuric acid may now be put into the jar. For this purpose the strongest sulphuric acid of commerce is to be boiled, with some crystals of sulphate of ammonia added, in a florence flask, supported on a retort stand over a jet of gas or other convenient source of heat. It is recommended to boil under a chimney, so that the noxious fumes rising from the acid may escape. To guard against the destructive effects of the acid in the event of the flask breaking by the heat, there should be placed beneath it a broad pan filled with ashes, or it should stand above a fireplace containing a sufficient quantity of cold ashes. A little sand put into the flask will lessen the risk of breaking. The object of boiling the acid is to expel the volatile acid impurities, which will otherwise impregnate the air inside of the jar and tarnish the works. When cool, the acid may be poured into the jar through a glass filler with a long stem inserted through the tube F, whose cover screws off. The stem of the filler should reach the bottom of the jar, to avoid splashing upon its sides or upon the works. The stem should also pass through a wider tube which does

not reach the acid, in order that it may be drawn out without touching the brass after it has been wetted by the acid. The acid may be poured in till the surface is about an inch below the lower end of the wide brass tube which hangs down the middle of the jar. It must at least reach the three platinum wires hanging from the works. If tubes suitable for filling in this manner cannot be obtained, the screws 5, 6, and 7 may be taken out, and the cover removed by lifting it vertically upwards till everything hanging from it is clear of the jar. The cover being held by a careful assistant, or supported on a suitable stand, so that the wires may hang freely, the acid may be poured into the jar in any way that can be trusted to cause no splashing, and to leave the sides of the jar untouched.

15. When the acid has been put in (and the works replaced if they have been removed), verify the adjustment of the needle to lie as represented in Fig. 148, II., the instrument being levelled, and all electrical influence guarded against by the connecting wire, as directed in sect. 2. Place the scale and charge the jar, as directed in sects. 4, 5, 6, and 7.

16. The charge sometimes suffers loss by shreds inside of the quadrants drawing it from the needle. It should be ascertained now whether this is taking place. Insulate alternately each pair of quadrants by raising the corresponding electrode, while the other pair are connected through their electrode with the cover. If the reflected image in either case keeps moving slowly along the scale—for instance, over twenty scale divisions in half an hour, the charge in the jar being at the same time kept constant by the use of the replenisher if necessary—the insulated pair of quadrants is receiving a charge from the needle. In that case the inside of the quadrants may be brushed with a light feather, after sliding them outwards as far as the slots allow and securing the needle in the position in which it was fixed in during transit, care being taken not to press upon the needle so as to bend it or the suspending wire. Without securing the needle, each quadrant may be drawn outwards and brushed, while the needle is deflected away from it by the screws *a*, *b*, or by any obvious means of keeping the needle deflected, care being taken not to strain the fibres.

17. Another possible source of loss of charge is want of insulation by the main glass stem. If the percentage of the charge lost from day to day be so considerable as to require much use of the replenisher to recover it, the glass stem should be cleaned with a wet sponge, rubbed with soap at first, or with a piece of hard silk ribbon, wet and soaped at first, then simply wet with clean water, which may be drawn round the stem to clean every part of it. The ribbon, being dried before a fire, may be used in the same manner to dry the stem.

18. The instrument being in position before the scale and charged as in sect. 15, make the connections described in sect. 9, with a single cell, and proceed to test the symmetrical suspension of the needle by the fibres. The conditions sought to be realized are, that in the level

position of the instrument the needle may hang with equal strain on the two fibres, and in a symmetrical position with regard to the four quadrants. It is plain that if these conditions be fulfilled the deflection produced by the same electric force in the level position of the instrument will be less than it will be in any position of the instrument which throws the greater part of the weight on one fibre, or brings the needle nearer to any part of the inner surface of the quadrants than it is in its symmetrical position, which is its position of greatest distance from all the quadrants. Compare the deflections produced upon the scale by the single cell, while the instrument is set at different levels by screwing one or more of the three feet on which it is supported. At each observation note the extreme range or difference of readings got by reversing the key. If the range diminishes as one side of the instrument is raised, the suspending fibre on that side must be drawn up, by turning very slightly the small pin *c* or *d*, round which it is wound, and another series of observations taken in the same manner, beginning with the instrument levelled. Instead of drawing up one fibre, the other may be let down, to keep the needle midway between the upper and under surfaces of the quadrants; and after each alteration of the suspension, it will be necessary to readjust the screws *a*, *b*, to make the needle hang as in Fig. 148, II., when undisturbed by electricity. It will be observed also that the charge of the jar is lost by touching these screws, unless the insulated key is used. They are reached without taking off the lantern by screwing out the vulcanite plug in the glass window in front of them.

19. In deflecting the instrument much from its level position the guard tube may be brought into contact with the wire hanging from the needle, and the movements of the latter thus interfered with by friction. When the needle vibrates freely, it will be observed that the image comes to rest in any position to which it may be deflected, after vibrating with constant period and gradually diminishing range on each side of this position of rest. The occurrence of friction is shown by the needle coming to rest abruptly, or vibrating more quickly than proper. The reading obtained in these circumstances is, of course, of no value. The quicker vibrations obtained in using the induction plate, as in sect. 10, must not be mistaken for vibrations indicating friction, from which they may be easily distinguished by their regularity.

20. If, as may possibly happen, the process of observing the deflections at different levels, and drawing up the fibre on that side which is being raised while getting less sensibility, should only lead the operator to draw up one fibre till it bears the whole weight, while the other is seen to hang loosely, he should adjust them as nearly as he can by the eye to bear an equal share of the weight, and examine the position of the needle by looking through the glass of the jar just below the rim, the two quadrants in front of the mirror being drawn out, and the lantern taken off to let in plenty of light. He will

probably find that the needle leans slightly downwards relatively to the quadrants on that side which he was drawing up while getting smaller deflections. To correct this is a delicate operation, which should only be attempted by a very careful operator. Though perfect symmetry of suspension is aimed at, it is not essential to the utility of the instrument. If it be desired to make the correction, first secure the needle as during transit ; take off the cover, and, while it is held by a careful assistant, or properly supported in a position in which it may be levelled, remove the lower guard tube (the wide brass tube hanging down the centre), after screwing out the small pin in its neck. It will be observed that the upper and narrower guard tube consists of two semi-cylindrical parts united. The part in front may now be removed by taking out the two screws which fasten it at the top, and the platinum wire which carries the needle may be examined. If it has got bent it must be straightened ; if not, it may be bent carefully just above the needle, so as to raise that end of the needle which was observed to hang lowest. If the cover be supported so that it may be levelled, the needle may be set free, and the operator may observe whether he has succeeded in making it hang parallel to the surfaces above and below it. The needle must not, however, be allowed to hang by the fibres while bending the platinum wire, or while removing or replacing the guard tubes.

21. The works being replaced, the process of observing the deflections at different levels and adjusting the tension of the fibres, as described in sect. 18, should be repeated, with the view of getting minimum sensibility in the level position.

22. The two unoccupied holes bored through the cover and flange of the jar are intended to receive the square-pointed keys when not in use.

Resistances.

These may be divided into two main classes, viz. (1) accurately known resistances of either fixed or variable amounts, and suitable for standard and other purposes ; (2) unknown resistances of either fixed or variable values, and generally used for larger current work, which may more properly be termed "rheostats."

Little here need be said of this latter class, but it may be remarked that in almost all cases such rheostats should be, and usually are, constructed so as to obtain a continuous variation of resistance either very gradually or in definite amounts, at a step from the maximum to 0 without breaking the circuit in which they are placed. The first-named class is, however, a very important one, and embraces fixed invariable resistances of low, medium, and high values, which are accurately known at a certain temperature, and in addition standard

adjustable resistances of known values in the form of plug or switch resistance boxes. For very accurate work the plug form is the best, unless high resistances with well-fitted switches are used, when any extra unknown contact resistance is quite negligible compared with that in use in the coil.

Up to 1884 the unit of resistance, in terms of which all standards were made, was (and still is called) the British Association or "B.A. unit." In that year a legal unit of resistance was settled on by the International Electrical Congress at their meeting in Paris, and was defined to be that of a column of pure mercury 106 cms. long, 1 sq. mm. in sectional area at 0° C., the small additional decimal required to be added for exactitude being left to stand over for further investigation. The true value has, by laborious and lengthy testing, been recently found and defined as the legal standard of resistance by Order in Council to be 106·243 cms., and is called the "Board of Trade True Ohm," which may be abbreviated to B.T.O. Hence we have—

1 B.T.O. = 1·01358 B.A. units
1 B.A.U. = 0·9866 B.T.O.

This B.T.O. is the unit of resistance which has been agreed to by the Governments of France, Germany, and the United States, and there is every reason to suppose that it is an extremely close approximation to 10^9 C.G S. units, and it is highly improbable that it will ever be altered again. It is the only unit which has as yet been legalized.

All new boxes are standardized in terms of this last unit, the B.T.O. But many of the older ones are in terms of the B.A U. and previous ohm, hence it is of importance in accurate work to know and correct for any differences so introduced.

Carbon Plate Rheostat.

A form of non-inductive rheostat that will be found very convenient and useful in experimental work which is suitable for currents ranging up to about 10 or 12 amps. is shown in Fig. 156. It merely consists of a pile of flat carbon plates (hard gas-retort carbon being most suitable for the purpose) arranged horizontally so that they can be compressed in a rectangular box by means of the milled-headed screw seen at the right-hand end. This works in a metal bush screwed to the end of the box, and its end presses up against a flat iron plate so as not to crack the carbon plates. The two terminals shown are connected respectively to the bush and the plate at the other end of the pile. It is convenient also to have a loose plate of brass, containing a third terminal, for inserting at any point of the pile so as to pick off any fraction of the full resistance. A fine graduation of

resistance can be obtained with such a carbon rheostat from a maximum resistance, when the plates are loose to a minimum when they are compressed.

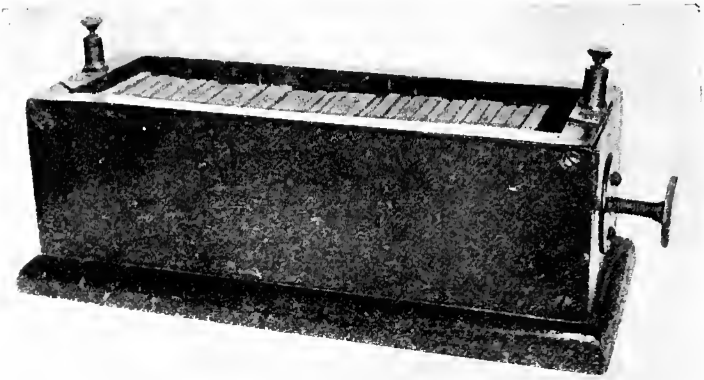


Fig. 156.

Non-inductive Wire Rheostat.

A non-inductive wire-wound rheostat for continuously varying the resistance without appreciable jumps is shown in Fig. 157. It consists of a split slate cylinder carried by two iron end standards having projecting lugs at the top, between which latter two parallel metal rods are fixed. The cylinder, which is doubly wound with suitable bare resistance wire in shallow grooves round its periphery, and connected to the two terminals on the base, is split in a horizontal plane, and the bottom part takes up any slack in the wire due to expansion on warming. The winding starts at one end and goes to the other, and then back again. The terminal resistance is varied by a spring contact block, which slides along the guide rods at the top, and short-circuits the turns as required. The rheostat shown, owing to the large cooling surface, will carry currents up to about 6 or 7 amps. comfortably.

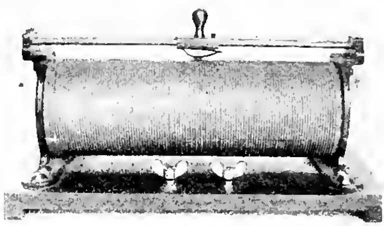


Fig. 157.

Carbonized Cloth Rheostat.

Another form of variable rheostat capable of carrying currents up to 2 or 3 amps. comfortably is shown in Figs. 158 and 159. A small vertical brass rod *h* is fixed to a suitable base at one end, and the other end is screwed to receive a milled-headed nut *n*. A tube of insulating material surrounds the rod, and over it is slipped a pile of discs of carbonized cloth, *c*, prepared by heating ordinary cloth to a high temperature in a vacuum, thereby carbonizing it without destroying it. Brass discs, p_1, p_2, p_3, having side projections to which a terminal is screwed, are inserted as shown, at two or three points in the pile, so as to

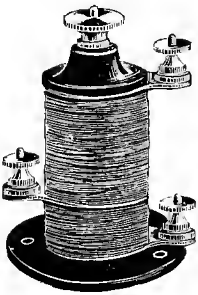


FIG. 158.

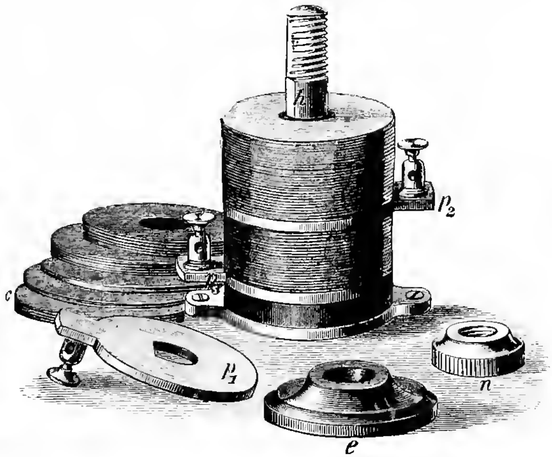


FIG. 159.

obtain fractions of the whole pile. A washer, *e*, which works along a flat, filed on the brass rod, is pressed against the column of discs by the nut, but cannot turn round. The base, if of metal, may form the bottom terminal plate. The action is precisely that mentioned in the case of the carbon rheostat. The resistance of such a column of discs, about 3 inches long and $1\frac{1}{2}$ inch diameter, may vary between $\frac{1}{2}$ ohm and 10 ohms.

The Wirt Wire Rheostat.

A most convenient form of continuously variable resistance is shown in Fig. 160, and is commonly known as the "Wirt" rheostat. It consists of a long rectangular-shaped sheet of stout but pliable

cardboard, over which is closely and neatly wound fairly thin silk-covered platinoid or other suitable wire. The card thus overwound is bent round over the outside of a light, hollow, drum-shaped metal frame provided with metal end discs, and is carried on a vertical spindle, about which it can make almost one complete turn. The spindle, which terminates in a nut at the top, is fixed to the centre of a square flat slab forming the base, and which itself rests on two wooden bars extending along two opposite sides, as seen. Two handles are provided on the top disc or end for the purpose of turning the drum. The terminals of the rheostat are on the right-hand side, only one of which is seen. The periphery of the drum is overwound, with the exception of a narrow strip at the lower end, with thin string, for the purpose of protecting the wire and keeping it in position. The insulation on the outside of the wire in this strip is carefully removed, and a spring carried by two brass pillars (seen to the left) presses against this strip of bared wires as the drum turns round, thus successively making contact with each turn of wire on the card. One terminal is connected to the spring and the other to one end of the wire on the card through the spindle and metal of the drum, while the other end is free or insulated. Hence, when the drum is up against the stop provided for arresting its motion, one way, no resistance is obtained between the terminals, but if it is turned right round so as to again be up against the stop, the whole of the wire or resistance is in. This, in the rheostat shown, is 380 ohms, and it will carry 1 amp.

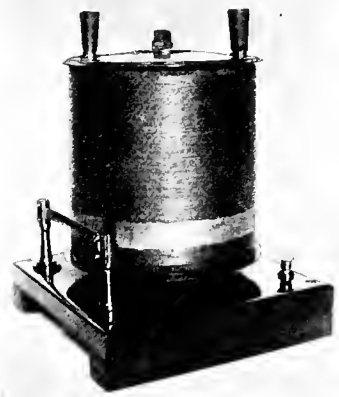


FIG. 160.

Standard Low Resistance.

The construction of these, accurate to, say, 1 part in 10,000, requires a considerable amount of care and labour, and in addition a somewhat special mode of procedure. Probably the best way is as follows: Assume a standard 0·1 ohm is required. Construct ten 1-ohm coils accurate to, say, 1 in 500, by comparison with a standard 1-ohm coil on the Foster bridge; then, since if these are put in parallel with one another, we shall have the reciprocal of the combined resistance equal to the sum of the reciprocals of the several resistances so placed

in parallel; we shall manifestly have constructed a resistance of $\frac{1}{10}$ or 0·1 ohm, which will be correct to 1 in 5000 very approximately. Of course this amount of accuracy requires that the standard 1-ohm coil used in the comparison is accurate to at least 1 in 500, which is easily obtained by constructing it originally in the same manner from ten 10-ohm coils in parallel. It will thus be seen that the greater the number of parallels the less does any error become in the final resistance.

Variable Standard known Resistance.

Fig. 161 shows diagrammatically a form of adjustable low resistance box, in which, by putting additional coils in parallel with each other by means of plugs, a number of low resistances can be obtained. The diagram represents only eight coils, each of 0·5 ohm resistance, one end of each being connected to the common terminal bar T_1PT_1. By connecting any circuit to T_1T_2, and plugging in a suitable manner, eight resistances can be obtained, varying from 0·5 to 0·0625 of an ohm. An extended range and greater fineness of graduation of resistance can be obtained by having more parallels, some of which are of considerably higher resistance than the rest. The plug P is for short-circuiting T_1T_2 when desired.

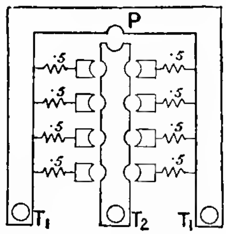


FIG. 161.

Adjustable low known resistances are made in other different forms of which Fig. 162 shows one. It consists of a suitable containing box, to the under side of the cover of which is attached a light non-metallic

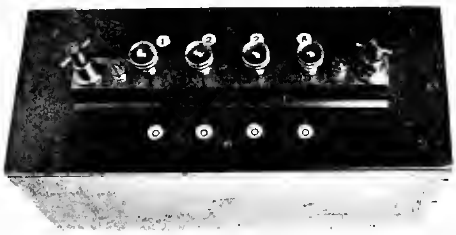


FIG. 162.

framework wound non-inductively with four separate coils. These are composed respectively of four or five lengths of some suitable thick high-resistance bare wire in parallel, and the combination, having the

desired resistance, connected to two adjacent massive. brass blocks, capable of being short-circuited by massive and well-fitting plugs.

FIG. 163.

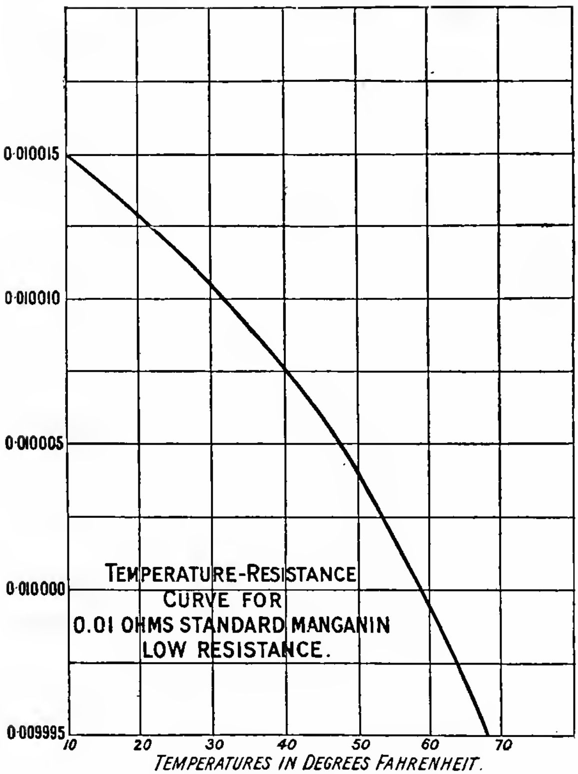


FIG. 164.

The figure shows a standard 1-ohm 4-plug box, capable of carrying 25 amps. without sensible heating. Two small terminals are fitted to the end blocks, in addition to the large current ones, for potential measurements of current when required.

Fixed Standards of Resistance take almost innumerable forms. Fig. 163 shows an unalterable standard of low resistance for carrying 100 amps. without appreciable heating. It consists of a thick rectangular sheet of manganin, divided into five strips of equal width by saw-cuts taken to within the width of each strip from the ends. The resistance thus consists of one continuous strip of equal width and thickness in a small compass, and kept rigid by wooden ends. Two large current terminals are attached to the extremities of the strip and two small potential ones to two points (found by trial), the resistance between which is exactly 0·01000 ohm. Thus a fall of potential of exactly 1 volt will occur between these small terminals when 100 amps. flow through the strip. The curve of variation of resistance, with temperature for this low resistance, is shown in Fig. 164, from which we see that the temperature coefficient of manganin is negative.

Standard 1-ohm Resistance.

Fig. 165 shows a fixed standard 1-ohm resistance, fitted up in such a way that its resistance can be found very accurately at the moment of comparing another with it. As shown, symbolically, in Fig. 166, it consists of an outer vessel, O, with an ebonite lid. Inside this is a hollow, closed vessel I, supported so as to be about $\frac{1}{2}$ inch from the side, top, and bottom of O. I has a cylindrical hole right through it, in which moves a thin ring, *d*, actuated by a rod *r*. A coil of insulated wire, *c*, is wound inside I, and its ends connected to two copper prongs brought through ebonite bushes, and which dip into two mercury cups in the metal pillars seen in Fig. 165; these pillars make good connection with the two terminal leads. One thermometer, T, is inserted in the interior of I, and another, *t*, inside with the coil *c*. Water fills all the spaces between O and the outsides of I. Thus, by using the stirrer *r* to agitate the

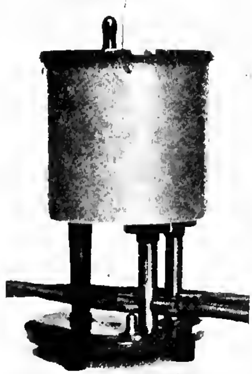


Fig. 165.

water, the temperature of *c* can be kept fairly uniform, and the resistance of the 1-ohm coil corrected, if necessary, for temperature. *f* is merely a foot for supporting O in conjunction with *p*.

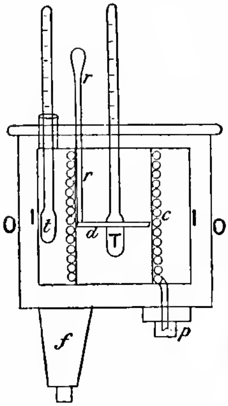


FIG. 166.

Carbon Megohm.

When very high resistances of a fixed nature, but of no great accuracy, are required, a carbon megohm, as it is termed, may be used conveniently. One is shown in Fig. 167.

FIG. 167.

It consists of an ebonite slab having a fairly deep groove (not seen) down one face; a graphite pencil is drawn down the groove, thereby making a conducting path, which is then adjusted to have 10^6 ohms resistance between the two terminals connected to its ends. A thin slab of ebonite is fixed over the groove to protect it and form the base of the megohm; such resistances have an appreciably negative temperature coefficient of resistance.

Resistance Boxes.

There are many different forms of adjustable resistance boxes, an ample supply of which apparatus is indispensable in experimental work. Without actually describing the method of constructing such resistances, which is in the province of an ordinary text-book, it may be remarked that all the coils in any resistance box are doubly wound, thereby making them non-inductive in themselves, and also ineffective in creating external magnetic disturbances, due to a current flowing through them, causing them to be magnetic. There is, of course, a separate resistance coil across each consecutive pair of brass blocks on the top, of the value in ohms stamped opposite each gap; hence, to insert a resistance between the terminals, remove the plug or plugs the numbers opposite which will total up to the required resistance. Great care should be used to ensure all plugs in making good contact, for a bad-fitting plug may cause a grave error in the summed resistance. To obtain good contact use only a very small downward pressure in inserting a plug, but at the same time give it a slight

turning motion, perhaps one-eighth to one-fourth turn. The plugs and holes being conical, this ensures good surface contact and a nil resistance. In removing a plug do the reverse exactly. In high-resistance boxes the ebonite top should be quite clean, especially in

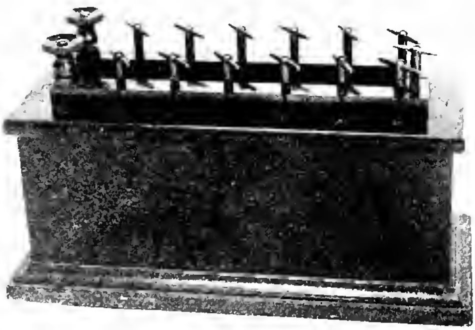


FIG. 168.

the gaps between blocks, for a layer of dust or dirt might have a conductivity comparable with that of the coil, in which case the resistance in the box would *not* be the correct or effective terminal resistance.

Fig. 168 shows a convenient adjustable plug resistance box, totalling 1110 ohms when all the plugs are out, and capable of 1 ohm variations. Some boxes, however, total 11,110 or 110 ohms, and some have these ranges by 0·1 ohm variations.

Wheatstone Bridges.

Metre Bridge.

There are many different forms of these. Fig. 169 represents one form of the simplest type—the so-called "*metre*" bridge. It consists of a straight uniform wire, 1 metre in length, stretched between two stout copper terminal blocks, terminating in angle-shaped stout copper strips as seen. There are three other straight copper strips in a line at the back of the bridge, thus forming four gaps in the line of strips. Terminals are fixed on the strips as shown, and those on either side of each gap are connected, by substantial connections, each to a small mercury cup just in front of it, on the base-board, as shown. By this means either ordinary wires leading to resistances can be

connected to the gap terminals, or resistances terminating in two stout legs, as seen in front of the bridge, can be connected up by inserting the legs in the cups. A metre scale is fixed close to the stretched wire, the ends of the two being in line. The scale acts as a guide to a spring tapping or sliding key, seen to the front of Fig. 169, and enables its position along the stretched wire to be observed. This wire should be of some hard material, such as platinum-iridium, so that it is not dented by the somewhat blunt knife-edge of the slider key, as the accuracy of a measurement on the metre bridge primarily

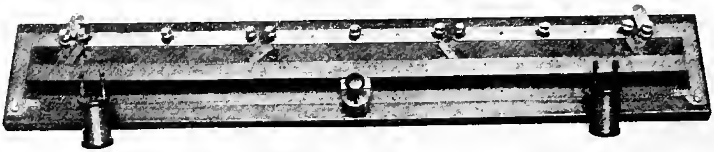


FIG. 169.

depends on the uniformity of the wire. Though only two gaps are essential, a four-gap bridge is very useful for a number of additional tests, and the two surplus gaps at the ends can always be closed, when not required, by stout copper connectors of the form indicated. More than one stretched wire is often employed to increase the range of sensibility of the bridge. The arrangement is then called a "multiple-metre bridge."

The four-gap metre bridge just considered can be applied in Professor Carey Foster's method of comparing two resistances which are nearly the same, by combining with them two additional similar resistances placed in the two middle gaps of this metre bridge. When, after a balance is obtained on the slide wire, the positions of the two outside coils which are being compared are interchanged and a fresh position of balance obtained for the slider, a comparison is effected in the manner seen on p. 63. With this particular construction it is possible to use the metre bridge for the comparison of higher resistances, by placing two coils of suitable resistance, one in each of the end gaps of the bridge, thus extending the length of the stretched wire, and making the resistance of the two portions on either side of the sliding key comparable with those being compared, this being necessary for obtaining accurate results.

Temperature Coefficient Bridge.

To avoid having to really touch the coils, and also to ensure having a compact and accurate arrangement, Messrs. Nalder Brothers & Co. devised the form of Carey-Foster bridge shown in Figs. 170, 171. It consists of a small ebonite table, T, resting on four legs, and carrying four pairs of massive copper bars or blocks, having a mercury cup at each of their ends, two of them being provided with terminals B_1, B_2,

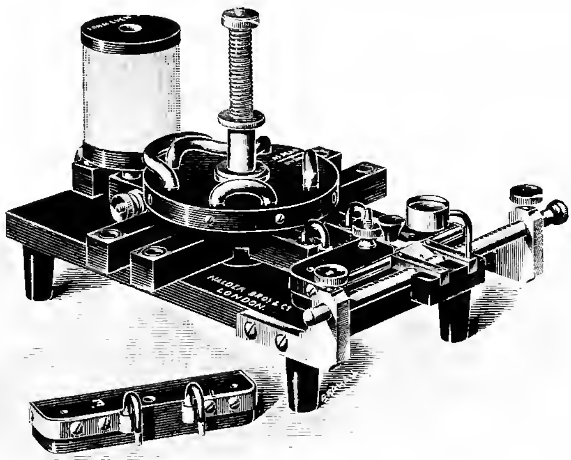


Fig. 170.

which go to the battery. An extra bar, with two cups and a common terminal G_1, is placed as shown, and goes to the galvanometer terminals. The four pairs of blocks are arranged in position as shown, so that one cup of each lies on a circle, and is equidistant from that on each side of it.

A switch disc, D, is fitted with massive connectors as shown, and can rotate about the centre of the cup circle, so making contact between various pairs of cups. The bridge wire, *ww*, is fixed to a removable slab of ebonite, A, which is clamped in position on the table by a pin and nut, *n*. The ends of *ww* are fixed to massive copper straps, which terminate in curved rods, seen in Fig. 170. These dip into the two cups as shown, a sliding key, K, moves along a graduated scale, *ss*, and carries a lens for reading *ss*. Contact with *ww* is made by depressing K.

The terminal G_2 is in electric connection with K, and goes to the other terminal of the galvanometer ; r_3, r_4 are the resistances to be

compared, which make contact with their two respective pairs of mercury cups; r_1, r_2 are a pair of interchangeable ratio coils wound on the same bobbin, so as to have the same temperature, and each of either 1, 10, 100, or 1000 ohms; the coils terminate in copper rods, which dip into the cups; r_3, r_4 respectively may preferably be built up in the same way.

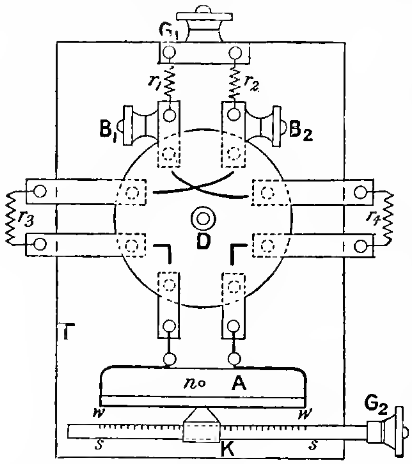


FIG. 171.

A set of bridge wires, A, should be at hand to suit the ratio coils in use.

By reference to Fig. 171 it will be at once evident that on rotating D through 180°, the positions of r_3 and r_4 will be interchanged with respect to the bridge wire *ww*, and r_1, r_2, which is the essence of Professor Carey Foster's method of comparison.

Other forms of metre or multi-metre bridges have been devised, with the object of making the ordinary form more compact and portable. In some types the wire is stretched round the edge of a circular disc 1 or 2 feet in diameter, the sliding key being carried at the end of a rotating arm.

Circular Multiple-Metre Bridge.

Fig. 172 shows a compact form of circular multi metre bridge. It consists of a slate cylinder, capped with metal ends, and mounted on a spindle, which can be rotated by the handle. The slate cylinder

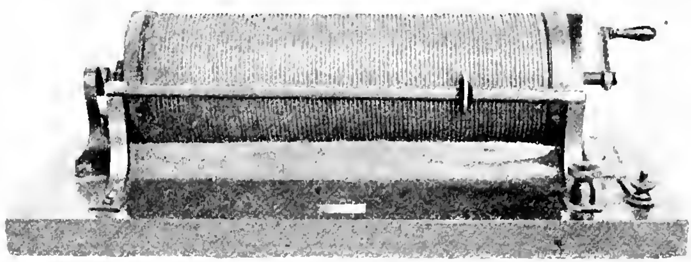


FIG. 172.

has a shallow screw-thread running down it, in which is wound the stretched wire, its ends being connected to the end caps, each of which is connected to a separate terminal across which the battery goes. A small pulley, having a V-groove which fits the wire, is rotated, and at the same time slides along a graduated rod carried by two upright springs, which press it towards the cylinder. This arrangement is connected to a terminal, and constitutes the sliding or rolling key of the bridge. Thus reading the position of the pulley on its rod gives the relative lengths of wire on the cylinder either side of it. It is to be noted that the contact resistance between pulley and wire and also between the rubbing connections at each end of the cylinder is of no consequence, providing it is not sufficiently great to seriously diminish the sensibility of the test.

Post-office Bridge.

Fig. 173 represents a still more compact and highly portable form of Wheatstone bridge, known as the "post-office" pattern (P.O. form). Here plug resistances are substituted for the stretched wire of the metre bridge, to form the two "proportional" arms of the bridge, each consisting of three coils of 10, 100, and 1000 ohms. These six coils form the back row of the box. The third, or "adjustable" arm includes the three remaining rows of resistances, totalling 11,110 ohms. The two spring tapping keys seen in front are for controlling the battery and galvanometer circuits; the connections of their under studs to the rest of the bridge inside the box are shown by white lines on the top. Double terminals are provided where two wires have to go to that point, in order to avoid putting two wires under one head in the case of a single terminal. Each has stamped, on the ebonite top and directly opposite to it, the name of that part of the bridge which should be connected to it, so that it is practically impossible to couple up wrongly. As the arrangement was primarily intended for telegraph

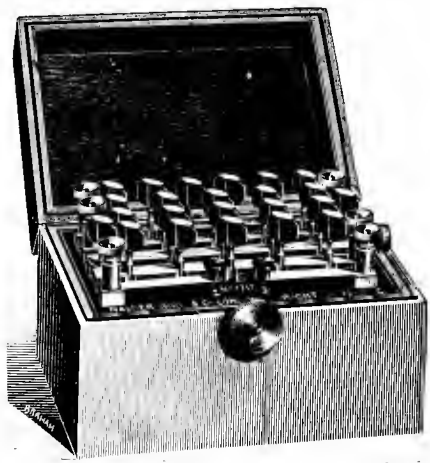


FIG. 173.

engineers, "line" means the telegraph wire, or for ordinary work the resistance to be measured. Plugs, when removed, should always be placed in the lid, and not on the usually dirty table. Moreover, the keys should be pressed lightly, or their platinum tipping will be damaged.

Standard Dial Bridge.

A more elaborate form of Wheatstone bridge is shown in Fig. 174, and is known as the *standard six-dial bridge.*

The proportional arms are in a separate box, and consist of five coils each, of 1, 10, 100, 1000, and 10,000 ohms respectively of the ordinary plug form.

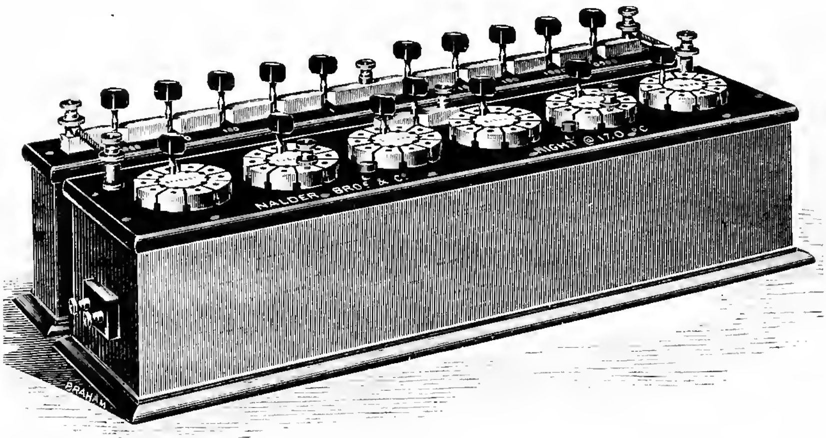


FIG. 174.

The "adjustable" arm consists of six dial resistances, each running from 0 to 9 in tenths, units, tens, hundreds, thousands, and ten-thousands of ohms. An infinity plug is placed just behind the two central dials, and an electrical thermometer is fitted to this box, consisting of a resistance usually of 100 or 1000 ohms, distributed over the interior, and connected to the two terminals shown. Its resistance is correct at the temperature stamped on the bridge ; hence, if measured at any other temperature, this temperature can at once be calculated, knowing the material of which it is made, and its temperature coefficient of resistance. The great advantage of this form of bridge is that the resistance of the adjustable arm is simply read off by inspection on glancing down the dials from left to right. It is used for very accurate standardizing purposes.

Thomson-Varley Slides.

Fig. 175 shows the general view, and Fig. 176 a part diagrammatic one, of an elaborate and beautiful but costly resistance, known as the Thomson-Varley slide coils. It is primarily intended for making very rapid tests on rapidly varying resistances, and also for testing resistances possessing capacity with which the ordinary Wheatstone bridge requires time in order to allow the current to arrive at its steady value after altering the resistances of the arms, and before making the final adjustments.

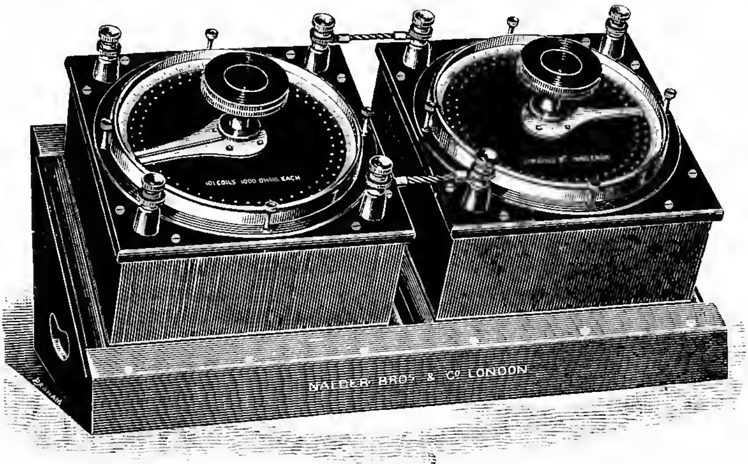


FIG. 175.

The arrangement consists of two boxes, A and B, of resistance coils, each fitted with a dial containing a circular double row of studs, to which the coils are connected. The left-hand box, A, contains one hundred and one coils of 1000 ohms, connected to the studs as shown; while the right-hand box contains one hundred coils of 20 ohms each, connected to their studs in the same way. An ordinary single-lever contact arm, *b*, makes contact with any *one* stud at one time when turned by its milled ebonite head.

A double-lever, composed of two contact arms side by side, but insulated from each other, makes contact with two alternate studs on box A at *one* and the same time as the lever is rotated. All the studs and contacts have platinum iridium faces, to prevent tarnishing and wear. From reference to Fig. 176 it will be seen that resistance B, whose sum = 2000 ohms, is connected between the two contact arms

of the left-hand lever, which makes contact between the alternate studs, giving also 2000 ohms between its two arms. Hence the box of coils B acts as a kind of vernier to any two successive coils in A, and the arrangement acts as a potentiometer, of which T_1, T_2 are the terminals of the extreme resistance (100,000 ohms), and the terminal T_3 the sliding intermediate contact. One disadvantage in the arrangement of coils shown is that the smallest in the vernier side is 20 ohms, and

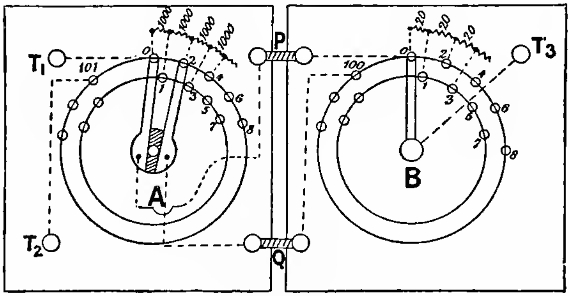


FIG. 176.

therefore the instrument has no such fine adjustment as the sliding key on a meter bridge wire. Another is that only a very sensitive high-resistance galvanometer can be used with it to obtain accurate results. Slightly modified forms of this slide resistance have been introduced by Dr. Muirhead, in one of which four dials of resistances are employed, and the same range obtained, three of the dials having eleven coils each, and the fourth ten coils.

Shunts.

The class of apparatus which goes by the above name is one that provides a simple and ready means of altering the sensitiveness of a galvanometer. Thus, for instance, if with a certain galvanometer we are using the only available source of E.M.F. and resistance in circuit, and then the deflection is too great to be read, it can be diminished to any desired extent by connecting across the terminals a wire or "bye-pass," or "*shunt*," as it is commonly termed, of a suitable resistance. A certain fraction of the total current which originally went through the galvanometer now is shunted past it, thus reducing the deflection.

Ordinary Shunt Box.

Sets of shunts are supplied in the form of "shunt boxes," one very common form of which is shown in Fig. 177, and symbolically, with the connections, etc., in Fig. 178. Usually such a box is supplied with, and only intended for use with, a particular galvanometer for the following reasons. Three resistances, r_1, r_2, r_3, having respectively $\frac{1}{9}$, $\frac{1}{99}$, and $\frac{1}{999}$, the resistance of the galvanometer itself, are mounted in a metal case with a plug top of the form shown, and connected to the blocks as there indicated. The blocks carrying the two terminals T_1, T_2 can,

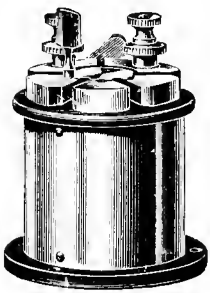


FIG. 177.

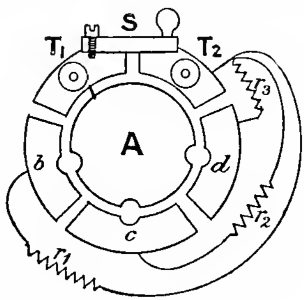


FIG. 178.

when desired, be short-circuited by the lever S. T_1 is permanently connected to the common block A, to which also either *b*, *c*, or *d* can be separately connected by the one plug supplied. According, therefore, as to whether the shunts r_1, r_2, or r_3 are plugged across the terminals T_1, T_2, to which the galvanometer is directly connected, so $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$ only of the total current flowing through the battery will pass through the galvanometer. The proof of this will be found on p. 237. If S is down, so as to short-circuit T_1, T_2, no current will go through the galvanometer, and to use the latter unshunted, lift *s* and have no plug in. T_1, T_2 are usually double terminals, enabling their lower halves to be permanently joined to the galvanometer.

High Insulation Shunt Box.

Fig. 179 shows a similarly arranged shunt box highly insulated, and for use with very highly insulated galvanometers. As can be seen, the plugs have long corrugated ebonite handles, to prevent leakage to earth through the operator when using the plugs. The brass blocks are each on corrugated ebonite pillars fixed to the ebonite top of the box containing the shunt coils, and this itself is supported by longer

similar pillars on an ebonite base, carried by three ebonite feet. The long-handled plug shown performs the function of S in the other type of box, and a spare plug is seen on the base. It will thus be evident that the insulation resistance of this shunt box will be exceedingly high.

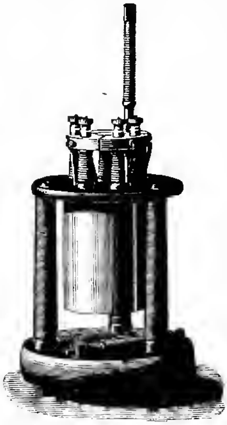


FIG. 179.

Constant Total Current Shunt Box.

As will be obvious at first sight, the insertion of a shunt across the terminals of any galvanometer reduces the total circuit resistance, from the fact that the current for a certain length of the circuit has an additional path to flow through. Thus it may happen that the total current increases by Ohm's law. There are two ways by which a shunt box can be arranged, so that the act of inserting a shunt box also adds a compensating resistance in the main circuit of such a value as to maintain the main current constant. This gives rise to what are known as "constant total current shunts." Fig. 180 shows the general form, and Fig. 181 a symbolical view, of one of the methods due to Mr. Kempe. s_1, s_2, s_3 are the three shunt coils, and r_1, r_2, r_3

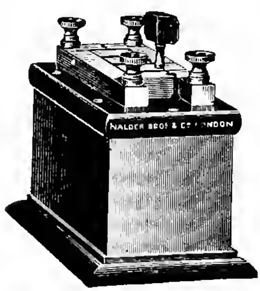


FIG. 180.

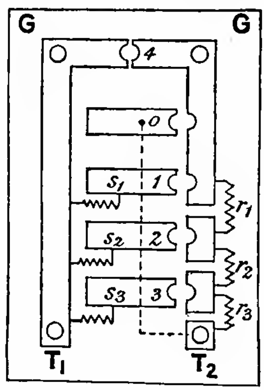


FIG. 181.

the respective main circuit resistances that are automatically added by the insertion of a plug in the holes 1, 2, and 3, so as to keep the combined resistance between the main terminals T_1, T_2 constant. The galvanometer is joined directly to terminals G, G. If the plug is

inserted in hole 4, the galvanometer is short-circuited, as by S in Fig. 178; but if the plug is in hole 0, the galvanometer is unshunted, and T_1, T_2 may be regarded as its terminals. s_1, s_2, s_3 shunt the usual fraction of current. The values of r_1, r_2, r_3, in terms of the galvanometer resistance and multiplying powers of the shunts, can be found algebraically.

Universal Shunt Box.

A new form of shunt box is that devised by Professor Ayrton and Mr. Mather, and commonly known as the "universal shunt box" for galvanometers. This is, perhaps, an inappropriate appellation for it in one sense of the word, for reasons which follow later on. Fig. 182 shows a general view of the latest form of this shunt box, and Fig. 183 a symbolical one of the connections of coils to the studs and

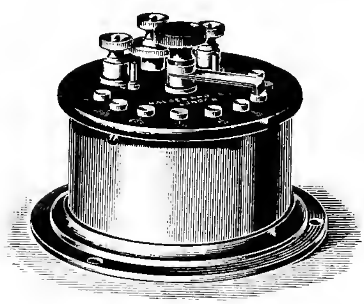


FIG. 182.

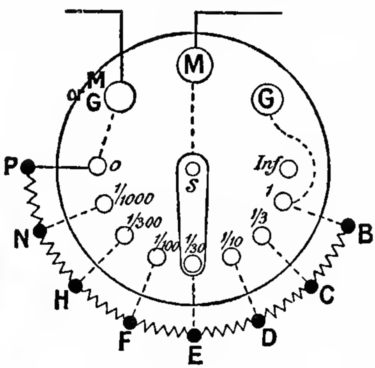


FIG. 183.

terminals. As seen, seven resistances, BC, CD, DE, etc., are enclosed in a suitable box and connected in the manner indicated to the eight contact studs fixed to the ebonite top. A contact lever s, in permanent connection with the middle terminal M, moves over the studs, and acts practically as a potentiometer arrangement. The two terminals marked G go direct to the galvanometer to be shunted, while those marked M go to the *main* circuit. Terminals G, G are permanently connected to studs marked Inf. and 1, or, in other words, to the extremities of the whole resistance BFP, which is permanently across the galvanometer terminals G. Intermediate studs are connected to intermediate points on this resistance, in such a way that if S equal the whole resistance between B and P, then that between C and P $= \frac{S}{3}$, between D and P $= \frac{S}{10}$, between E and P $= \frac{S}{30}$, and so

on. Thus, if G = galvanometer current when s is on stud 1, it will be $\frac{G}{3}, \frac{G}{10}, \frac{G}{30}, \ldots$ etc., when s is put to studs marked $\frac{1}{3}, \frac{1}{10}, \frac{1}{30}, \ldots$ respectively. One great advantage of this form of shunt is that its shunting constants are correct for ballistic work, as the damping remains constant with whatever shunt is used ; also that intermediate shunts are provided over the old-form, enabling a deflection of at least one-third of the whole scale to be worked to instead of only one-tenth, say. Owing to the high resistance which the lever s controls, no practical error is introduced through any slightly bad contact between it and the studs, and, on the other hand, it keeps itself clean, and is much more convenient than plugs. This form of shunt must have as high a resistance as possible compared with that of the galvanometer, and for this reason they are made with different resistances to suit galvanometers having any resistance up to a suitable limit.

Referring to Fig. 183, it will be seen that the arrangement will also act as a potentiometer, giving the respective definite fractions of the total volts across MM.

For the detailed theory of this "universal shunt," see *Electrician*, vol. xxxii. p. 627.

Keys.

In consequence of the multiplicity of designs, not only amongst the various types of keys, but also in any one type in itself, it will only be possible here to notice some of the most commonly used apparatus of this kind. Speaking generally of the various keys, etc., it may be remarked that for elementary experimental work *strength* is chiefly

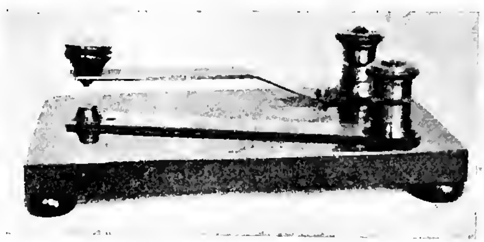


FIG. 184.

required for the average manipulator, and not so much high finish and high insulation resistance. The last named is, however, of great importance in more advanced practical work, particularly in capacity tests, and considerable care must be taken to keep the keys clean and

free from dust. All keys must be so constructed as to introduce *no* appreciable extra *resistance* into the circuit in which they are inserted. All tapped contact surfaces are or should be tipped with platinum to prevent oxidation and tarnishing, and so improving the contact. Such *keys* should *always* be *tapped lightly* to avoid damaging this platinum facing.

Fig. 184 shows a most simple and useful form of single-way spring tapping-key, mounted, in the instance shown, on an ebonite base with four ebonite feet. The little finger-knob on the end of the spring lever is also of ebonite. Thus it will be seen that the key will have a very considerable insulation resistance.

Fig. 185 represents a high insulation (H.I.) two-way spring tapping-key. As seen, the two spring levers are carried at the end of a horizontal

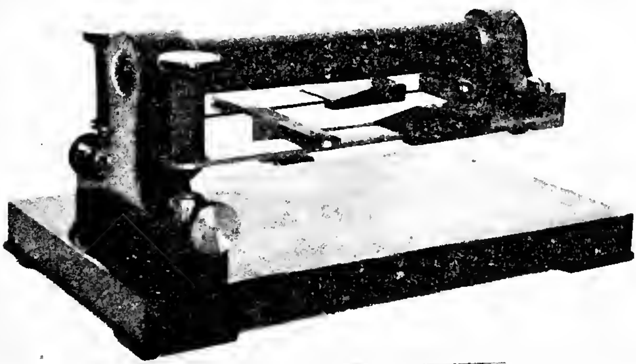


FIG. 185.

polished ebonite rod, itself supported by an ebonite standard fixed to the ebonite base. The two levers are provided with finger-knobs of considerable length, in order to prevent leakage of current from levers to earth through finger and body. The two levers run from a common brass terminal block carried at the end of the rod, and are each provided with an ebonite wedge capable of sliding along them and under the brass cross-piece shown, thus keeping the levers permanently pressed when required without having to hold them down.

Fig. 186 shows a four-way plug key on an ebonite base, and is a very convenient piece of apparatus in many different tests.

A single-lever "charge and discharge" spring tapping-key is shown in Fig. 187. It is a H.I. key, and its principal use lies in condenser work, for which it is primarily intended, though it is useful for numerous other tests. The figure explains itself, but it will be observed that corrugated ebonite pillars are used to support the brasswork on the

ebonite base fitted with ebonite feet. The chief objects of this form of pillar are that the effective length of the path for leakage is considerably

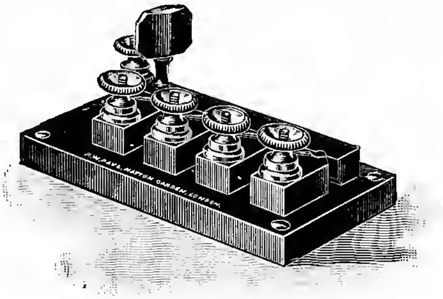


Fig. 186.

increased for a given length of pillar, and also touching them accidentally with the fingers only dirties the edges of the corrugations without doing so to the bottoms of the grooves. Thus a high insulation

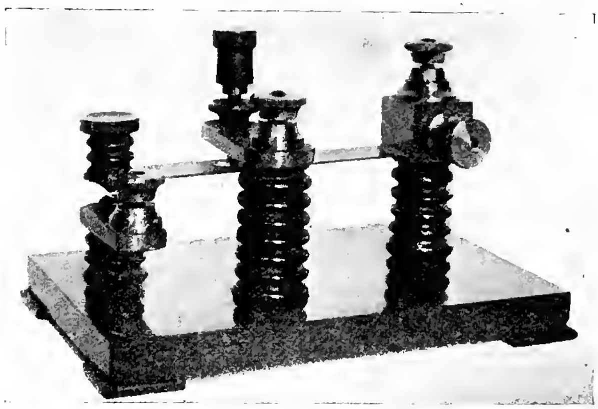


Fig. 187.

resistance is obtained. The spring lever, when pressed, makes contact on the extreme left-hand block only, and when released springs up against and makes contact with the block carried by the middle pillar.

Fig. 188 shows an ordinary form of H.I. pillar reversing key. Here the two spring levers can be kept permanently depressed by ebonite cams carried on independent pillars; when released, the levers spring up against their intermediate contacts. The key can also be used for charge and discharge work, etc.

An extremely simple and useful form of mercury reversing key is shown in Fig. 189. It consists of either a wood or ebonite base containing four small mercury cups forming a square, and which are connected by copper strips to the four terminals. To an ebonite block

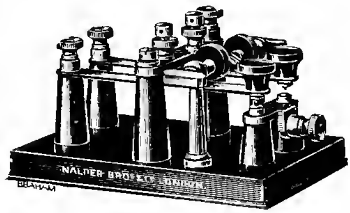


FIG. 188.

of the shape shown, which insulates them from each other, are fixed a couple of copper angle plates, their outer vertical edges terminating in prongs or legs the same distances apart as any two cups. The ebonite block slips up and down the pin, about which it can rotate ; thus, when all the prongs are in the mercury cups in one position, the current

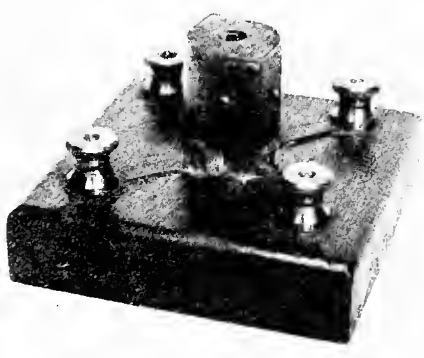


FIG. 189.

flows in one direction, and when the block is raised and turned through 90°, the current flows in the opposite direction. As a simple instance of connection, a battery would go across one pair of diagonal terminals, and the circuit in which the current had to be reversed across the other pair of diagonal terminals.

Fig. 190 shows what is commonly known as a Pohl's commutator, which can be used either as a single or two-way key or as a reversing key. It consists of an ebonite base on four legs or feet, and containing six mercury cups, A–F, spaced as shown (Fig. 191). Each of these cups is

connected to its own separate terminal by the side of it. An ebonite rod is supported by and rocks on two copper legs dipping into C and D and pivoted in them. The cups, through the medium of these legs,

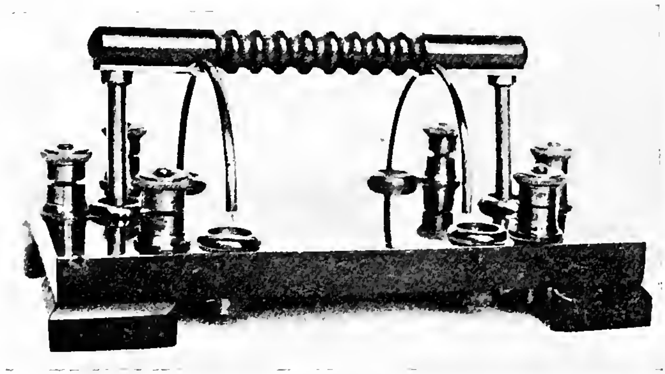


FIG. 190.

are in good electrical connection with two curved copper rods, seen in Fig. 190, which dip into either A and B on the one side or E and F on the other, as the cross-bar is rocked. This connects A to C and B to D on the first side, or C to E and D to F on the other at will, thereby constituting a two-way key. To use it as a reversing key, connect A to F and B to E by independent connectors; then, if a battery is across C and D, the current in any circuit across AB or EF only will be reversed on rocking the key over.

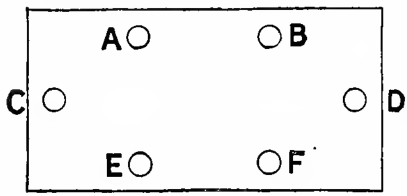


FIG. 191.

The key shown in Fig. 190 has a high insulation when clean. Mercury keys and commutators of this description possess the disadvantage, unless screwed permanently down to the table, of spilling, and the author has found it to be almost invariably the case that a student first commences by turning such a key upside down to look at the bottom, perfectly oblivious of the fact that mercury tends to gravitate somewhat rapidly.

In some tests, as, for example, with the "make" and "break" method of carrying out Rowland's permeability test on iron, etc., the current should be broken quickly to avoid chiefly the destructive spark on breaking an induction current. Fig. 192 shows a good form of quick-break switch, which can be used for all currents up to about

30 amps. A description of this switch, which is intended for electrical engineering work, is perhaps superfluous here. Suffice it to say that

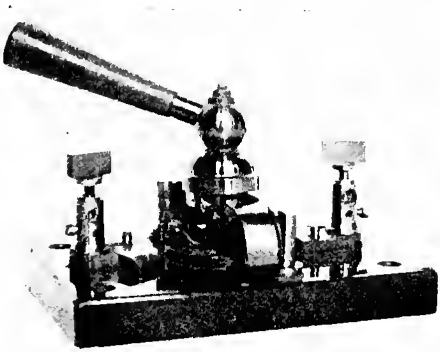


Fig. 192.

the contact lever, actuated by the handle and its boss, is thrust very rapidly away from the terminal blocks shown, after it has reached a certain point close to the edge of each block.

Voltameters.

Mixed Gas.—A convenient form of "mixed gas" voltameter is shown in Fig. 193. It consists of a wide-mouthed glass jar, closed by an indiarubber cork, through which passes two glass tubes (air-tight). The left-hand one has a small cylindrical bulb in its middle region, and the lower end dips under the surface of the dilute sulphuric acid contained in the jar, the upper terminating in a thistle-headed funnel. A stop-tap is fitted to the right-hand tube, the lower end of which is well above the liquid surface, whilst the upper bends over into the funnel. Two platinum wires, connected to the terminals on the base, pass through the cork and carry two parallel sheets of platinum foil, which are immersed in the acid. When a current of electricity is sent through the voltameter with the tap shut, decomposition ensues, and the gas collects over the liquid, forcing it down and some of it up the left-hand tube. The rate at which the liquid column rises between two fixed scratches is proportional to the current. The volume between the scratches is determined accurately once for all. The funnel is for the purpose of catching any spray that may be forced through the right-hand tube when the tap is turned to allow the collected gas to

escape. The tap, stopper, and all that passes through it must be perfectly air-tight.

Copper.—Fig. 194 shows a copper voltameter, which consists preferably of a cubical-shaped glass box, fitted with a hollow frame-shaped top. On two opposite sides of this frame are fixed substantial

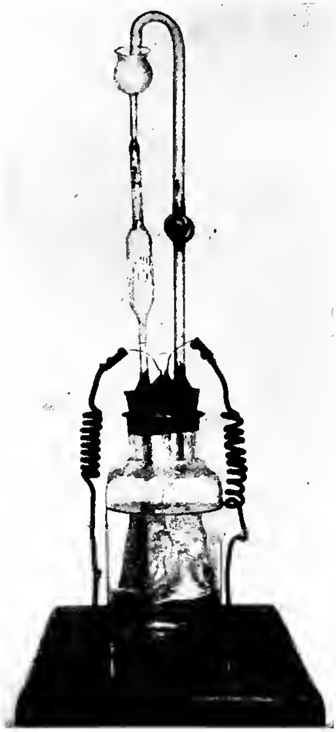


Fig. 193.

brass or copper strips, carrying spring clamps of the shape shown, and the terminals. The former are spaced equally on the strips to the number required, but a clamp on one side is opposite the centre of a gap on the other. The plates, which are of thin sheet copper for the cathode and thick sheet copper for the anode, and as pure as possible, each have a projecting lug on either side at one end. Hence to insert one or more plates in the voltameter, they are put in, and the lugs pressed in between the sides of the clamp. In this way good contact is obtained, and the insertion or withdrawal of a plate is only the work of a moment. It will at once be evident that all the plates connected

with one row of clamps will form the cathode, when perforce all on the other side will form the anode, of the voltameter.

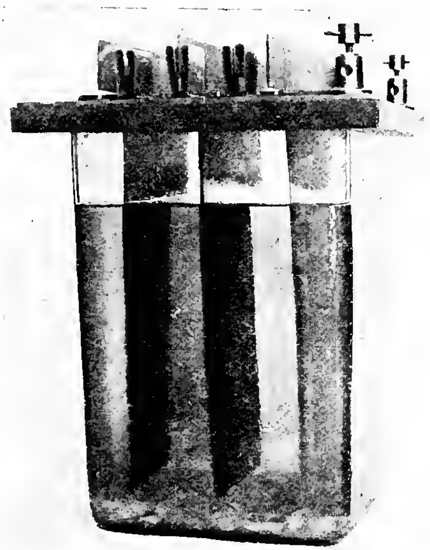


FIG. 194.

Silver.—A simple and convenient form of silver voltameter is represented in Fig. 195, and consists of a thin platinum bowl, *b*, from 3 to 4 inches diameter, and $1\frac{1}{4}$ to 2 inches deep. This merely rests on the edge of a circular hole cut in a brass or copper plate, *a*, screwed to the wooden base B. By this arrangement, *b*, which makes good contact with *a*, and therefore with the terminal T_2 fixed to *a*, can be easily removed. *b*, which forms the cathode of the voltameter, is made of platinum instead of silver, in order that, after an experiment is completely finished, the silver deposited on the inside of it can, by pouring in a little nitric acid, be dissolved off to form silver nitrate, which can be poured back into a stock bottle and used over and over again on future occasions. In this way the cathode can be made thin and light,

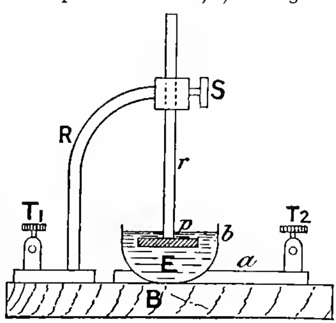


FIG. 195.

and its weight consequently determined more accurately; whereas, if it was silver, the deposit could not safely be redissolved, and hence the bowl would grow thicker and heavier. A plate of pure silver, *p*, about 1 inch smaller in diameter than *b*, is fixed to the end of a metal rod, *r*, and dips into a solution of silver nitrate, E, contained in the bowl; *r* can be clamped by a set-screw, S, to a metal standard, R, making electric contact with a brass plate carrying the other terminal, T_1. This silver plate forms the anode, and is, during an experiment, temporarily wrapped in a good filter-paper to prevent particles of silver oxide and other impurities being deposited on the bowl and causing an error in the true weight of deposited silver.

The electrolyte should consist of a 15 per cent. to 30 per cent. solution of silver nitrate in pure water.

A voltameter such as the above is suitable for measuring accurately any current up to 2 amps. with a 30 per cent. solution. For 1 amp. a 15 per cent. solution might be used. The maximum current that may be used so as to obtain a good adherent deposit with any platinum bowl is approximately 1 amp. per 6 square inches of surface.

Magnetic Apparatus.

A simple arrangement for enabling the distribution of magnetism in a long bar magnet to be easily determined ballistically is shown in Fig. 196. A framework composed of two rectangular plates joined by two small rods on opposite sides, is capable of sliding along the bar magnet, this latter working through rectangular holes in the plates. One plate has a lug with a set screw, by means of which the frame can be clamped in any position on the magnet. A rectangular coil of fine silk-covered wire, the ends of which are attached to two terminals fixed to the flange of the coil, allows the magnet to pass freely through its centre. Thus when the frame is clamped the little coil can slip freely along it between the two end plates, and in so doing cuts the magnetic lines of force straying out of the magnet's sides at that position.

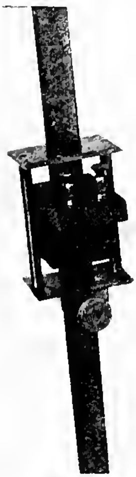


FIG. 196.

Fig. 197 shows a very simple piece of apparatus for enabling the quantity M.H. to be found for a small magnet. It merely consists of a mahogany base, on three legs to ensure steadiness, to which is fixed a brass rod or standard, bent over at the top. A small aluminium stirrup is suspended from this cross-piece by means of a fibre of cocoon silk. A glass shade, let into a circular groove in the base, covers the standard and suspension

in, draught-tight. Hence a magnet placed in the stirrup is able to oscillate quite freely from torsional resistance of the fibre and air-draughts.

Dip Circle.

A form of "dip circle" of a very simple nature, but fairly well suited for the use of elementary students is shown in Fig. 198. It consists of the containing box, having a glass back and front and a hinged lid at the top. A hollow ring-shaped scale, degree divided, is

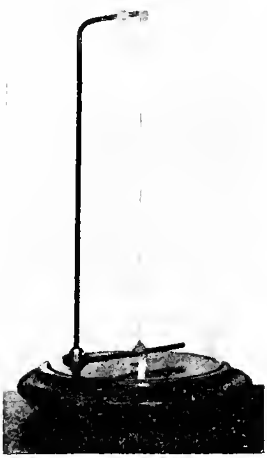


Fig. 197.

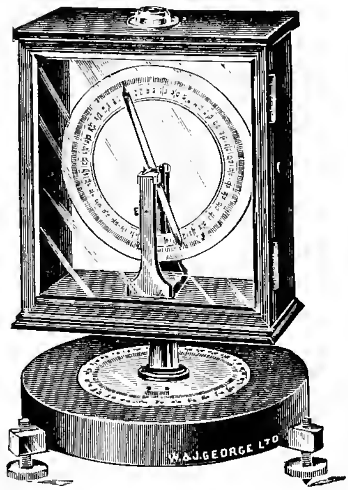


Fig. 198.

supported by the wooden sides. The box is carried by a stout central pivot, to which is also fixed another circular degree-divided scale, the whole being capable of turning about the vertical pin on a base, supported on three levelling screws, and fitted with the small spirit-level seen in the figure. The central pivot terminates above in two limbs, one on either side of the scale, and supporting at their tops two agate knife-edges on a level with the scale centre. On these agate edges rests the delicate steel spindle of the somewhat lengthy magnetic needle. Two forks, actuated by turning the milled head, seen in front of the box, rise, and lift the needle off the agates when the instrument is not in use or when carried about. The horizontal scale indicates the angle through which the box is turned at any time. Most elaborate dip circles are used when very accurate results are required, and the student must refer to other books for a description of them.

Magnetometers.

Fig. 199 represents a form of magnetometer well suited for comparing magnetic moments of magnets with a fair degree of accuracy as well as for other tests. It consists of a long base terminating in an enlarged square portion, and supported on three levelling screws. The square part supports a circular degree-divided scale and a metal standard bent over at the top. From this latter hangs a short magnetized needle at the end of a fine silk fibre. To the needle is fixed a light aluminium pointer which moves over the scale, a circular mirror being placed in the centre to avoid errors due to parallax. The moving system is enclosed under a glass shade, and a metre scale is

Fig. 199.

fixed to the long part of the base so as to read distances from the centre of the needle to that of a bar magnet sliding along the metre scale as a straight-edge. It should be noted that the magnetic axis of the bar magnet ought to pass through the centre of the magnetometer needle as it slides along the metre scale.

A more delicate form of magnetometer with its bench is shown in Fig. 200. On a long bench, about 1 inch thick and supported on arms away from the wall with sides vertical, is a bracket at about its centre. A tripod form of magnetometer rests on this bracket, and consists of a highly magnetized magnetic needle, with a plane mirror attached, suspended by a fine fibre from a torsion head at the top of a brass tube fixed to the top of a wooden box protecting the needle. This needle box is closed by a hinged door containing a panel of ordinary "looking-glass," with a circular disc of the silvering removed to enable the moving mirror to be seen. A long fixed metre scale is provided some short distance in front of the magnetometer, and is illuminated by two gas-jets. The reflection of this scale in the moving plane mirror is read by a small telescope under the scale, and the

deflections of the needle thus determined. A simple but special form of sliding table slides along the bench, and can carry the magnet, for

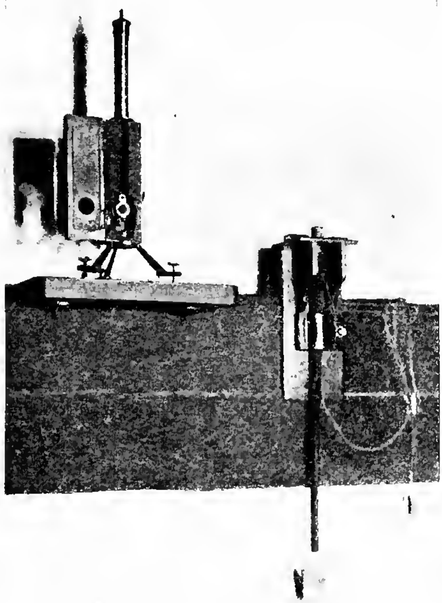


FIG. 200.

which the quantity $\frac{M}{H}$ may be required, on the top, or a long magnetizing solenoid, as used in Ewing's permeability tests, in a clamp at the side, as shown in Fig. 200.

Secondary Battery.

Portable E.M.F.'s up to the value of 12 volts or more are often needed in testing work, and in some cases are required to give an appreciable current as well. Fig. 201 shows an easily portable arrangement, consisting of a stout box containing six Headland electric secondary cells, each capable of giving about 5 amps. at 2 volts pressure for several hours, and weighing only some 5 lbs. each complete. This "make" of cell is most suited to the work because

of its extreme lightness and longevity for a given output and rough usage respectively. The six cells are in simple series, and each junction is joined by a lead strip to a terminal, of which there are consequently seven, three being the other side. Hence any number of

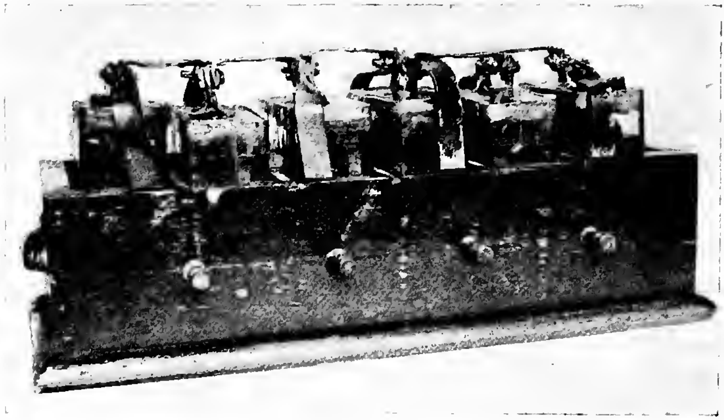


FIG. 201.

cells from one to six can easily be obtained. It may be pointed out that the cells are rendered sufficiently non-spillable by running melted paraffin wax on to the acid surface, with a cork inserted at one corner to act as a bung-hole.

Thermo E.M.F.

Sometimes, in experimental work, a very small and constant E.M.F. is required having a low internal resistance. This, with the exception of the latter condition, can easily be obtained by a potentiometer arrangement in the usual way. Fig. 202 shows a simple piece of apparatus in the form of a thermo-couple, which will give a low E.M.F., and which has a low internal resistance. It consists of two copper cans supported side by side on two brackets, and connected together by a rod of bismuth soldered to them, as shown. Each can contains water, the left cold and the right hot, and boiled if necessary by a Bunsen burner underneath, as shown. A thermometer dips into each can, and the two cans are electrically connected to the terminals on the base by means of wires. Thus, with boiling water one side (100° C.) and cold the other (about 10° C.), an E.M.F. of 3600 microvolts, or 0·0036 volt, with an internal resistance of a small fraction

of an ohm, can easily be obtained. The constancy of the E.M.F. depends on that of the difference of temperature of the two cans.

It is convenient to have some form of cell for experimental purposes in which the effective sizes or areas of the plates and their distance apart can easily be altered. A good form of cell for this purpose is that shown in Fig. 203. It is merely a special form of Daniell cell. The copper plate can slide up or down in the clamp, and also horizontally along the two guide-rods. The zinc plate can only slide up or down in its porous pot, containing either $ZnSO_4$ or dilute H_2SO_4, and be clamped at any desired height. The copper plate dips into a trough of copper sulphate. In this way both the amount of plate immersed and their distance apart can be varied within wide limits, and the E.M.F. and internal resistance experimentally investigated at each position.

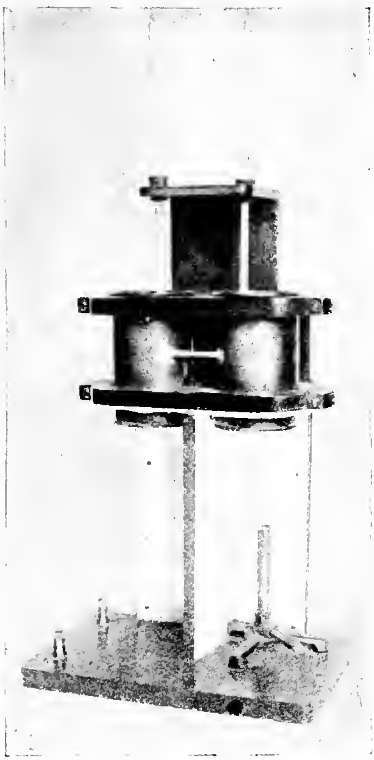


FIG. 202.

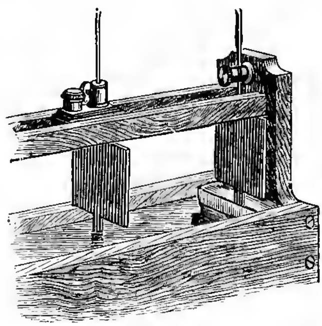


FIG. 203.

Standards of E.M.F.

The two principal kinds of *standard cells* used in this country, and indeed in a large proportion of the Continent as well, are the Board of Trade (B.O.T.) Clark cell, devised and patented by Dr. Muirhead,

and the Carhart-Clark cell, devised by Professor H. S. Carhart. The general outside appearance of a double cell of this latter type is shown in Fig. 204, and consists of two distinct Carhart-Clark cells, mounted in a brass containing case fitted with an ebonite top carrying the two pairs of terminals, one pair for each cell. A delicate thermometer, having a range from 0° to 30° C., has its bulb placed inside the brass case close to the two cells, and its stem bent round horizontally and secured to the ebonite cover. As seen, the two zinc terminals are placed one side and the two mercury terminals the other side of the stem. The Carhart-Clark standard cell attains the

FIG. 204.

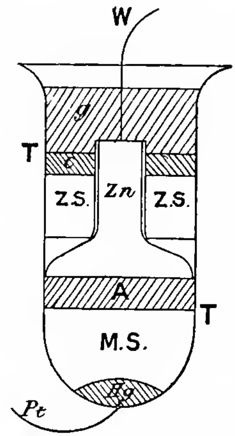


FIG. 205.

E.M.F. corresponding to its temperature almost instantaneously. Its temperature coefficient is only 0·038 per cent. per degree Centigrade, as against 0·078 per cent. in the B.O.T. form. It is thoroughly portable, and can be handled with considerable impunity.

The construction of a Carhart cell is shown in Fig. 205. It consists of a test-tube, T, having a hemispherical closed end, through which is fused a thin platinum wire, Pt; a fairly large globule of pure mercury, Hg, is then placed at the bottom of T. The usual mercurous sulphate paste, M.S, is then inserted over the mercury, and an asbestos disc or diaphragm, A, placed on top. Zn is a circular zinc rod of the form shown, which is pressed down so as to bed the paste M.S all round the mercury Hg. The ordinary zinc sulphate liquid paste, Z S, is then inserted; then a cork, *c*; and finally a sealing, *g*, of marine glue or other

suitable material ; a copper wire, W, soldered to the zinc rod, forms the Zn terminal of the cell. As the zinc Zn is in solution of varying density, it will be acted on at the upper portions, to prevent which Professor Carhart protects the zinc rod by a glass tube in the upper portions. The portability is attained by the paste M.S hemming in the globule of mercury all round the upper parts. A cell of this description, if used with only the feeblest current, never short-circuited or exposed to great variations of temperature, will have an E.M.F. of 1·4345 volt at 15° C., and its E.M.F. E at other temperatures $t°$ C., will be given as $E = \{1{\cdot}4345(1 - 0{\cdot}00038)(t - 15°)\}$ volts.

The Board of Trade cell, otherwise the ordinary Latimer-Clark standard cell, has an E.M.F. which is taken by the B.O.T. to be 1·4345 legal volts at 15° C. ; the cell consists of zinc or an amalgam of zinc with mercury, and of mercury in a neutral saturated solution of zinc sulphate, and mercurous sulphate in water, with mercurous sulphate in excess. The cell is portable, rapidly attains the E.M.F. corresponding to its temperature, and has a temperature coefficient of 0·078 per cent. per degree Centigrade. A detailed description of the manufacture and preparation of this cell will be found on p. 107. The principle use of such standards of E.M.F. is to balance an unknown E.M.F. in a potentiometer arrangement, such as Clark's, etc., in order to determine the value of the unknown. All such cells should be used in series with high resistances while balancing, which may temporarily be removed when balance is almost obtained, so as to increase the sensitiveness of the arrangement.

Standard Daniell cells and numbers of others have been devised by various authors, but the above two types will be the most frequently met with by far.

Thermo-Electric Generator.

Fig. 206 shows a thermo-electric generator designed by Mr. H. Barringer Cox, and capable of giving a current 3·5 amps. on short circuit and 4·5 volts on open circuit, and having an internal resistance of about 1·2 ohms under working conditions.

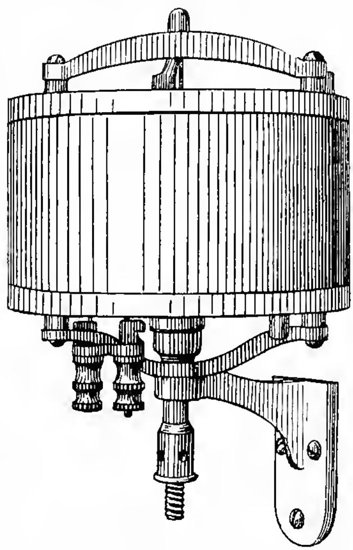
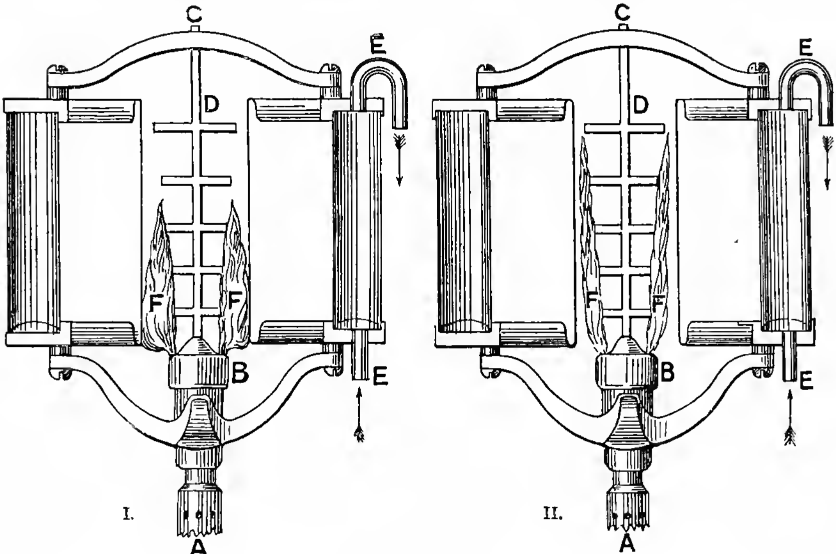


FIG. 206.

It consists of a very large number of pairs, or couples, as they are usually termed, of dissimilar metallic strips connected in series, but arranged

so that one set of alternate junctions are at the inner surface of a hollow cylindrical vessel, and the other alternate set of junctions on the outer edge. Thus the lengths of the strips run radially. The outer junctions are kept cool by ordinary tap-water flowing through the water-jacket which surrounds them, and forming the outside of the generator ; the inner junctions are heated by a central flame from a Bunsen burner, which is guided close to the inner walls of the generator by a "deflector" of the form shown in Fig. 207. Thus one set of junctions being heated and the other cooled, a potential difference is set up between the extreme ends of the series which are connected to the two terminals shown, and marked positive and negative. The generator



I. II.

FIG. 207.

should be fixed to the wall by means of the bracket to which it is attached, and in a situation free from draughts of air ; referring to Fig. 207, connect the gas-burner A by small lead piping to a source of supply, and the water-tubes EE of the water-jacket to a water-tap in a similar manner, so that the cold water enters at the bottom inlet and leaves by the top outlet. The deflector rod D must rest in the centre of the burner B. *To start the generator* working, turn on the water, and obtain a small gentle stream from the overflow ; then light the gas and see that a flame similar in appearance to that in Fig. 207, I. is obtained. After the generator has thus been running for about 10 minutes, feel the outside of it to see if it feels cool ; if otherwise, slightly increase the flow of water, of which *only a small quantity is necessary.*

The temperature of the generator should be about 70° to 80° Fahr., but never above 150° Fahr. *To stop the generator*, turn off the gas, and a few minutes later the water, but *always allow this to remain in the machine.*

Precautions to be observed.—See that the flame is like that shown in Fig. 207, I., and not as in Fig. 207, II., which is dangerous to the generator. The correct flame can be observed by looking up from underneath, and can be obtained by regulating the amount of gas. On no account must the machine be used without the stream of water, and the flame must never be allowed to strike or impinge against the inside wall of the machine.

Electrolytic Resistance Cells.

A somewhat rough but extremely useful piece of apparatus for use with the method of measuring the electrolytic conductivity of electrolytes, devised by Drs. W. Stroud and J. B. Henderson, is shown in Fig. 208.

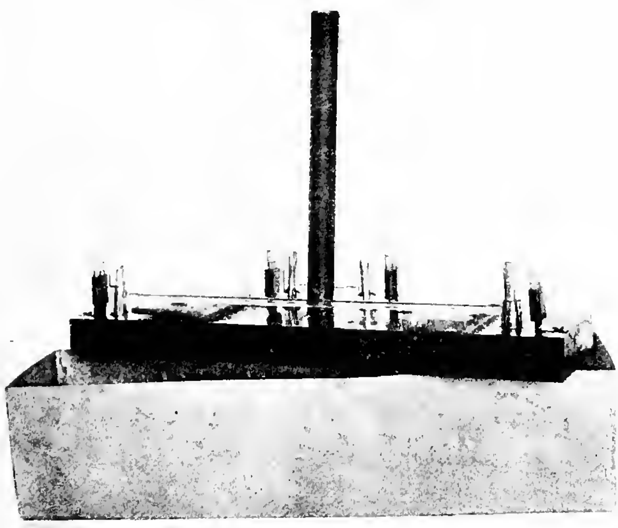


Fig. 208.

It consists of a rectangular metal trough containing paraffin oil, in which is immersed a thick, heavy rectangular board fitted with a vertical rod or handle, by means of which the board can be gently lifted up and down in the oil, so as to ensure uniformity of temperature

throughout the oil. Supported on this wooden board, by being let into it, are two electrolytic cells of special form, devised by the authors of the method Each cell consists of two vertical thick-walled test-tubes, T, T, about 6 cms. high and about 1·25 cm. diameter, having necks, *n*, halfway up their sides. Into these necks fit the well-ground ends of a glass tube, *t*, of nearly uniform bore. With the exception that in one cell *t* is about 30 cms. long and in the other only about 5 or 6 cms., the two cells are as nearly alike in all other respects as it is possible to get them. The horizontal tubes *t* are about 0·6 cm. diameter externally, and have a bore that will give a convenient resistance with the electrolyte tested. Exactly similar electrodes, consisting of platinum foil bent in a cylindrical form so as to fit the tubes T, are employed, a short piece of platinum wire being welded to each, and making contact with an adjacent U-shaped mercury cup, seen by the side of each vertical tube T. By this means the electrodes are easily removable. The connections to the circuit are made with the remaining limb of each U-tube mercury cup. It is found necessary to use oil in the bath, as with water the apparent resistance of the electrolyte depends on the direction of the current owing to leakage over the surface of the glass and through the water. This, however, is non-existent when an insulating liquid is used in the bath. The temperature of the bath, and therefore of the electrolyte, is read by a delicate thermometer.

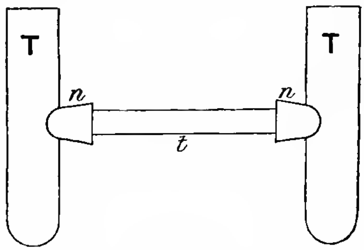


FIG. 209.

Kohlrausch Bridge.

Fig. 210 represents a specially arranged piece of apparatus for measuring the electrolytic resistance of electrolytes by means of alternating currents, and is known as a Kohlrausch bridge. A description of it will be more easily understood by reference to the symbolical diagram of it shown in Fig. 211, in which all dotted lines are electrical connections between the various terminals (indicated by large circles) under the base. The arrangement is essentially that of a Wheatstone bridge, and is as follows :—

The stretched wire, AB (about 24 cms. long), together with the resistance coil, *r*, fixed under the base, form two arms of the bridge, or, strictly speaking, as shown, CB is one arm, and CA + *r* the other. R is a third arm, and perforce the electrolytic cell the fourth arm, which goes across terminals E_1, E_2. S is a two way lever switch

controlling the battery, which is connected across terminals B_1, B_2. When a telephone is used which is connected across G_1G_2, the switch S is put to stud marked "Tel." This inserts an alternating current obtained from an induction coil, I, across the bridge. If, however, it

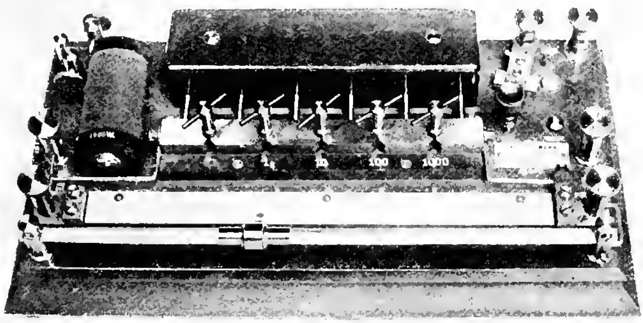


FIG. 210.

is desired to employ a galvanometer instead across G_1G_2, then S is put over to "Galv." This cuts out I, and simply puts the battery direct on to the bridge. R is a set of resistance coils of the values in ohms shown, which can be inserted by taking out the plugs shown.

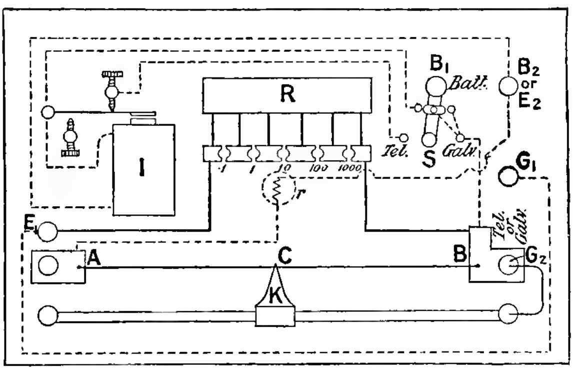


FIG. 211.

A sliding contact or key, K, can make contact at any point (such as C) on the wire AB, which is stretched over a graduated scale, and its position thereby noted when balance is obtained on the telephone or galvanometer.

The scale is graduated in ohms, so that when balance is *obtained the scale-reading of* K × R = *the resistance measured.* Fig. 212 shows a convenient form of electrolytic cell for use with the Kohlrausch bridge. It consists of a glass tube, about 2 cms. diameter, closed at the top and bottom by indiarubber corks, c, c.

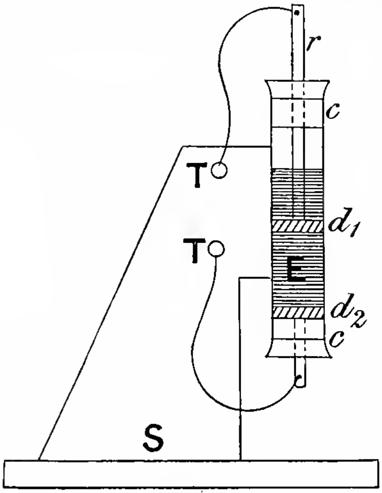


FIG. 212.

Through these corks pass suitable metallic rods, r, one end of each being connected to terminals, T, T, on the stand, S. The other ends terminate in circular metal discs, d_1, d_2, fitting the tube closely, of which d_2 is temporarily fixed during a test, while d_1 can be moved up and down to some convenient distance from d_2.

The temperature of the solution can be obtained very approximately by means of a delicate thermometer, either by passing it through the cork at c by the side of r, and then allowing its bulb to dip into the solution *above* the top disc, or by measuring its temperature just before a series of

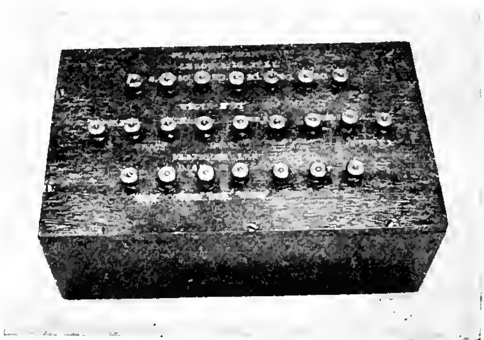


FIG. 213.

tests and just after, and taking the mean. The temperature will probably be found to alter very little in the brief interval of the test.

For liquid resistances not greater than 1000 ohms roughly,

Drs. Stroud and Henderson found that the Kohlrausch method was greatly improved and made more sensitive by using their special form of balancing electrolytic cell (*vide* p. 343) instead of such as the above. By means of it dead silence could be obtained in the telephone, while without their cell there was always a feeble buzz to be heard. For resistances above 1000 ohms, the balancing cell appeared of no avail.

An arrangement which will be found useful in testing the relation between the resistance of a conductor and its length, diameter, and the

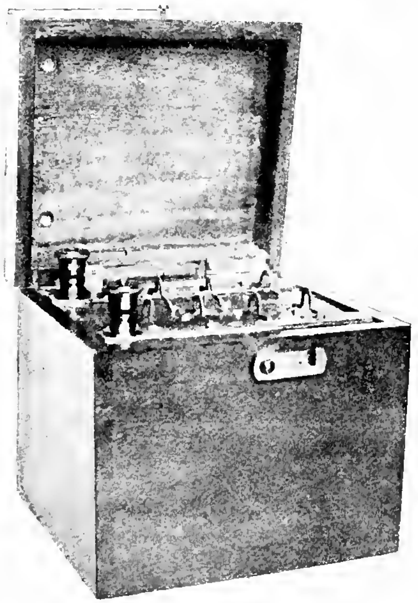


FIG. 214.

material of which it is made, is shown in Fig. 213. It consists of a box containing a number of coils of insulated wire connected severally to the pairs of terminals seen on the top. The top row of terminals is connected to coils of the same material and diameter, but different lengths, the bottom row to coils of the same material and length, but different diameters, whilst the middle row are coils of the same diameter and length, but different materials. In this way a most instructive comparison can be made.

Fig. 214 shows a box containing four coils of insulated wire arranged for proving the laws of combination of resistances in series

and parallel. Referring to Fig. 215, which represents a symbolical plan of the top or mercury switchboard, two terminals, T, T, are fixed to the ends of two copper bars, each containing four mercury cups in a line. Four coils, A, B, C, and D, are connected respectively to four pairs of mercury cups, 1 and 5, 2 and 6, 3 and 7, and 4 and 8. All the cups are so spaced that all the copper wire connectors will just connect cups 1, 2, 3, or 4 to the top terminal bar cups, the same

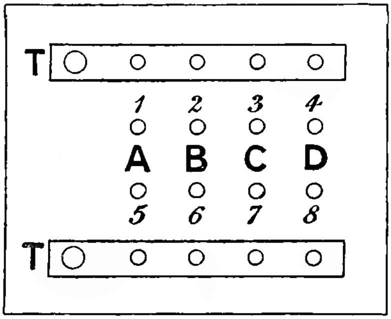


FIG. 215.

the other side, and in addition will connect 2 and 5, or 1 and 6, 3 and 6, and so on. Thus to put—

A and B in series, connect 1 to bar, 2 to 5, and 6 to other bar.

A and D in series, connect 1 to bar, 2 to 5, 3 to 6, 4 to 7, and 8 to other bar.

A and B in parallel, connect 1 and 2 to bar, and 5 and 6 to other bar.

A, B, C, and D in parallel, connect 1, 2, 3, and 4 to one bar, and 5, 6, 7, and 8 to the other, and so on.

Permeability Ring.

For experimenting on the magnetic qualities of iron, for instance, ballistically, it is desirable and convenient to have a neatly wound specimen at all times ready to test in the laboratory for students. Fig. 216 shows such an arrangement for use with Rowland's ballistic method. It merely consists of, say, a welded soft wrought-iron ring neatly and uniformly overwound with a suitable gauge of insulated copper wire, connected to the pair of terminals seen at the top. A short, uniformly wound coil of thin insulated copper wire is wound

over a part of the first coil, and its ends connected to the bottom pair

Fig. 216.

Fig. 217.

of terminals. This is the "search coil," and the other the "magnetizing coil." The ring is cleated down to a mahogany baseboard.

Damping Coil.

Fig. 217 represents a very simple piece of apparatus known as a "damping coil," by the use of which, in the vicinity of a galvanometer, an enormous amount of time can be saved in bringing the needle and spot of light quickly to rest, especially in ballistic work. It merely consists of a flat bobbin wound with fine insulated wire and connected to the two terminals shown, mounted, together with the coil, on a small wooden base. A spring clamp is attached to the back of this, which slips, when pushed, up or down a rod having a suitable foot. The coil contains no iron or other magnetic material, but when used near a galvanometer in conjunction with a single cell and key, it acts magnetically on the needle, and by tapping the key so as to make circuit at the right instant, the spot of light can be brought to rest in about two swings. A reversing key used with the coil expedites matters considerably.

Temperature Coefficient.

Fig. 218 shows a piece of apparatus for determining the variation of resistance of different metals with temperature, and hence their temperature coefficient. It consists of an outer copper vessel containing

water, and supported on a tripod with a Bunsen burner underneath. An inner copper vessel is immersed in this outer vessel, and contains a

FIG. 218.

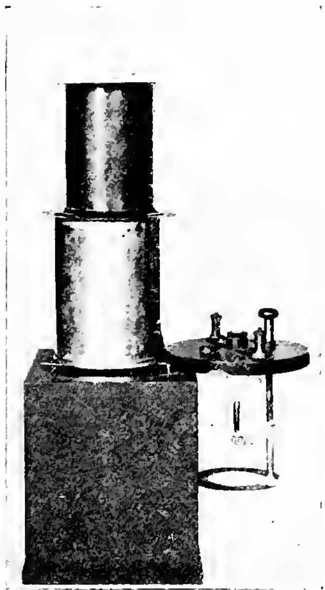


FIG. 219.

hollow frame of insulating material, over which is wound one or more coils of insulated wire of different materials, their ends being connected to the respective pairs of terminals on the lid or cover to which the frame is fixed. A thermometer dips into the centre of the inner vessel, which is provided with a stirrer, for the purpose of preserving uniformity of temperature of the liquid (usually oil) in which the coils and frame are immersed.

Electro-Calorimeter.

An electro-calorimeter for measuring the amount of heat developed by an electric current in a given time and obtaining "Joule's equivalent," is seen in Fig. 219. It consists of a hollow cubical box from the

top of which hangs, quite freely inside, a tin can by its flange. Inside this tin can hangs quite freely a copper can, the flanges of the two cans being separated by a felt ring. The copper can is closed by a wooden cover carrying two terminals connected to a coil, which is heated by the passage of the current. A thermometer, very finely divided over a short range, passes through the lid into the water contained in the copper can. A stirrer of the form shown is provided in order to enable a uniformity of temperature to be obtained throughout the water. It will thus be seen that very little heat is lost through conduction, and, suitable precautions being taken, not much is lost by radiation from the copper calorimeter.

Condensers.

Fig. 220 shows a general view of the Kelvin standard air Leyden condenser, and Figs. 221 and 222 a plan and sectional elevation of the

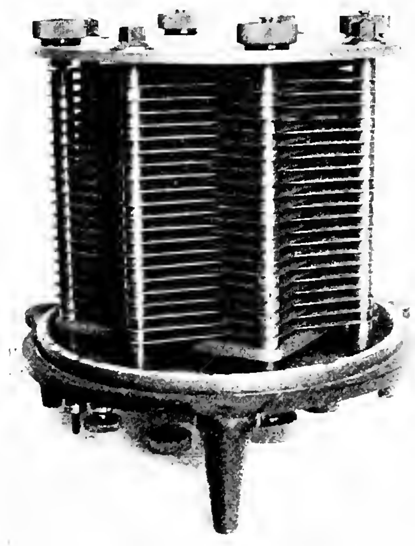


FIG. 220.

same. The instrument is formed by two mutually insulated metallic pieces, which we shall call A and B, constituting the two systems of the air condenser, or Leyden. The systems are composed of parallel

plates, each set bound together by four long metal bolts. The two extreme plates of set A are circles of much thicker metal than the rest, which are all squares of thin sheet brass. The set B are all squares, the bottom one of which is of much thicker metal than the others, and the plates of this system are one less in number than the plates of system A. The four bolts binding together the plates of each system pass through well-fitted holes in the corners of the squares; and the distance from plate to plate of the same set is

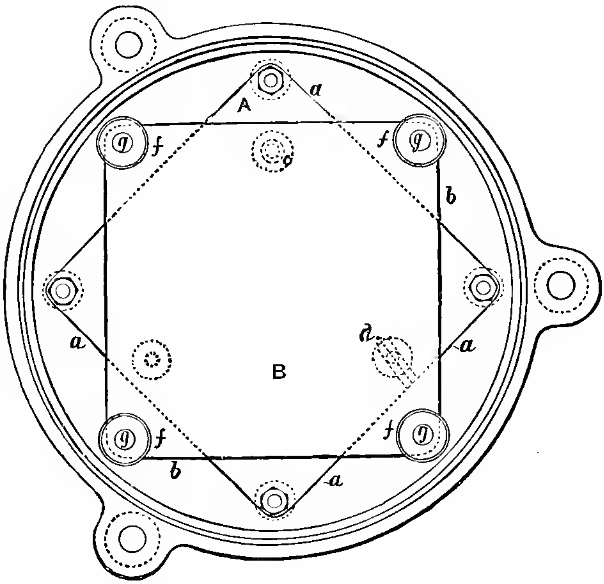


FIG 221.

regulated by annular distance pieces, which are carefully made to fit the bolt, and are made exactly the same in all respects. Each system is bound firmly together by screwing home nuts on the ends of the bolts, and thus the parallelism and rigidity of the entire set is secured.

The two systems are made up together, so that every plate of B is between two plates of A, and every plate of A, except the two end ones, which only present one face to those of the opposite set, is between two plates of B. When the instrument is set up for use, the system B rests by means of the well-known "hole, slot, and plane arrangement,"[1] engraved on the under side of its bottom plate, on three glass columns, which are attached to three metal screws working

[1] Thomson and Tait's "Natural Philosophy," § 198, example 3.

through the sole plate of system A. These screws can be raised or lowered at pleasure, and by means of a gauge the plates of system B can be adjusted to exactly midway between, and parallel to, the plates of system A. The complete Leyden stands upon three vulcanite feet attached to the lower side of the sole plate of system A.

In order that the instrument may not be injured in carriage, an arrangement, described as follows, is provided, by which system B can be lifted from off the three glass columns and firmly clamped to the top and bottom plates of system A.

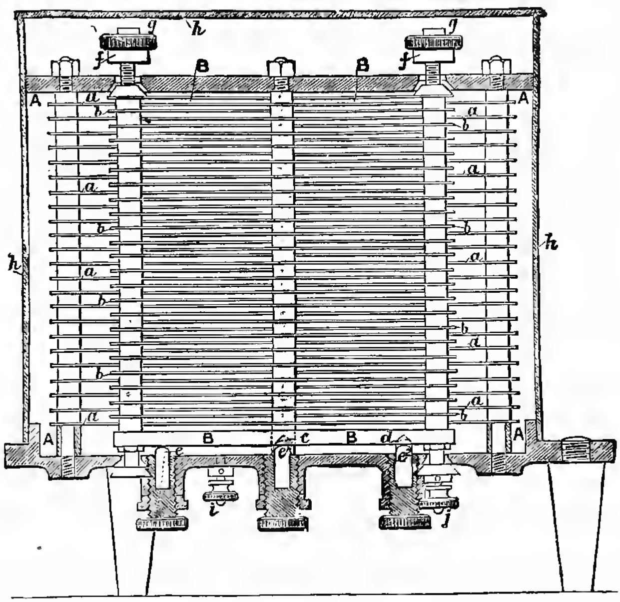


FIG. 222.

The bolts fixing the corners of the plates of system B are made long enough to pass through wide conical holes cut in the top and bottom plates of system A, and the nuts at the top end of the bolts are also conical in form, while conical nuts are also fixed to their lower ends below the base plate of system A. Thumbscrew nuts, *f*, are placed upon the upper ends of the bolts after they pass through the holes in the top plate of system A.

When the instrument is set up ready for use, these thumbscrews are turned up against fixed stops, *g*, so as to be well clear of the top plate of system A; but when the instrument is packed for carriage, they are screwed down against the plate until the conical nuts

mentioned above are drawn up into the conical holes in the top and bottom plates of system A ; system B is thus raised off the glass pillars, and the two systems are securely locked together so as to prevent damage to the instrument.

A dust-tight cylindrical metal case, *h*, which can be easily taken off for inspection, covers the two systems, and fits on to a flange on system A. The whole instrument rests on three vulcanite legs attached to the base plate on system A ; and two terminals are provided, one, *i*, on the base of system A, and the other, *j*, on the end of one of the corner bolts of system B.

Fig. 223 shows a standard adjustable condenser in general view,

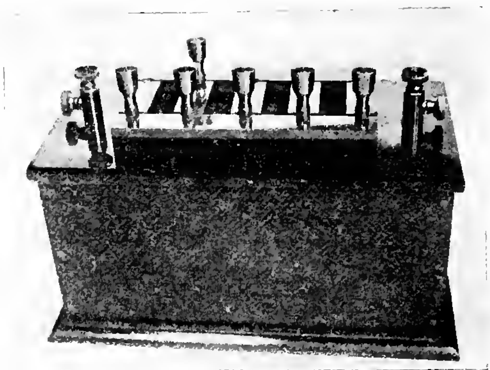


FIG. 223.

and Fig. 224 a plan of the top with the connections of the five condensers inside the box. E and C are the two double terminals of the

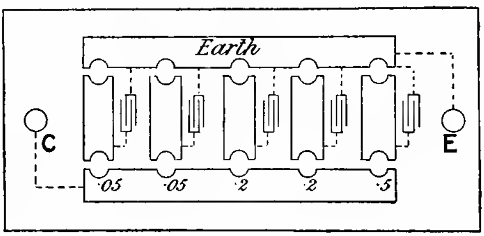


FIG. 224.

condenser, and from the diagrammatic figure it will be noticed that the manipulation of the plugs to insert capacity is just the reverse to that of an ordinary resistance box, in that you plug in the front set of

holes to increase the terminal capacity, since capacities in parallel sum up like resistances in series.

It will also be seen that a plug inserted in a back hole short-circuits that particular condenser, and any of the others that may be in, providing a plug is also in the front hole of the same condenser.

Hughes' Magnetic Balance.

A form of Hughes' magnetic balance, which is useful for rapidly determining the relative magnetic qualities of two or more samples of material, is shown in Fig. 225. It consists of a long bench or table supported on two standards on a base fitted with levelling screws. At one end (the right-hand) of this raised bench is a magnetizing solenoid,

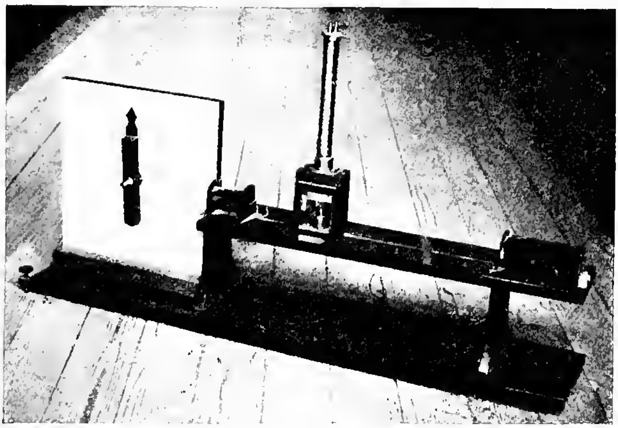


FIG. 225.

in which is inserted the specimen to be tested, and at the other a small balancing solenoid to neutralize the inductive effect of the magnetizing coil by itself on a magnetometer needle suspended between the two coils. A permanent steel compensating magnet (seen to the extreme left) can be turned round a pivot passing through the centre of a vertical scale fixed to the base, and its deflections made to neutralize the various stages of magnetization of the specimen itself. The method is a "zero" one, and the indications or position of the magnetometer needle is read by forming an image of a vertical pin, fixed just in front of the needle box, in the moving plane mirror attached to the needle, which is half covered by a fixed plane mirror immediately in front of it. Thus the zero position of the needle is

when the two images coincide. Professor S. P. Thompson has shown that, when the distance from needle to centre of compensating magnet is 2·3 times the latter's length, the angle through which the magnet is turned (up to 60°) is proportional to the magnetic force due to the iron core or specimen in the magnetizing coil.

Hughes' Induction Balance.

Fig. 226 shows a Hughes' induction balance, together with the interrupter, telephone, and other accessories. It consists of two pairs of flat circular coils supported as shown, one pair at each end of a baseboard. The bottom coil of each pair is permanently fixed on the

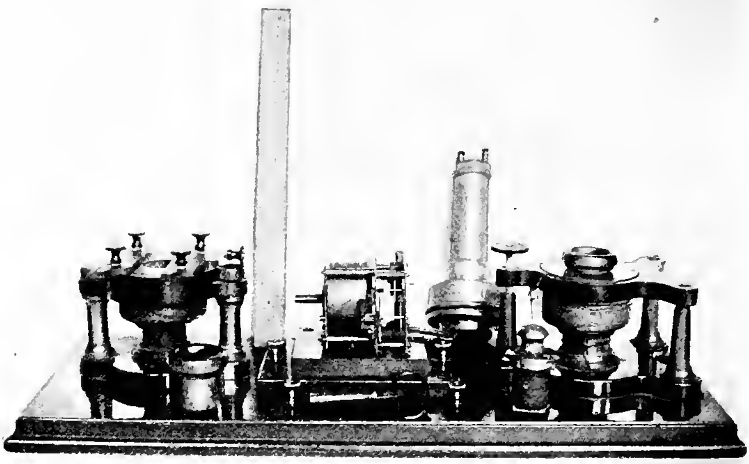


FIG. 226.

top of a short central pedestal. These are in series, and constitute the primary coils of the balance, being connected to two terminals marked P on the baseboard. The other coil of each pair, which is supported close over its respective primary, is a flat circular coil of similar size, but having a hole through its centre large enough to receive a wooden cup as shown. These latter coils, constituting the secondary of the balance, go to terminals S. The left-hand one is permanently fixed on the two pillars seen in the figure, while the right-hand one is supported on three pillars, the two on the extreme right

forming a kind of axis or fulcrum, about which the coil can be raised or lowered by a set screw working on the third pillar being pulled downwards by an elastic band seen on the hook. A current interrupter, consisting of a spring strip pressing against, and making electrical contact with, the teeth of a toothed wheel, which is driven by clock-work, is seen with its case off between the two pairs of coils, and is for the purpose of "making and breaking" the primary circuit in which it is inserted. A Bell's telephone is seen on the right of it, and a long wedge-shaped metal strip of zinc fixed to a millimeter scale on the left. This can slide between the adjustable guides seen on the left-hand top coil. Circular discs of various metals are provided for placing in the right-hand cup. It should be understood that the four coils are as nearly alike in every detail as it is possible to make them, both as regards size, form, wire, number of turns, etc.

FIG. 227.

The Permeameter.

A useful piece of apparatus, known as the "permeameter," is illustrated in Fig. 227, by means of which the magnetic quality of different materials can easily and rapidly be found. The method is essentially a workshop one, and the principle of it is due to Professor S. P. Thompson.

The arrangement consists of a somewhat massive hollow rectangular-shaped block of good soft wrought iron forged to the shape shown. Inside this is a magnetizing solenoid wound on a thin brass tube with thin flanges or ends, its length being just that between the insides of the block ends. The sufficiently long rod of magnetic material to be tested, having its lower end faced quite true, passes freely but closely through the top end of the block, down through the solenoid, and beds on the carefully "faced" inside of the bottom end of the block. The protruding end of the rod has a metal pin through it, which is caught by a double hook on the lower end of an ordinary spring balance, the top end of which is suspended by a gut cord passing over a fixed pulley and attached to the lever shown. This permeameter is also fitted with an arrangement for testing the

specimen ballistically. It consists of a small flat coil fixed to a brass plate, which slides backwards and forwards between guides. The rod passes through this coil and beds as before on the block, at the same time keeping the coil back against the force of two spring strips on the outside of the block. Immediately the rod is suddenly pulled up the coil flies out, and a circuit joined to its terminals will receive an electro-magnetic impulse proportional to the field just broken. In this way the ballistic and traction methods can be made to check one another in the final results obtained.

Standard Magneto-Inductor.

In testing-rooms, where there may be masses of iron or dynamos in the immediate vicinity, the value of H, the horizontal component of the earth's magnetic force, may vary considerably from its orthodox

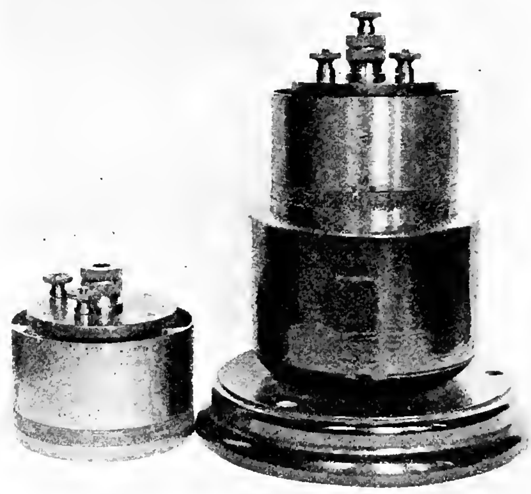


FIG. 228.

value for that year. In such cases the ordinary earth coil, or earth inductor, as it is variously termed, is of little or no use in evaluating ballistic quantities. A simple and neat form of permanent standard magneto inductor has been devised by Mr. W. Hibbert for use in such

cases, and is represented in perspective in Fig. 228, and in symbolical sectional elevation in Fig. 229.

The instrument consists of a hard permanent steel magnet, NS, fixed at the lower end to a highly permeable cast-steel pole-piece, P, of the form of a cup, and at the other end to a flat circular disc, D, of similar material. A circular gap, *g*, is thus formed between D and P, just wide enough to allow a brass tube to slip freely through.

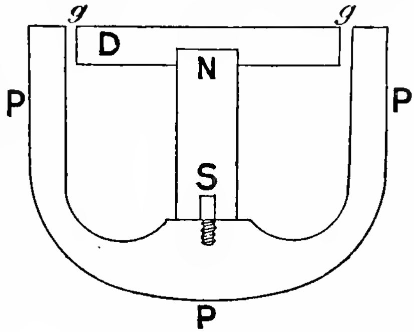


Fig. 229.

A coil of fine wire is wound on the tube, which in slipping through the gap cuts the lines of force flowing across, and gives an electro-magnetic impulse to any circuit connected to the coil terminals fixed to the top of the tube. Owing to the pole-pieces forming a nearly closed magnetic circuit, the magnet is quite permanent, and the total lines of force through the gap are constant. This number is usually about 20,000, so that the magneto-inductor is much more convenient than earth coils, which usually are relied on. In Fig. 228 one tube is just shown in position for slipping through the gap, to do which lightly span the milled head on its top and turn slightly, when the tube will drop by its own weight. A spare tube is shown on the left having a different number of turns, which is often very convenient.

Earth Coil or Inductor.

A form of ordinary earth inductor or coil is shown in Fig. 230. It consists of a hollow wooden ring wound with a large number of turns of insulated copper wire, the ends of which are connected to the two spindles which support the coil. These spindles work in bearings carried on the wooden framework, composed of two sides held rigidly together by a back. The two terminals of the inductor shown on the top of this back are connected to the two bearings respectively, and thence to the coil. The advantage of this construction is that the coil will rotate freely, no matter which side the inductor rests on. When resting on its back, or up against a wall, the spring stop seen is pushed out against the spring, and stops the coil rotating. This arrangement, however, is not to be recommended, as it only ends in

damaging the bearings of the coil, thus producing bad contact with the coil.

Fig. 231 shows a model of a two-pole dynamo made for the purpose of roughly investigating the distribution of the magnetic field under various conditions.

It consists of an iron casting forming the two pole-pieces at the top, the limbs at the sides, and yoke at the bottom. The limbs are each wound with a coil connected to the centre and one of the outside terminals seen on the top, and hence either coil can be used separately to magnetize or both together by connecting to the two outside

FIG. 230.

FIG. 231.

terminals. Four iron rings are provided of the same axial thickness, but having different external and internal diameters. These are to represent unwound armature cores of both the *ring* and *drum* types, and the effect on the flow of the lines of force between the pole-pieces by using them will, to a certain extent, show the effect of both the length of air-gap and sectional area of iron in the core on the magnetic distribution generally. The whole is fixed and let into a baseboard, so that a sheet of thin cardboard will lie flat on the top. The rings are held concentrically with the pole-pieces by slipping over a central pin and wooden rings when necessary. The general distribution of the magnetic field can then be found by sprinkling iron filings on the sheet of cardboard laid on the top of this representative machine.

TABLES OF CONSTANTS

TABLE VI.

TEMPERATURE COEFFICIENTS AND SPECIFIC RESISTANCES OF PURE METALS AND ALLOYS DETERMINED BY PROFESSORS J. A. FLEMING AND J. DEWAR.

Metals, pure, soft, and annealed.	Specific resistance, ρ, in microhms, per c.c. at 0° C.	Mean temperature coefficient, α, between 0° and 100° C	Alloys, usual proportions.	Specific resistance, ρ, in microhms, per c.c. at 0° C.	Temperature coefficient, α, at 15° C.
Platinum ...	10·917	0·003669	Platinum-silver...	31·582	0·000243
Gold ...	2·197	0·003770	,, -iridium	30·896	0·000822
Palladium ...	10·219	0·003540	,, -rhodium	21·142	0·00143
Silver ...	1·468	0·004000	Gold-silver ...	6·280	0·00124
Copper ...	1·561	0·004280	Manganese steel	67·148	0·00127
Aluminium ..	2·665	0·004350	Nickel steel ...	29·452	0·00201
Iron[1] ...	9·065	0·006250	German silver ...	29·982	0·000273
Nickel ...	12·323	0·006220	Platinoid ...	41·731	0·000310
Tin	13·048	0·004400	Manganin ...	46·678	0·00000
Magnesium...	4·355	0·003810	Silverene ...	2·064	0·00285
Zinc ...	5·751	0·004060	Aluminium-silver	4·641	0·00238
Cadmium ...	10·023	0·004190	,, -copper	2·904	0·00381
Lead ...	20·380	0·004110	,, -bronze[2]	12·300	0·0010
Thallium ...	17·633	0·003980	Reostene[2] ...	76·468	0·00110
Mercury ...	94·070	0·000720	Brass[2]	7·2	—
			Nickelin[2] ...	38·50	—
			Arc lamp carbon[2]	4400–8600	0·00052

[1] Approximately pure. [2] Not determined by Fleming and Dewar.

TABLE VII.

SPECIFIC HEATS (REGNAULT).

Aluminium	0·20566	Pewter	0·05623
Cobalt	0·10696	Brass	0·09391
Nickel	0·11095	Glass	0·19768
Iron	0·11379	Platinum	0·03243
Zinc	0·09555	Gold	0·03244
Copper	0·09515	Mercury	0·03332
Silver	0·05701	Tin[1]	0·0559
Lead	0·03140	Steel[1]	0·118
Bismuth	0·03084	Iridium[1]	0·0303
Antimony	0·05077	Graphite[1]	0·254–0·467

[1] Not by Regnault.

TABLE VIII.

Electro-chemical Equivalents, Specific Gravities, etc.

Metal.	Atomic weight.	Chemical equivalent $= \frac{\text{at. wt.}}{\text{valency}}$	Electro-chemical equivalent, grams per coulomb.	Weight in grams per hour deposited by 1 amp.	Specific gravity, grams per c.c.
Aluminium ...	27·0	9·0	0·00009317	0·3354	2·6
Copper (monad)	63·1	63·1	0·00065735	2·3665	8·9
,, (dyad) ...	63·1	31·6	0·00032867	1·1832	8·9
Gold	196·7	65·6	0·00067806	2·4410	19·3
Iron (dyad) ...	56	28	0·00028986	1·0435	7·85
Lead	206·4	103·2	0·0010714	3·8571	11·4
Nickel	58·6	29·3	0·00030538	1·0994	8·5
Silver	108	108	0·0011180	4·0249	10·5
Tin (dyad) ...	117·8	58·9	0·00061077	2·1988	7·3
Zinc	65	32·5	0·00033644	1·2112	7·1
Hydrogen ...	1	1	0·000010352	0·03738	0·000090

TABLE IX.

Relation between E.M.F. and Temperature of Clark Standard Cell—made according to Regulations.

Temperature, ° C.	E.M.F. in legal volts.	Temperature, ° C.	E.M.F. in legal volts.	Temperature, ° C.	E.M.F. in legal volts.
6	1·444	13	1·436	20	1·428
7	1·443	14	1·435	21	1·427
8	1·442	15	1·434	22	1·426
9	1·441	16	1·433	23	1·425
10	1·440	17	1·432	24	1·424
11	1·438	18	1·431	25	1·423
12	1·437	19	1·430	26	1·422

TABLE X.

E.M.F. and Internal Resistance of Cells.

Name of cell.	E.M.F., legal volts.	Approximate resistance, legal ohms.
Bichromate	1·75–1·98	About 2
Daniell	1·07–1·140	From 0·5 to 6
Grove	1·76–1·934	From 0·15–0·30
Bunsen	1·73–1·942	About same
Leclanché	1·402–1·650	From about 1·5 to 3
Clark (Standard) ...	1·4340	Almost anything up to 2000
Secondary	2·10	From 0·0002 upwards

TABLE XI.

Relation between Practical and C.G.S. (Absolute) Units.

	Practical units.	Absolute C.G.S. units.		Dimensions.	
		Electro-magnetic.	Electrostatic.	Electro-static.	Electro-magnetic.
Quantity ...	1 coulomb	10^{-1}	$v \times 10^{-1} = 3 \times 10^{9}$	$M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}$	$M^{\frac{1}{2}}L^{\frac{1}{2}}$
Current ...	1 ampere	10^{-1}	$v \times 10^{-1} = 3 \times 10^{9}$	—	—
Potential ...	1 volt	10^{8}	$10^{8} \div v = \frac{1}{3} \times 10^{-2}$	$M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}$	$M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2}$
Resistance ...	1 ohm	10^{9}	$\frac{1}{v^2} \times 10^{9} = \frac{1}{9} \times 10^{-11}$	$L^{-1}T$	LT^{-1}
Capacity ...	1 farad	10^{-9}	$v^2 \times 10^{-9} = 9 \times 10^{11}$	L	$L^{-1}T^{2}$
Self-induction ...	1 secohm	10^{9}	—	—	L
Mutual induction	1 secohm	10^{9}	—	—	L
Power	1 watt	10^{7}	—	—	—
Work	1 joule	10^{7}	—	—	—

v = velocity of light = 3×10^{10} cms. per second.

TABLE XII.

Molecular Conductivity $\frac{\kappa}{\mu}$ of Different Electrolytes at 18° C. in Aqueous Solution containing μ Equivalents (F. Kohlrausch).

μ = strength per litre in gram-equivalents.
κ = conductivity as compared to mercury at 0° C.

	KCl.	NaCl.	HCl.	$\frac{1}{2}K_2SO_4$.	$\frac{1}{2}MgSO_4$.	$\frac{1}{2}H_2SO_4$.
Approximate limiting values.	$10^8\frac{\kappa}{\mu}$ 1230.	$10^8\frac{\kappa}{\mu}$ 1040.	$10^8\frac{\kappa}{\mu}$ 3550.	$10^8\frac{\kappa}{\mu}$ 1290.	$10^8\frac{\kappa}{\mu}$ 1100.	$10^8\frac{\kappa}{\mu}$ 3800.
μ = 0·0001	1215	1026	3500	1125	1034	3580
2	1210	1021	3490	1240	1015	3520
5	1201	1016	3480	1224	976	3440
0·001	1193	1008	3460	1207	935	3350
2	1185	999	3455	1181	881	3250
5	1165	981	3445	1140	790	3050
0·01	1147	962	3416	1098	715	2860
2	1123	938	3390	1044	632	2650
5	1083	897	3330	959	534	2340
0·1	1047	865	3240	897	467	2090
2	1009	826	3140	832	408	1960
5	958	757	3020	736	330	1900
1	917	696	2780	672	271	1820
2	864	604	2340	—	202	1700
5	—	398	1420	—	82	1270
10	—	—	600	—	—	655
20	—	—	—	—	—	147
50	—	—	—	—	—	30

Useful Data.

Base of hyperbolic or Napierian logarithms $\epsilon = 2{\cdot}71828$.

To convert *common into Napierian logarithms*, multiply by 2·30258.

To convert *Napierian into common logarithms*, multiply by 0·43429.

Ratio of circumference of circle to its diameter, $\pi = 3{\cdot}14159$.

$\log \pi = 0{\cdot}49715$.

Angle subtended by arc equal to radius = Radian = $57^\circ\ 17'\ 45''$ $= 57^\circ{\cdot}2958$.

Length of circumference of circle of radius $r = 2\pi r = \pi d$.

Area of circumference of circle of radius $r = \pi r^2 = \frac{\pi d^2}{4} = {\cdot}7854 d^2$.

Metre in inches	39·37043
Foot in centimetres	30·4797
Inch in centimetres	2·54
Square inch in square cms.	6·451
1 lb. (avoir.) in grams	453·593
Equivalent of 1 *watt* in foot-lbs. per second ...	0·7373

1 weber = 3×10^{10} electrostatic C.G.S. units = 10^8 electromagnetic C.G.S. units of induction.

Joule's equivalent = 1390 lb.-cent. units = $4{\cdot}156 \times 10^7$ ergs per gram ° C.

Revolution per second in radians per second, 6·2832.

Acceleration due to gravity, g, at Greenwich in ft.-seconds, 32·1908.

A bismuth-copper element junctions at 0° C. and 100° C., E.M.F. = 0·005476 volt.

Density of mercury, 13·596 grms. per c.c.

" platinum, 21·45 " "

LOGARITHMS, ANTILOGARITHMS, ETC.

LOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0044	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7

LOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	7479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

ANTILOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
·01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
·02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
·03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
·04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
·05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
·06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
·07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
·08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
·09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
·10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
·11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
·12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
·13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
·14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
·15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
·16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
·17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
·18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
·19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
·20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
·21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
·22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
·23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
·24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
·25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
·26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
·27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
·28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
·29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
·30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
·31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
·32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
·33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
·34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
·35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
·36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
·37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
·38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
·39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
·40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
·41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
·42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
·43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
·44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
·45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
·46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
·47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
·48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
·49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6

ANTILOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
·51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
·52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
·53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
·54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
·55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
·56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
·57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
·58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
·59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
·60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
·61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
·62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
·63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
·64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
·65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
·66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
·67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
·68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
·69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
·70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
·71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
·72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
·73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
·74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
·75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
·76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
·77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
·78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
·79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
·80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
·81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
·82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
·83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
·84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
·85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
·86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
·87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
·88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
·89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	12	14	16
·90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
·91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
·92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
·93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
·94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
·95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
·96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
·97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
·98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
·99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

NATURAL SINES.

	·0°	·1°	·2°	·3°	·4°	·5°	·6°	·7°	·8°	·9°
0°	0000	0017	0035	0052	0070	0087	0105	0122	0140	0157
1	0175	0192	0209	0227	0244	0262	0279	0297	0314	0332
2	0349	0366	0384	0401	0419	0436	0454	0471	0488	0506
3	0523	0541	0558	0576	0593	0610	0628	0645	0663	0680
4	0698	0715	0732	0750	0767	0785	0802	0819	0837	0854
5	0871	0889	0906	0924	0941	0958	0976	0993	1011	1028
6	1045	1063	1080	1097	1115	1132	1149	1167	1184	1201
7	1219	1236	1253	1271	1288	1305	1323	1340	1357	1374
8	1392	1409	1426	1444	1461	1478	1495	1513	1530	1547
9	1564	1582	1599	1616	1633	1650	1668	1685	1702	1719
10	1736	1754	1771	1788	1805	1822	1840	1857	1874	1891
11	1908	1925	1942	1959	1977	1994	2011	2028	2045	2062
12	2079	2096	2113	2130	2147	2164	2181	2198	2215	2232
13	2250	2267	2284	2300	2317	2334	2351	2368	2385	2402
14	2419	2436	2453	2470	2487	2504	2521	2538	2554	2571
15	2588	2605	2622	2639	2656	2672	2689	2706	2723	2740
16	2756	2773	2790	2807	2823	2840	2857	2874	2890	2907
17	2924	2940	2957	2974	2990	3007	3024	3040	3057	3074
18	3090	3107	3123	3140	3156	3173	3190	3206	3223	3239
19	3256	3272	3289	3305	3322	3338	3355	3371	3387	3404
20	3420	3437	3453	3469	3486	3502	3518	3535	3551	3567
21	3584	3600	3616	3633	3649	3665	3681	3697	3714	3730
22	3746	3762	3778	3795	3811	3827	3843	3859	3875	3891
23	3907	3923	3939	3955	3971	3987	4003	4019	4035	4051
24	4067	4083	4099	4115	4131	4147	4163	4179	4195	4210
25	4226	4242	4258	4274	4289	4305	4321	4337	4352	4368
26	4384	4399	4415	4431	4446	4462	4478	4493	4509	4524
27	4540	4555	4571	4586	4602	4617	4633	4648	4664	4679
28	4695	4710	4726	4741	4756	4772	4787	4802	4818	4833
29	4848	4863	4879	4894	4909	4924	4939	4955	4970	4985
30	5000	5015	5030	5045	5060	5075	5090	5105	5120	5135
31	5150	5165	5180	5195	5210	5225	5240	5255	5270	5284
32	5299	5314	5329	5344	5358	5373	5388	5402	5417	5432
33	5446	5461	5476	5490	5505	5519	5534	5548	5563	5577
34	5592	5606	5621	5635	5650	5664	5678	5693	5707	5721
35	5736	5750	5764	5779	5793	5807	5821	5835	5850	5864
36	5878	5892	5906	5920	5934	5948	5962	5976	5990	6004
37	6018	6032	6046	6060	6074	6088	6101	6115	6129	6143
38	6157	6170	6184	6198	6211	6225	6239	6252	6266	6280
39	6293	6307	6320	6334	6347	6361	6374	6388	6401	6414
40	6428	6441	6455	6468	6481	6494	6508	6521	6534	6547
41	6561	6574	6587	6600	6613	6626	6639	6652	6665	6678
42	6691	6704	6717	6730	6743	6756	6769	6782	6794	6807
43	6820	6833	6845	6858	6871	6884	6896	6909	6921	6934
44	6947	6959	6972	6984	6997	7009	7022	7034	7046	7059

NATURAL SINES.

	·0°	·1°	·2°	·3°	·4°	·5°	·6°	·7°	·8°	·9°
45°	7071	7083	7096	7108	7120	7133	7145	7157	7169	7181
46	7193	7206	7218	7230	7242	7254	7266	7278	7290	7302
47	7314	7325	7337	7349	7361	7373	7385	7396	7408	7420
48	7431	7443	7455	7466	7478	7490	7501	7513	7524	7536
49	7547	7558	7570	7581	7593	7604	7615	7627	7638	7649
50	7660	7672	7683	7694	7705	7716	7727	7738	7749	7760
51	7771	7782	7793	7804	7815	7826	7837	7848	7859	7869
52	7880	7891	7902	7912	7923	7934	7944	7955	7965	7976
53	7986	7997	8007	8018	8028	8039	8049	8059	8070	8080
54	8090	8100	8111	8121	8131	8141	8151	8161	8171	8181
55	8192	8202	8211	8221	8231	8241	8251	8261	8271	8281
56	8290	8300	8310	8320	8329	8339	8348	8358	8368	8377
57	8387	8396	8406	8415	8425	8434	8443	8453	8462	8471
58	8480	8490	8499	8508	8517	8526	8536	8545	8554	8563
59	8572	8581	8590	8599	8607	8616	8625	8634	8643	8652
60	8660	8669	8678	8686	8695	8704	8712	8721	8729	8738
61	8746	8755	8763	8771	8780	8788	8796	8805	8813	8821
62	8829	8838	8846	8854	8862	8870	8878	8886	8894	8902
63	8910	8918	8926	8934	8942	8949	8957	8965	8973	8980
64	8988	8996	9003	9011	9018	9026	9033	9041	9048	9056
65	9063	9070	9078	9085	9092	9100	9107	9114	9121	9128
66	9135	9143	9150	9157	9164	9171	9178	9184	9191	9198
67	9205	9212	9219	9225	9232	9239	9245	9252	9259	9265
68	9272	9278	9285	9291	9298	9304	9311	9317	9323	9330
69	9336	9342	9348	9354	9361	9367	9373	9379	9385	9391
70	9397	9403	9409	9415	9421	9426	9432	9438	9444	9449
71	9455	9461	9466	9472	9478	9483	9489	9494	9500	9505
72	9511	9516	9521	9527	9532	9537	9542	9548	9553	9558
73	9563	9568	9573	9578	9583	9588	9593	9598	9603	9608
74	9613	9617	9622	9627	9632	9636	9641	9646	9650	9655
75	9659	9664	9668	9673	9677	9681	9686	9690	9694	9699
76	9703	9707	9711	9716	9720	9724	9728	9732	9736	9740
77	9744	9748	9751	9755	9759	9763	9767	9770	9774	9778
78	9781	9785	9789	9792	9796	9799	9803	9806	9810	9813
79	9816	9820	9823	9826	9829	9833	9836	9839	9842	9845
80	9848	9851	9854	9857	9860	9863	9866	9869	9871	9874
81	9877	9880	9882	9885	9888	9890	9893	9895	9898	9900
82	9903	9905	9907	9910	9912	9914	9917	9919	9921	9923
83	9925	9928	9930	9932	9934	9936	9938	9940	9942	9943
84	9945	9947	9949	9951	9952	9954	9956	9957	9959	9960
85	9962	9963	9965	9966	9968	9969	9971	9972	9973	9974
86	9976	9977	9978	9979	9980	9981	9982	9983	9984	9985
87	9986	9987	9988	9989	9990	9990	9991	9992	9993	9993
88	9994	9995	9995	9996	9996	9997	9997	9997	9998	9998
89	9998	9999	9999	9999	9999	9999	9999	9999	9999	9999

NATURAL TANGENTS.

	·0°	·1°	·2°	·3°	·4°	·5°	·6°	·7°	·8°	·9°
0°	·0000	0017	0035	0052	0070	0087	0105	0122	0140	0157
1	·0175	0192	0209	0227	0244	0262	0279	0297	0314	0332
2	·0349	0367	0384	0402	0419	0437	0454	0472	0489	0507
3	·0524	0542	0559	0577	0594	0612	0629	0647	0664	0682
4	·0699	0717	0734	0752	0769	0787	0805	0822	0840	0857
5	·0875	0892	0910	0928	0945	0963	0981	0998	1016	1033
6	·1051	1069	1086	1104	1122	1139	1157	1175	1192	1210
7	·1228	1246	1263	1281	1299	1317	1334	1352	1370	1388
8	·1405	1423	1441	1459	1477	1495	1512	1530	1548	1566
9	·1584	1602	1 20	1638	1655	1673	1691	1709	1727	1745
10	·1763	1781	1799	1817	1835	1853	1871	1890	1908	1926
11	·1944	1962	1980	1998	2016	2035	2053	2071	2089	2107
12	·2126	2144	2162	2180	2199	2217	2235	2254	2272	2290
13	·2309	2327	2345	2364	2382	2401	2419	2438	2456	2475
14	·2493	2512	2530	2549	2568	2586	2605	2623	2642	2661
15	·2679	2698	2717	2736	2754	2773	2792	2811	2830	2849
16	·2867	2886	2905	2924	2943	2962	2981	3000	3019	3038
17	·3057	3076	3096	3115	3134	3153	3172	3191	3211	3230
18	·3249	3269	3288	3307	3327	3346	3365	3385	3404	3424
19	·3443	3463	3482	3502	3522	3541	3561	3581	3600	3620
20	·3640	3659	3679	3699	3719	3739	3759	3779	3799	3819
21	·3839	3859	3879	3899	3919	3939	3959	3979	4000	4020
22	·4040	4061	4081	4101	4122	4142	4163	4183	4204	4224
23	·4245	4265	4286	4307	4327	4348	4369	4390	4411	4431
24	·4452	4473	4494	4515	4536	4557	4578	4599	4621	4642
25	·4663	4684	4706	4727	4748	4770	4791	4813	4834	4856
26	·4877	4899	4921	4942	4964	4986	5003	5029	5051	5073
27	·5095	5117	5139	5161	5184	5206	5228	5250	5272	5295
28	·5317	5340	5362	5384	5407	5430	5452	5475	5498	5520
29	·5543	5566	5589	5612	5635	5658	5681	5704	5727	5750
30	·5774	5797	5820	5844	5867	5890	5914	5938	5961	5985
31	·6009	6032	6056	6080	6104	6128	6152	6176	6200	6224
32	·6249	6273	6297	6322	6346	6371	6395	6420	6445	6469
33	·6494	6519	6544	6569	6594	6619	6644	6669	6694	6720
34	·6745	6771	6796	6822	6847	6873	6899	6924	6950	6976
35	·7002	7028	7054	7080	7107	7133	7159	7186	7212	7239
36	·7265	7292	7319	7346	7373	7400	7427	7454	7481	7508
37	·7536	7563	7590	7618	7646	7673	7701	7729	7757	7785
38	·7813	7841	7869	7898	7926	7954	7983	8012	8040	8069
39	·8098	8127	8156	8185	8214	8243	8273	8302	8332	8361
40	·8391	8421	8451	8481	8511	8541	8571	8601	8632	8662
41	·8693	8724	8754	8785	8816	8847	8878	8910	8941	8972
42	·9004	9036	9067	9099	9131	9163	9195	9228	9260	9293
43	·9325	9358	9391	9424	9457	9490	9523	9556	9590	9623
44	·9657	9691	9725	9759	9793	9827	9861	9896	9930	9965

NATURAL TANGENTS.

	·0°	·1°	·2°	·3°	·4°	·5°	·6°	·7°	·8°	·9°
45°	1·0000	0035	0070	0105	0141	0176	0212	0247	0283	0319
46	1·0355	0392	0428	0464	0501	0538	0575	0612	0649	0686
47	1·0724	0761	0799	0837	0875	0913	0951	0990	1028	1067
48	1·1106	1145	1184	1224	1263	1303	1343	1383	1423	1463
49	1·1504	1544	1585	1626	1667	1708	1750	1792	1833	1875
50	1·1918	1960	2002	2045	2088	2131	2174	2218	2261	2305
51	1·2349	2393	2437	2482	2527	2572	2617	2662	2708	2753
52	1·2799	2846	2892	2938	2985	3032	3079	3127	3175	3222
53	1·3270	3319	3367	3416	3465	3514	3564	3613	3663	3713
54	1·3764	3814	3865	3916	3968	4019	4071	4124	4176	4229
55	1·4281	4335	4388	4442	4496	4550	4605	4659	4715	4770
56	1·4826	4882	4938	4994	5051	5108	5166	5224	5282	5340
57	1·5399	5458	5517	5577	5637	5697	5757	5818	5880	5941
58	1·6003	6066	6128	6191	6255	6319	6383	6447	6512	6577
59	1·6643	6709	6775	6842	6909	6977	7045	7113	7182	7251
60	1·7321	7391	7461	7532	7603	7675	7747	7820	7893	7966
61	1·8040	8115	8190	8265	8341	8418	8495	8572	8650	8728
62	1·8807	8887	8967	9047	9128	9210	9292	9375	9458	9542
63	1·9626	9711	9797	9883	9970	$\bar{0}057$	$\bar{0}145$	$\bar{0}233$	$\bar{0}323$	$\bar{0}413$
64	2·0503	0594	0686	0778	0872	0965	1060	1155	1251	1348
65	2·1445	1543	1642	1742	1842	1943	2045	2148	2251	2355
66	2·2460	2566	2673	2781	2889	2998	3109	3220	3332	3445
67	2·3559	3673	3789	3906	4023	4142	4262	4383	4504	4627
68	2·4751	4876	5002	5129	5257	5386	5517	5649	5782	5916
69	2·6051	6187	6325	6464	6605	6746	6889	7034	7179	7326
70	2·7475	7625	7776	7929	8083	8239	8397	8556	8716	8878
71	2·9042	9208	9375	9544	9714	9887	$\bar{0}061$	$\bar{0}237$	$\bar{0}415$	$\bar{0}595$
72	3·0777	0961	1146	1334	1524	1716	1910	2106	2305	2506
73	3·2709	2914	3122	3332	3544	3759	3977	4197	4420	4646
74	3·4874	5105	5339	5576	5816	6059	6305	6554	6806	7062
75	3·7321	7583	7848	8118	8391	8667	8947	9232	9520	9812
76	4·0108	0408	0713	1022	1335	1653	1976	2303	2635	2972
77	4·3315	3662	4015	4374	4737	5107	5483	5864	6252	6646
78	4·7046	7453	7867	8288	8716	9152	9594	$\bar{0}045$	$\bar{0}504$	$\bar{0}970$
79	5·1446	1929	2422	2924	3435	3955	4486	5026	5578	6140
80	5·6713	7297	7894	8502	9124	9758	$\bar{0}405$	$\bar{1}066$	$\bar{1}742$	$\bar{2}432$
81	6·3138	3859	4596	5350	6122	6912	7920	8548	9395	$\bar{0}264$
82	7·1154	2066	3002	3962	4947	5958	6996	8062	9158	$\bar{0}285$
83	8·1443	2636	3863	5126	6427	7769	9152	0579	$\bar{2}052$	$\bar{3}572$
84	9·5144	9·677	9·845	10·02	10·20	10·39	10·58	10·78	10·99	11·20
85	11·43	11·66	11·91	12·16	12·43	12·71	13·00	13·30	13·62	13·95
86	14·30	14·67	15·06	15·46	15·89	16·35	16·83	17·34	17·89	18·46
87	19·08	19·74	20·45	21·20	22·02	22·90	23·86	24·90	26·03	27·27
88	28·64	30·14	31·82	33·69	35·80	38·19	40·92	44·07	47·74	52·08
89	57·29	63·66	71·62	81·85	95·49	114·6	143·2	191·0	286·5	573·0

SQUARES OF NUMBERS FROM 1 TO 10000, CORRECT TO FOUR SIGNIFICANT FIGURES.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	1000	1020	1040	1061	1082	1102	1124	1145	1166	1188	3	5	7	9	11	13	15	17	19
11	1210	1232	1254	1277	1300	1323	1346	1369	1392	1416	3	5	7	10	12	14	17	19	21
12	1440	1464	1488	1513	1538	1563	1588	1613	1638	1664	3	5	8	10	13	15	18	20	23
13	1690	1716	1742	1769	1796	1823	1850	1877	1904	1932	3	6	9	11	14	17	19	22	25
14	1960	1988	2016	2045	2074	2103	2132	2161	2190	2220	3	6	9	12	15	18	21	24	27
15	2250	2280	2310	2341	2372	2403	2434	2465	2496	2528	4	7	10	13	16	19	22	25	28
16	2560	2592	2624	2657	2690	2723	2756	2789	2822	2856	4	7	10	14	17	20	24	27	30
17	2890	2924	2958	2993	3028	3063	3098	3133	3168	3204	4	7	11	14	18	21	25	28	32
18	3240	3276	3312	3349	3386	3423	3460	3497	3534	3572	4	8	12	15	19	23	26	30	34
19	3610	3648	3686	3725	3764	3803	3842	3881	3920	3960	4	8	12	16	20	24	28	32	36
20	4000	4040	4080	4121	4162	4203	4244	4285	4326	4368	5	9	13	17	21	25	29	33	37
21	4410	4452	4494	4537	4580	4623	4666	4709	4752	4796	5	9	13	18	22	26	31	34	39
22	4840	4884	4928	4973	5018	5063	5108	5153	5198	5244	5	9	14	18	23	27	32	36	41
23	5290	5336	5382	5429	5476	5523	5570	5617	5664	5712	5	10	15	19	24	27	33	38	43
24	5760	5808	5856	5905	5954	6003	6052	6101	6150	6200	5	10	15	20	25	30	35	40	45
25	6250	6300	6350	6401	6452	6503	6554	6605	6656	6708	6	11	16	21	26	31	36	41	46
26	6760	6812	6864	6917	6970	7023	7076	7129	7182	7236	6	11	16	22	27	32	38	43	48
27	7290	7344	7398	7453	7508	7563	7618	7673	7728	7784	6	11	17	22	28	33	39	44	50
28	7840	7896	7952	8009	8066	8123	8180	8237	8294	8352	6	12	18	23	29	35	40	46	52
29	8410	8468	8526	8585	8644	8703	8762	8821	8880	8940	6	12	18	24	30	36	42	48	54
30	9000	9060	9120	9181	9242	9303	9364	9425	9486	9548	7	13	19	25	31	37	43	49	55
31	9610	9672	9734	9797	9860	9923	9986	1005*	1011*	1018*	7	13	19	26	32	38	45	51	57
32	1024	1030	1037	1043	1050	1056	1063	1069	1076	1082	1	1	2	3	3	4	5	5	6
33	1089	1096	1102	1109	1116	1122	1129	1136	1142	1149	1	1	2	3	4	4	5	6	6
34	1156	1163	1170	1176	1183	1190	1197	1204	1211	1218	1	2	2	3	4	4	5	6	6
35	1225	1232	1239	1246	1253	1260	1267	1273	1282	1289	1	2	2	3	4	4	5	6	7
36	1296	1303	1310	1318	1325	1332	1340	1347	1354	1362	1	2	2	3	4	5	5	6	7
37	1369	1376	1384	1391	1399	1406	1414	1421	1429	1436	1	2	2	3	4	5	5	6	7
38	1444	1452	1459	1467	1475	1482	1490	1498	1505	1513	1	2	2	3	4	5	6	6	7
39	1521	1529	1537	1544	1552	1560	1568	1576	1584	1592	1	2	3	3	4	5	6	6	7
40	1600	1608	1616	1624	1632	1640	1648	1656	1665	1673	1	2	3	3	4	5	6	7	7
41	1681	1689	1697	1706	1714	1722	1731	1739	1747	1756	1	2	3	3	4	5	6	7	8
42	1764	1772	1781	1789	1798	1806	1815	1823	1832	1840	1	2	3	4	4	5	6	7	8
43	1849	1858	1866	1875	1884	1892	1901	1910	1918	1927	1	2	3	4	5	5	6	7	8
44	1936	1945	1954	1962	1971	1980	1989	1998	2007	2016	1	2	3	4	5	5	6	7	8
45	2025	2034	2043	2052	2061	2070	2079	2088	2098	2107	1	2	3	4	5	6	7	7	8
46	2116	2125	2134	2144	2153	2162	2172	2181	2190	2200	1	2	3	4	5	6	7	8	9
47	2209	2218	2228	2237	2247	2256	2266	2275	2285	2294	1	2	3	4	5	6	7	8	9
48	2304	2314	2323	2333	2343	2352	2362	2372	2381	2391	1	2	3	4	5	6	7	8	9
49	2401	2411	2421	2430	2440	2450	2460	2470	2480	2490	1	2	3	4	5	6	7	8	9
50	2500	2510	2520	2530	2540	2550	2560	2570	2581	2591	1	2	3	4	5	6	7	8	9
51	2601	2611	2621	2632	2642	2652	2663	2673	2683	2694	1	2	3	4	5	6	7	8	9
52	2704	2714	2725	2735	2746	2756	2767	2777	2788	2798	1	2	3	4	5	6	8	9	10
53	2809	2820	2830	2841	2852	2862	2873	2884	2894	2905	1	2	3	4	6	7	8	9	10
54	2916	2927	2938	2948	2959	2970	2981	2992	3003	3014	1	2	3	5	6	7	8	9	10

Squares from 1 to 3 contain 1 figure.
,, ,, 4 to 9 ,, 2 figures.
,, ,, 10 to 31 ,, 3 ,,
,, ,, 32 to 99 ,, 4 ,,

Squares from 100 to 316 contain 5 figures.
,, ,, 317 to 999 ,, 6 ,,
,, ,, 1000 to 3162 ,, 7 ,,
,, ,, 3163 to 10000 ,, 8 ,,

* The differences for squares from 3171 to 3199 are 1, 1, 2, 3, 3, 4, 5, 5, 6.

SQUARES OF NUMBERS FROM 1 TO 10000, CORRECT TO FOUR SIGNIFICANT FIGURES.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
55	3025	3036	3047	3058	3069	3080	3091	3102	3114	3125	1	2	3	5	6	7	8	9	10
56	3136	3147	3158	3170	3181	3192	3204	3215	3226	3238	1	2	4	5	6	7	8	9	10
57	3249	3260	3272	3283	3295	3306	3318	3329	3341	3352	1	2	4	5	6	7	8	9	11
58	3364	3376	3387	3399	3411	3422	3434	3446	3457	3469	1	2	4	5	6	7	8	10	11
59	3481	3493	3505	3516	3528	3540	3552	3564	3576	3588	1	3	4	5	6	7	8	10	11
60	3600	3612	3624	3636	3648	3660	3672	3684	3697	3709	1	3	4	5	6	7	9	10	11
61	3721	3733	3745	3758	3770	3782	3795	3807	3819	3832	1	3	4	5	6	8	9	10	11
62	3844	3856	3869	3881	3894	3906	3919	3931	3944	3956	1	3	4	5	6	8	9	10	11
63	3969	3982	3994	4007	4020	4032	4045	4058	4070	4083	1	3	4	5	7	8	9	10	12
64	4096	4109	4122	4134	4147	4160	4173	4186	4199	4212	1	3	4	5	7	8	9	10	12
65	4225	4238	4251	4264	4277	4290	4303	4316	4330	4343	1	3	4	5	7	8	9	11	12
66	4356	4369	4382	4396	4409	4422	4436	4449	4462	4476	1	3	4	5	7	8	9	11	12
67	4489	4502	4516	4529	4543	4556	4570	4583	4597	4610	2	3	4	6	7	8	10	11	12
68	4624	4638	4651	4665	4679	4692	4706	4720	4733	4747	2	3	4	6	7	8	10	11	12
69	4761	4775	4789	4802	4816	4830	4844	4858	4872	4886	2	3	4	6	7	8	10	11	13
70	4900	4914	4928	4942	4956	4970	4984	4998	5013	5027	2	3	4	6	7	9	10	11	13
71	5041	5055	5069	5084	5098	5112	5127	5141	5155	5170	2	3	4	6	7	9	10	12	13
72	5184	5198	5213	5227	5242	5256	5271	5285	5300	5314	2	3	5	6	7	9	10	12	13
73	5329	5344	5358	5373	5388	5402	5417	5432	5446	5461	2	3	5	6	8	9	10	12	13
74	5476	5491	5506	5520	5535	5550	5565	5580	5595	5610	2	3	5	6	8	9	11	12	14
75	5625	5640	5655	5670	5685	5700	5715	5730	5746	5761	2	3	5	6	8	9	11	12	14
76	5776	5791	5806	5822	5837	5852	5868	5883	5898	5914	2	3	5	6	8	9	11	12	14
77	5929	5944	5960	5975	5991	6006	6022	6037	6053	6068	2	3	5	6	8	9	11	13	14
78	6084	6100	6115	6131	6147	6162	6178	6194	6209	6225	2	3	5	6	8	10	11	13	14
79	6241	6257	6273	6288	6304	6320	6336	6352	6368	6384	2	3	5	7	8	10	11	13	14
80	6400	6316	6432	6448	6464	6480	6496	6512	6529	6545	2	3	5	7	8	10	11	13	15
81	6561	6577	6593	6610	6626	6642	6659	6675	6691	6708	2	3	5	7	8	10	12	13	15
82	6724	6740	6757	6773	6790	6806	6823	6839	6856	6872	2	3	5	7	8	10	12	13	15
83	6889	6906	6922	6939	6956	6972	6989	7006	7022	7039	2	3	5	7	9	10	12	14	15
84	7056	7073	7090	7106	7123	7140	7157	7174	7191	7208	2	4	5	7	9	10	12	14	15
85	7225	7242	7259	7276	7293	7310	7327	7344	7362	7379	2	4	5	7	9	10	12	14	16
86	7396	7413	7430	7448	7465	7482	7500	7517	7534	7552	2	4	5	7	9	11	12	14	16
87	7569	7586	7604	7621	7639	7656	7674	7691	7709	7726	2	4	5	7	9	11	12	14	16
88	7744	7762	7779	7797	7815	7832	7850	7868	7885	7903	2	4	5	7	9	11	13	14	16
89	7921	7939	7957	7974	7992	8010	8028	8046	8064	8082	2	4	6	7	9	11	13	14	16
90	8100	8118	8136	8154	8172	8190	8208	8226	8245	8263	2	4	6	7	9	11	13	15	16
91	8281	8299	8317	8336	8354	8372	8391	8409	8427	8446	2	4	6	7	9	11	13	15	17
92	8464	8482	8501	8519	8538	8556	8575	8593	8612	8630	2	4	6	8	9	11	13	15	17
93	8649	8668	8686	8705	8724	8742	8761	8780	8798	8817	2	4	6	8	10	11	13	15	17
94	8836	8855	8874	8892	8911	8930	8949	8968	8987	9006	2	4	6	8	10	11	13	15	17
95	9025	9044	9063	9082	9101	9120	9139	9158	9178	9197	2	4	6	8	10	12	14	15	17
96	9216	9235	9254	9274	9293	9312	9332	9351	9370	9390	2	4	6	8	10	12	14	16	18
97	9409	9428	9448	9467	9487	9506	9526	9545	9565	9584	2	4	6	8	10	12	14	16	18
98	9604	9624	9643	9663	9683	9702	9722	9742	9761	9781	2	4	6	8	10	12	14	16	18
99	9801	9821	9841	9860	9880	9900	9920	9940	9960	9980	2	4	6	8	10	12	14	16	18

Squares from 1 to 3 contain 1 figure.
,, ,, 4 to 9 ,, 2 figures.
,, ,, 10 to 31 ,, 3 ,,
,, ,, 32 to 99 ,, 4 ,,

Squares from 100 to 316 contain 5 figures.
,, ,, 317 to 999 ,, 6 ,,
,, ,, 1000 to 3162 ,, 7 ,,
,, ,, 3163 to 10000 ,, 8 ,,

RECIPROCALS OF NUMBERS FROM 1 TO 9999.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	$\bar{9}901$	$\bar{9}804$	$\bar{9}709$	$\bar{9}615$	$\bar{9}524$	$\bar{9}434$	$\bar{9}346$	$\bar{9}259$	$\bar{9}174$	9	18	27	36	45	54	63	72	81
11	9091	9009	8929	8850	8772	8696	8621	8547	8475	8403	8	16	23	31	38	46	53	61	68
12	8333	8264	8197	8130	8065	8000	7937	7874	7813	7752	6	13	19	26	32	38	45	51	57
13	7692	7634	7576	7519	7463	7407	7353	7299	7246	7194	5	11	16	22	27	32	38	43	49
14	7143	7092	7042	6993	6944	6897	6849	6803	6757	6711	5	9	14	19	23	28	33	37	42
15	6667	6623	6579	6536	6494	6452	6410	6369	6329	6289	5	9	13	17	21	25	29	34	38
16	6250	6211	6173	6135	6098	6061	6024	5988	5952	5917	4	8	11	15	19	22	26	30	33
17	5882	5848	5814	5780	5747	5714	5682	5650	5618	5587	3	6	10	13	16	19	23	26	29
18	5556	5525	5495	5464	5435	5405	5376	5348	5319	5291	3	6	8	11	14	17	20	23	26
19	5263	5236	5208	5181	5155	5128	5102	5076	5051	5025	3	5	8	10	13	16	18	21	23
20	5000	4975	4950	4926	4902	4878	4854	4831	4808	4785	2	5	7	10	12	14	17	19	21
21	4762	4739	4717	4695	4673	4651	4630	4608	4587	4566	2	4	6	9	11	13	15	17	19
22	4545	4525	4505	4484	4464	4444	4425	4405	4386	4367	2	4	6	8	10	11	13	15	17
23	4348	4329	4310	4292	4274	4255	4237	4219	4202	4184	2	3	5	7	9	11	12	14	16
24	4167	4149	4132	4115	4098	4082	4065	4049	4032	4016	2	4	5	7	9	10	12	14	15
25	4000	3984	3968	3953	3937	3921	3906	3891	3876	3861	1	3	4	6	7	9	10	12	13
26	3846	3831	3817	3802	3788	3774	3759	3745	3731	3717	1	2	4	5	7	8	9	11	12
27	3704	3690	3676	3663	3650	3636	3623	3610	3597	3584	1	2	4	5	6	8	9	10	12
28	3571	3559	3546	3534	3521	3509	3497	3484	3472	3460	2	3	4	5	6	8	9	10	11
29	3448	3436	3425	3413	3401	3390	3378	3367	3356	3344	1	3	4	5	6	7	8	9	11
30	3333	3322	3311	3300	3289	3279	3268	3257	3247	3237	1	3	4	5	6	7	8	9	10
31	3226	3215	3205	3195	3185	3175	3165	3155	3145	3135	2	3	4	5	6	7	8	9	10
32	3125	3115	3106	3096	3086	3077	3067	3058	3049	3040	1	2	3	4	5	6	7	8	9
33	3030	3021	3012	3003	2994	2985	2976	2967	2959	2950	1	2	3	4	4	5	6	7	8
34	2941	2933	2924	2915	2907	2899	2890	2882	2874	2865	0	1	2	3	4	5	5	6	7
35	2857	2849	2841	2833	2825	2817	2809	2801	2793	2785	1	2	3	3	4	5	6	7	7
36	2778	2770	2762	2755	2747	2740	2732	2725	2717	2710	1	2	3	3	4	5	6	6	7
37	2703	2695	2688	2681	2674	2667	2660	2653	2646	2639	1	2	3	3	4	5	5	6	7
38	2632	2625	2618	2611	2604	2597	2591	2584	2577	2571	0	1	2	2	3	4	4	5	6
39	2564	2558	2551	2545	2538	2532	2525	2519	2513	2506	1	2	2	3	4	4	5	6	6
40	2500	2494	2488	2481	2475	2469	2463	2457	2451	2445	1	1	2	2	3	4	4	5	5
41	2439	2433	2427	2421	2415	2410	2404	2398	2392	2387	1	2	2	3	3	4	5	5	6
42	2381	2375	2370	2364	2358	2353	2347	2342	2336	2331	1	1	2	2	3	3	4	5	5
43	2326	2320	2315	2309	2304	2299	2294	2288	2283	2278	1	1	2	2	3	3	4	4	5
44	2273	2268	2262	2257	2252	2247	2242	2237	2232	2227	0	1	1	2	2	3	3	4	4
45	2222	2217	2212	2208	2203	2198	2193	2188	2183	2179	1	1	2	2	3	3	4	4	5
46	2174	2169	2165	2160	2155	2151	2146	2141	2137	2132	0	0	1	1	2	2	3	3	4
47	2128	2123	2119	2114	2110	2105	2101	2096	2092	2088	0	1	1	2	2	2	3	3	4
48	2083	2079	2075	2070	2066	2062	2058	2053	2049	2045	1	1	2	2	2	3	3	4	4
49	2041	2037	2033	2028	2024	2020	2016	2012	2008	2004	0	1	1	2	2	2	3	3	4
50	2000	1996	1992	1988	1984	1980	1976	1972	1969	1965	0	1	1	1	2	2	3	3	3
51	1961	1957	1953	1949	1946	1942	1938	1934	1931	1927	1	1	1	2	2	3	3	3	4
52	1923	1919	1916	1912	1908	1905	1901	1898	1894	1890	1	1	1	2	2	3	3	3	4
53	1887	1883	1880	1876	1873	1869	1866	1862	1859	1855	0	1	1	1	2	2	2	3	3
54	1852	1848	1845	1842	1838	1835	1832	1828	1825	1821	1	1	1	2	2	2	3	3	3

Reciprocals from 2 to 10 = 0·
" " 11 to 100 = 0·0

Reciprocals from 101 to 1000 = 0·00
" " 1001 to 9999 = 0·000

NOTE.—Numbers in difference columns to be subtracted, not added.

RECIPROCALS OF NUMBERS FROM 1 TO 9999.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
55	1818	1815	1812	1808	1805	1802	1799	1795	1792	1789	1	1	1	2	2	2	3	3	3
56	1786	1783	1779	1776	1773	1770	1767	1764	1761	1757	1	1	1	1	2	2	2	3	3
57	1754	1751	1748	1745	1742	1739	1736	1733	1730	1727	0	1	1	1	1	2	2	2	3
58	1724	1721	1718	1715	1712	1709	1706	1704	1701	1698	0	0	1	1	1	1	2	2	2
59	1695	1692	1689	1686	1684	1681	1678	1675	1672	1669	1	1	1	2	2	2	2	3	3
60	1667	1664	1661	1658	1656	1653	1650	1647	1645	1642	0	1	1	1	2	2	2	2	3
61	1639	1637	1634	1631	1629	1626	1623	1621	1618	1616	0	1	1	1	1	2	2	2	2
62	1613	1610	1608	1605	1603	1600	1597	1595	1592	1590	0	1	1	1	1	2	2	2	2
63	1587	1585	1582	1580	1577	1575	1572	1570	1567	1565	1	1	1	1	2	2	2	2	3
64	1563	1560	1558	1555	1553	1550	1548	1546	1543	1541	0	0	0	1	1	1	1	2	2
65	1538	1536	1534	1531	1529	1527	1524	1522	1520	1517	1	1	1	1	2	2	2	2	2
66	1515	1513	1511	1508	1506	1504	1502	1499	1497	1495	1	1	1	1	1	2	2	2	2
67	1493	1490	1488	1486	1484	1481	1479	1477	1475	1473	0	0	0	0	1	1	1	1	2
68	1471	1468	1466	1474	1462	1460	1458	1456	1453	1451	0	1	1	1	1	2	2	2	2
69	1449	1447	1445	1443	1441	1439	1437	1435	1433	1431	0	1	1	1	1	1	2	2	2
70	1428	1427	1425	1422	1420	1418	1416	1414	1412	1410	0	0	0	0	1	1	1	1	1
71	1408	1406	1404	1403	1401	1399	1397	1395	1393	1391	1	1	1	1	1	2	2	2	2
72	1389	1387	1385	1383	1381	1379	1377	1376	1374	1372	0	0	0	1	1	1	1	1	1
73	1370	1368	1366	1364	1362	1361	1359	1357	1355	1353	0	0	0	0	0	1	1	1	1
74	1351	1350	1348	1346	1344	1342	1340	1339	1337	1335	0	0	0	1	1	1	1	1	1
75	1333	1332	1330	1328	1326	1325	1323	1321	1319	1317	0	0	0	0	0	1	1	1	1
76	1316	1314	1312	1311	1309	1307	1305	1304	1302	1300	0	0	0	1	1	1	1	1	1
77	1299	1297	1295	1294	1292	1290	1289	1287	1285	1284	0	0	0	0	1	1	1	1	1
78	1282	1280	1279	1277	1276	1274	1272	1271	1269	1267	0	1	1	1	1	1	1	2	2
79	1266	1264	1263	1261	1259	1258	1256	1255	1253	1252	0	1	1	1	1	1	1	2	2
80	1250	1248	1247	1245	1244	1242	1241	1239	1238	1236	0	0	0	0	1	1	1	1	1
81	1235	1233	1232	1230	1229	1227	1225	1224	1222	1221	0	0	1	1	1	1	1	1	1
82	1220	1218	1217	1215	1214	1212	1211	1209	1208	1206	0	0	0	1	1	1	1	1	1
83	1205	1203	1202	1200	1199	1198	1196	1195	1193	1192	1	1	1	1	1	1	1	2	2
84	1190	1189	1188	1186	1185	1183	1182	1181	1179	1178	0	0	0	0	0	1	1	1	1
85	1176	1175	1174	1172	1171	1170	1168	1167	1166	1164	0	0	0	0	0	0	0	1	1
86	1163	1161	1160	1159	1157	1156	1155	1153	1152	1151	0	0	0	1	1	1	1	1	1
87	1149	1148	1147	1145	1144	1143	1142	1140	1139	1138	0	1	1	1	1	1	1	1	1
88	1136	1135	1134	1133	1131	1130	1129	1127	1126	1125	0	0	1	1	1	1	1	1	1
89	1124	1122	1121	1120	1119	1117	1116	1115	1114	1112	0	0	0	0	0	1	1	1	1
90	1111	1110	1109	1107	1106	1105	1104	1103	1101	1100	0	0	0	1	1	1	1	1	1
91	1099	1098	1096	1095	1094	1093	1092	1091	1089	1088	0	0	1	1	1	1	1	1	1
92	1087	1086	1085	1083	1082	1081	1080	1079	1078	1076	0	0	0	0	1	1	1	1	1
93	1075	1074	1073	1072	1071	1070	1068	1067	1066	1065	0	0	0	0	0	0	0	0	1
94	1064	1063	1062	1060	1059	1058	1057	1056	1055	1054	0	0	0	0	0	1	1	1	1
95	1053	1052	1050	1049	1048	1047	1046	1045	1044	1043	0	0	0	0	1	1	1	1	1
96	1042	1041	1040	1038	1037	1036	1035	1034	1033	1032	0	0	0	0	0	0	1	1	1
97	1031	1030	1029	1028	1027	1026	1025	1024	1022	1021	1	1	1	1	1	1	1	1	1
98	1020	1019	1018	1017	1016	1015	1014	1013	1012	1011	0	0	0	0	0	0	1	1	1
99	1010	1009	1008	1007	1006	1005	1004	1003	1002	1001	0	0	0	0	1	1	1	1	1

Reciprocals from 2 to 10 = 0·
,, ,, 11 to 100 = 0·0
Reciprocals from 101 to 1000 = 0·00
,, ,, 1001 to 9999 = 0·000

NOTE.—Numbers in difference columns to be subtracted, not added.

TABLE OF DOUBLED SQUARE ROOTS FOR

	0	100	200	300	400	500	600	700	800	900	
0	0·000	20·00	28·28	34·64	40·00	44·72	48·99	52·92	56·57	60·00	0
1	2·000	20·10	28·35	34·70	40·05	44·77	49·03	52·95	56·60	60·03	1
2	2·828	20·20	28·43	34·76	40·10	44·81	49·07	52·99	56·64	60·07	2
3	3·464	20·30	28·50	34·81	40·15	44·86	49·11	53·03	56·67	60·10	3
4	4·000	20·40	28·57	34·87	40·20	44·90	49·15	53·07	56·71	60·13	4
5	4·472	20·49	28·64	34·93	40·25	44·94	49·19	53·10	56·75	60·17	5
6	4·899	20·59	28·71	34·99	40·30	44·99	49·23	53·14	56·78	60·20	6
7	5·292	20·69	28·77	35·04	40·35	45·03	49·27	53·18	56·82	60·23	7
8	5·657	20·78	28·84	35·10	40·40	45·08	49·32	53·22	56·85	60·27	8
9	6·000	20·88	28·91	35·16	40·45	45·12	49·36	53·25	56·89	60·30	9
10	6·325	20·98	28·98	35·21	40·50	45·17	49·40	53·29	56·92	60·33	10
11	6·633	21·07	29·05	35·27	40·55	45·21	49·44	53·33	56·96	60·37	11
12	6·928	21·17	29·12	35·33	40·60	45·25	49·48	53·37	56·99	60·40	12
13	7·211	21·26	29·19	35·38	40·64	45·30	49·52	53·40	57·03	60·43	13
14	7·483	21·35	29·26	35·44	40·69	45·34	49·56	53·44	57·06	60·46	14
15	7·746	21·45	29·33	35·50	40·74	45·39	49·60	53·48	57·10	60·50	15
16	8·000	21·54	29·39	35·55	40·79	45·43	49·64	53·52	57·13	60·53	16
17	8·246	21·63	29·46	35·61	40·84	45·48	49·68	53·55	57·17	60·56	17
18	8·485	21·73	29·53	35·67	40·89	45·52	49·72	53·59	57·20	60·60	18
19	8·718	21·82	29·60	35·72	40·94	45·56	49·76	53·63	57·24	60·63	19
20	8·944	21·91	29·66	35·78	40·99	45·61	49·80	53·67	57·27	60·66	20
21	9·165	22·00	29·73	35·83	41·04	45·65	49·84	53·70	57·31	60·70	21
22	9·381	22·09	29·80	35·89	41·09	45·69	49·88	53·74	57·34	60·73	22
23	9·592	22·18	29·87	35·94	41·13	45·74	49·92	53·78	57·38	60·76	23
24	9·798	22·27	29·93	36·00	41·18	45·78	49·96	53·81	57·41	60·79	24
25	10·000	22·36	30·00	36·06	41·23	45·83	50·00	53·85	57·45	60·83	25
26	10·198	22·45	30·07	36·11	41·28	45·87	50·04	53·89	57·48	60·86	26
27	10·392	22·54	30·13	36·17	41·33	45·91	50·08	53·93	57·52	60·89	27
28	10·583	22·63	30·20	36·22	41·38	45·96	50·12	53·96	57·55	60·93	28
29	10·770	22·72	30·27	36·28	41·42	46·00	50·16	54·00	57·58	60·96	29
30	10·954	22·80	30·33	36·33	41·47	46·04	50·20	54·04	57·62	60·99	30
31	11·136	22·89	30·40	36·39	41·52	46·09	50·24	54·07	57·65	61·02	31
32	11·314	22·98	30·46	36·44	41·57	46·13	50·28	54·11	57·69	61·06	32
33	11·489	23·07	30·53	36·50	41·62	46·17	50·32	54·15	57·72	61·09	33
34	11·662	23·15	30·59	36·55	41·67	46·22	50·36	54·18	57·76	61·12	34
35	11·832	23·24	30·66	36·61	41·71	46·26	50·40	54·22	57·79	61·16	35
36	12·000	23·32	30·72	36·66	41·76	46·30	50·44	54·26	57·83	61·19	36
37	12·166	23·41	30·79	36·72	41·81	46·35	50·48	54·30	57·86	61·22	37
38	12·329	23·49	30·85	36·77	41·86	46·39	50·52	54·33	57·90	61·25	38
39	12·490	23·58	30·92	36·82	41·90	46·43	50·56	54·37	57·93	61·29	39
40	12·649	23·66	30·98	36·88	41·95	46·48	50·60	54·41	57·97	61·32	40
41	12·806	23·75	31·05	36·93	42·00	46·52	50·64	54·44	58·00	61·35	41
42	12·961	23·83	31·11	36·99	42·05	46·56	50·68	54·48	58·03	61·38	42
43	13·115	23·92	31·18	37·04	42·10	46·60	50·71	54·52	58·07	61·42	43
44	13·266	24·00	31·24	37·09	42·14	46·65	50·75	54·55	58·10	61·45	44
45	13·416	24·08	31·30	37·15	42·19	46·69	50·79	54·59	58·14	61·48	45
46	13·565	24·17	31·37	37·20	42·24	46·73	50·83	54·63	58·17	61·51	46
47	13·711	24·25	31·43	37·26	42·28	46·78	50·87	54·66	58·21	61·55	47
48	13·856	24·33	31·50	37·31	42·33	46·82	50·91	54·70	58·24	61·58	48
49	14·000	24·41	31·56	37·36	42·38	46·86	50·95	54·74	58·28	61·61	49
50	14·142	24·49	31·62	37·42	42·43	46·90	50·99	54·77	58·31	61·64	50

LORD KELVIN'S STANDARD ELECTRIC BALANCES.

	0	100	200	300	400	500	600	700	800	900	
51	14·283	24·58	31·69	37·47	42·47	46·95	51·03	54·81	58·34	61·68	51
52	14·422	24·66	31·75	37·52	42·52	46·99	51·07	54·85	58·38	61·71	52
53	14·560	24·74	31·81	37·58	42·57	47·03	51·11	54·88	58·41	61·74	53
54	14·697	24·82	31·87	37·63	42·61	47·07	51·15	54·92	58·45	61·77	54
55	14·832	24·90	31·94	37·68	42·66	47·12	51·19	54·95	58·48	61·81	55
56	14·967	24·98	32·00	37·74	42·71	47·16	51·22	54·99	58·51	61·84	56
57	15·100	25·06	32·06	37·79	42·76	47·20	51·26	55·03	58·55	61·87	57
58	15·232	25·14	32·12	37·84	42·80	47·24	51·30	55·06	58·58	61·90	58
59	15·362	25·22	32·19	37·89	42·85	47·29	51·34	55·10	58·62	61·94	59
60	15·492	25·30	32·25	37·95	42·90	47·33	51·38	55·14	58·65	61·97	60
61	15·620	25·38	32·31	38·00	42·94	47·37	51·42	55·17	58·69	62·00	61
62	15·748	25·46	32·37	38·05	42·99	47·41	51·46	55·21	58·72	62·03	62
63	15·875	25·53	32·43	38·11	43·03	47·46	51·50	55·24	58·75	62·06	63
64	16·000	25·61	32·50	38·16	43·08	47·50	51·54	55·28	58·79	62·10	64
65	16 125	25·69	32·56	38·21	43·13	47·54	51·58	55·32	58·82	62·13	65
66	16·248	25·77	32·62	38·26	43·17	47·58	51·61	55·35	58·86	62·16	66
67	16·371	25·85	32·68	38·31	43·22	47·62	51·65	55·39	58·89	62·19	67
68	16·492	25·92	32·74	38·37	43·27	47·67	51·69	55·43	58·92	62·23	68
69	16·613	26·00	32·80	38·42	43·31	47·71	51·73	55·46	58·96	62·26	69
70	16·733	26·08	32·86	38·47	43·36	47·75	51·77	55·50	58·99	62·29	70
71	16·852	26·15	32·92	38·52	43·41	47·79	51·81	55·53	59·03	62·32	71
72	16·971	26·23	32·98	38·57	43·45	47·83	51·85	55·57	59·06	62·35	72
73	17·088	26·31	33·05	38·63	43·50	47·87	51·88	55·61	59·09	62·39	73
74	17·205	26·38	33·11	38·68	43·54	47·92	51·92	55·64	59·13	62·42	74
75	17·321	26·46	33·17	38·73	43·59	47·96	51·96	55·68	59·16	62·45	75
76	17·436	26·53	33·23	38·78	43·63	48·00	52·00	55·71	59·19	62·48	76
77	17·550	26·61	33·29	38·83	43·68	48·04	52·04	55·75	59·23	62·51	77
78	17·664	26·68	33·35	38·88	43·73	48·08	52·08	55·79	59·26	62·55	78
79	17·776	26·76	33·41	38·94	43·77	48·12	52·12	55·82	59·30	62·58	79
80	17·889	26·83	33·47	38·99	43·82	48·17	52·15	55·86	59·33	62·61	80
81	18·000	26·91	33·53	39·04	43·86	48·21	52·19	55·89	59·36	62·64	81
82	18·111	26·98	33·59	39·09	43·91	48·25	52·23	55·93	59·40	62·67	82
83	18·221	27·06	33·65	39·14	43·95	48·29	52·27	55·96	59·43	62·71	83
84	18·330	27·13	33·70	39·19	44·00	48·33	52·31	56·00	59·46	62·74	84
85	18·439	27·20	33·76	39·24	44·05	48·37	52·35	56·04	59·50	62·77	85
86	18·547	27·28	33·82	39·29	44·09	48·41	52·38	56·07	59·53	62·80	86
87	18·655	27·35	33·88	39·34	44·14	48·46	52·42	56·11	59·57	62·83	87
88	18·762	27·42	33·94	39·40	44·18	48·50	52·46	56·14	59·60	62·86	88
89	18·868	27·50	34·00	39·45	44·23	48·54	52·50	56·18	59·63	62·90	89
90	18·974	27·57	34·06	39·50	44·27	48·58	52·54	56·21	59·67	62·93	90
91	19·079	27·64	34·12	39·55	44·32	48·62	52·57	56·25	59·70	62·96	91
92	19·183	27·71	34·18	39·60	44·36	48·66	52·61	56·28	59·73	62·99	92
93	19·287	27·78	34·23	39·65	44·41	48·70	52·65	56·32	59·77	63·02	93
94	19·391	27·86	34·29	39·70	44·45	48·74	52·69	56·36	59·80	63·06	94
95	19·494	27·93	34·35	39·75	44·50	48·79	52·73	56·39	59·83	63·09	95
96	19·596	28·00	34·41	39·80	44·54	48·83	52·76	56·43	59·87	63·12	96
97	19·698	28·07	34·47	39·85	44·59	48·87	52·80	56·46	59·90	63·15	97
98	19·799	28·14	34·53	39·90	44·63	48·91	52·84	56·50	59·93	63·18	98
99	19·900	28·21	34·58	39·95	44·68	48·95	52·88	56·53	59·97	63·21	99
100	20·000	28·28	34·64	40·00	44·72	48·99	52·92	56·57	60·00	63·25	100

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THE END.

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